Date: 05.06.2014   Max. Marks: 40
Time: 3 hours   Weightage: 40%

Q1a). Find the l.c.m and gcd of 360, 75600 and 324000.
b). Magna Carta was signed on June 15, 1215 which day of the week did it happen?   [2+2]

Q2a). Find the number of positive integers in the range 500 through 2000 that are divisible by 3 or 7.
b). How many incongruent solutions does $87x \equiv 57 \pmod{105}$ have? Find them using Euclidean Algorithm.   [2+2]

Q3a). Find the last two digits of the number 7^{777}.
b). Find whether $2^{11} - 1$ is a prime number or not?   [2+2]

Q4a). Is {-3, 34, 8, 12,-1,-11}, modulus 6, a complete residue system? Justify your answer.
b) if a is an even integer, b is an odd integer then show
$\gcd(a, b) = \gcd(a/2, b)$ [1+3]

Q5.a) Find two values of n for which $\sigma(n) = 42$.
b) Show that for n=30 where $n = p_1 p_2 \ldots \ldots p_r$

$$\sum_{d|n} \mu(d)\phi(d) = (2 - p_1)(2 - p_2)\ldots\ldots(2 - p_r)$$ [2+2]

Q6 Does the quadratic congruence $x^2 \equiv 10 \pmod{13^3}$ have a solution? If yes solve it.   [1+3]

Q7a) Show $3^{2n} + 7$ is a multiple of 8.
b) Prove $(n^5 - 5n^3 + 4n)$ is divisible by 120.   [2+2]

Q8. Find if $28x + 91y = 119$ is solvable? if yes using continued fractions give the general Solution.   [4]

Q9. Three farmers equally divide the rice that they have grown .One goes to the market where a 9 pound weight is used. Another goes to a market which uses 11 pound weight and third to a market where 13 pound weight is used. Each sells as many full measures as possible and when the three return home ,first has 1 pound , second 2 pounds and third 6 pounds left. Find the total amount they took to the market   [4]

Q10. Mrs. Denver bought 10 pizzas, 15 cheeseburgers, and 20 sandwiches for a pool party. The total bill was $73.50.Find the general solution. [4]

BITS Pilani, Dubai Campus
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Fourth & Third Year – First Semester 2013 – 2014

MATH C231/ MATH F231 – Number Theory
Test 1 (Closed Book)

Date: 13.03.2014   **Max. Marks: 25**
Time: 50 Minutes   **Weightage: 25%**

Q1a. Find the quotient and remainder when -78 is divided by 11.
    b. Find the number of positive integer's ≤ 2059 and not divisible by 24.   [1+1]

Q2a. Prove if $a \mid b$ and $c \mid d$ then $ac \mid bd$.
    b. Prove if a is any arbitrary integer then a^3-a is divisible by 2.   [1+2]

Q3a. Find the number of leap years after 1900 and not exceeding 2000.
    b. Find the g.c.d and l.c.m of 300, 2160, 5040 using prime factorization.   [2+2]

Q4a. Find the g.c.d of (3k+2, 5k+3)
    b. Show g. c. d (a+ c b, b) = g. c. d (a, b)   [2+2]

Q5. A postal clerk has 14 and 21 cent stamps to sell. What combination of these may be
    used to mail a package requiring postage of $7.77?   [3]

Q6. Find whether 2,4,6,8,10,12,14,16,18,20,22 is a complete residue system mod 11
    or not?   [3]

Q7. Show that the following congruence has a solution and then find the complete set of
    mutually incongruent solution of
    $9x \equiv 12 (\mod 15)$ [3]

Q8. Solve the Diophantine equation 2x- 3y+4z=5   [3]

Name .... ID:..

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Second, Third & Fourth Year – Second Semester 2013 – 2014

MATH C231– Number Theory
Quiz 2(Closed Book)

Date: 11.04.2014   **Max. Marks: 7**
Time: 20 Minutes   **Weightage: 07%**

Q1.   Find the value of the Legendre Symbol $(\frac{16}{17})$.   [1]

Q2 Find the quotient if the continued fraction is given by $[1; 2, 3, 4, 5]$.   [1]

Q3.Does the quadratic congruence $x^2 \equiv 25 \pmod{32}$ has a solution ?   [1]

Q4.For the continued fraction [1;1, 1, 1, 1, 1, 1] find the fractions C_k for all values of k.   [2]

Q5. For what values of p and q will $\left(\dfrac{3}{p}\right) = \left(\dfrac{3}{q}\right)$, where p, q are odd primes [2]