Dubai International Academic City, Dubai
Fourth Year – Second Semester 2012 – 2013
MATH C231 – Number Theory
Comprehensive Examination

Date: 30.05.2013 **Max. Marks: 40**
Time: 3 hours **Weightage: 40%**

Q1 (a). Find the g.c.d of (1071, 462). Use the Euclidean Algorithm to obtain integers x and y satisfying, g.c.d of (1071, 462) = 1071x+462y
b). Find the g.c.d and l.c.m of (105,140,350) using prime factorization. [2+2]

Q2. (a)Verify that d (n) =d (n+1) =d (n+2) =d (n+3) for n= 3655.
b). Show the Fermat number F_3 is a prime. [2+2]

Q3 (a) Find the residue when 2^{117} is divided by 117
b) Show that (6a +1) and (6a -1) are relatively prime. [2+2]

Q4 a). Find consecutive integers for which $\mu(n)$ is zero.
b) For what value of n is -1 a quadratic residue (mod n) [2+2]

Q5.a) .Find the value of the Legendre symbol $(-\frac{242}{661})$.
b) If n= $p_1^{k_1} . p_2^{k_2} ..... p_r^{k_r}$ show that for n=20

$$\sum \frac{\mu^2(d)}{\phi(d)} = \frac{n}{\phi(n)}$$ [2+2]

Q6. Find the number which is a multiple of 11 that leaves remainder 1 when divided by 2,3,5,7. [3]

Q7. Does the following quadratic congruence have a solution? If yes solve it. [4]

$x^2 = 14 (\text{mod}\, 5^3)$

Q8. Solve 56x + 85y = 25 using continued fractions. Give the general Solution. [3]

Q9.Prove there are infinitely many primes of the form 6n+5. [3]

Q10.A student returning from Europe changes his euros and swiss francs into US money. If she receives $ 46.26 .For each euro she received $1.11 and 83 cents for each swiss franc .How much of each type of currency did she exchange? [3]

Q11(a). If gcd(a,b) = 1 , Prove gcd (a+b , a-b) = 1 or 2 [2+2]
b) If σ(n)=126 find n.

30/5/13

Marking Scheme
Comprehensive

Q1.

(a) $1071 = 2\times462 + 147$

$462 = 3\times147 + 21$

$147 = 7\times21 + 0$

g.c.d $= 21$ (1)

$21 = 462 - 3\times147$

$= 462 - 3\times(1071 - 2\times462)$

$= 462 - 3\times1071 + 6(462)$

$= 7\times462 - 3\times1071$

$x = -3, \quad y = 7$ (1)

(b) $105 = 5\times3\times7$

$140 = 2^2\times5\times7$

$350 = 2\times5^2\times7$

g.c.d $= 2^0\times3^0\times5\times7 = 35$ (1)

l.c.m $= 2^2\times3\times5^2\times7 = 2100$ (1)

2a) $3655 = 5\times17\times43$

$d(n) = d(3655) = d(5)d(17)d(43)$

$= 2\cdot2\cdot2 = 8$ (1/2)

$d(n+1) = d(3656) = d(2^3\cdot457) = (3+1)\cdot2 = 8$ (1/2)

$d(n+2) = d(3657) = d(3\times23\times53)$

$d(3)d(23)d(53) = 8$ (1/2)

$d(n+3) = d(3655+3) = d(3658) = d(2\times 31\times 59)$

$= d(2)\,d(31)\,d(59) = 2\times 2\times 2 = 8$

(b) $f_3 = 2^{2^3}+1 = 257$

$\sqrt{257} = 16.\ldots$ (1)

Primes before 16 are 2, 3, 5, 7, 11 and 13

257 is not divisible by any of the above (1)

so 257 is prime

5a) $2^7 = 128 \equiv 11 \pmod{117}$

$2^{117} = 2^{7\times 16+5} = (2^7)^{16}\cdot 2^5 \equiv 11^{16}\,2^5 = (11^2)^8(2^5)$ (1)

[As $11^2 \equiv 4 \pmod{117}$]

$2^{117} \equiv (4)^8\cdot 2^5 = (4^4)^2(2^5)$

$\equiv (22)^2\,2^5$

[As $4^4 \equiv 22 \pmod{117}$]

$= 11^2\,2^7 \equiv 2^9 \equiv 11\cdot 4 = \underline{44}$ (1)

3b)

$$6a+1 = (6a-1) + 2$$

$$6a-1 = 2(3a-1) + 1$$

$$2 = 2 \times 1 + 0$$ (1)

$$3a-1 =$$

$$g.c.d = 1$$ (1)

4a) $\mu(8)$ and $\mu(9)$

$\mu(8) = \mu(2^3)$ as $2^2 \nmid 2^3$ (1)

$\mu(9) = \mu(3^2)$ as $3^2 \nmid 3^2$ (1)

4b) $\left(\frac{-1}{p}\right) = (-1)^{\frac{p-1}{2}} = 1$ then for the congruence

$$x^2 \equiv -1 \pmod{p}$$ (1)

will have -1 as residue if

$$\left(\frac{-1}{p}\right) = 1$$ (1)

for $p = 5 = n$.

5a) $\left(\frac{-242}{661}\right) = \left(-\frac{2\times 11^2}{661}\right)$

$= \left(-\frac{1}{661}\right)\left(\frac{2}{661}\right)\left(\frac{11^2}{661}\right)$ 1

$= (-1)^{\frac{661-1}{2}} \cdot 2^{\frac{661^2-1}{8}} \cdot 1$ 1

$= 1$

b) $n = 20 = 2^2 \times 5$

$d = 1, 2, 4, 5, 10, 20$

$\sum \frac{\mu^2(d)}{\phi(d)} = \frac{\mu^2(1)}{\phi(1)} + \frac{\mu^2(2)}{\phi(2)} + \frac{\mu^2(4)}{\phi(4)} + \frac{\mu^2(5)}{\phi(5)}$

$+ \frac{\mu^2(10)}{\phi(10)} + \frac{\mu^2(20)}{\phi(20)}$ 1/2

$\mu(1) = 1$

$\mu(2) = (-1)$

$\mu(4) = 0$

$\mu(5) = -1$

$\mu(10) = \mu(2)\mu(5) = 1$

$\mu(20) = \mu(2^2 \times 5) = \mu(4) \cdot \mu(5) = 0$ 1

$\phi(1) = 1$

$\phi(2) = 1$

$\phi(4) = 2$

$\phi(5) = 4$

$\phi(10) = \phi(2)\phi(5) = 4$

$\phi(20) = 8$

$$\sum \frac{\mu^2(d)}{\phi(d)} = \frac{1^2}{1} + \frac{(-1)^2}{1} + 0 + \frac{(-1)^2}{4} + \frac{1}{4} + 0$$

$$= 1 + 1 + \frac{1}{4} + \frac{1}{4} = \frac{5}{2}$$

$$\frac{n}{\phi(n)} = \frac{20}{\phi(20)} = \frac{20}{8} = \frac{5}{2}$$ 1/2

L.H.S = RHS

Q6). $x \equiv 0 \pmod{11}$

$x \equiv 1 \pmod{2}$

$x \equiv 1 \pmod{3}$

$x \equiv 1 \pmod{5}$

$x \equiv 1 \pmod{7}$ 1/2

$c_1 = 0$, $c_2 = 1$, $c_3 = 1$, $c_4 = 1$, $c_5 = 1$

$n_1 = 210$, $n_2 = 1155$, $n_3 = 770$, $n_4 = 462$, $n_5 = 330$

$\bar{n}_1 = 1$, $\bar{n}_2 = 1$, $\bar{n}_3 = 2$, $\bar{n}_4 = 3$, $\bar{n}_5 = 1$ 1

$x_0 = c_1 n_1 \bar{n}_1 + c_2 n_2 \bar{n}_2 + c_3 n_3 \bar{n}_3 + c_4 n_4 \bar{n}_4 + c_5 n_5 \bar{n}_5$ 1

$= 0 + 1 \times 1155 \times 1 + 1 \times 770 \times 2 + 1 \times 462 \times 3 + 1 \times 330 \times 1$

$= 1155 + 1540 + 1386 + 330 = 4411$

$\equiv 2101 \pmod{2310}$

1/2

Q 4)

$$x^2 \equiv 14 \pmod{5^3}$$

$$\left(\frac{14}{125}\right) = \left(\frac{2\cdot 7}{125}\right) = \left(\frac{2}{125}\right)\left(\frac{7}{125}\right)$$

$$= \left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{2}{5}\right)\left(\frac{7}{5}\right)\left(\frac{7}{5}\right)\left(\frac{7}{5}\right)$$

$$\left(\frac{2}{5}\right) = 1 \qquad \left(\frac{7}{5}\right) = \left(\frac{2}{5}\right) = 1$$

so $\left(\frac{14}{125}\right) = 1$ soln exists 1

$$x^2 \equiv 14 \pmod{5^3}$$

I

$k = 1$

$$x^2 \equiv 14 \pmod 5$$

$$x_0 = 4$$

$$x_0^2 = a + b5^1 = 14 + b.5$$

$$b = 4$$

$$2x_0y_0 \equiv -b \pmod 5$$

$$2\cdot 4\cdot y_0 \equiv -4 \pmod 5$$

$$y_0 = 2$$

$$x_1 = 4 + 2.5 = 24 \ 14$$

1

II

x_0 is now 17.

$$x_0^2 = 14 + b \cdot 5^2$$

$$\frac{17^2 - 14}{25} = b$$ 1/2

$$b = 11$$

$$2 \cdot 17 \cdot y_0 \equiv -11 \pmod 5$$

$$34\, y_0 \equiv -11 \pmod 5$$

$$y_0 = 1$$ 1/2

$$x_2 = 17 + 1 \cdot 5^2 = 42$$ 1

Q8) $56x + 85y = 25$

$1 \div \frac{85}{56} = \gcd(85, 56) = 1$

$$\frac{56}{85} = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{13 + \frac{1}{2}}}}}$$ 1

$C_0 = 0$

$C_1 = 1$

$C_2 = \frac{1}{2}$

$C_3 = \frac{3}{5}$

$C_4 = \frac{27}{41}$

$C_5 = \frac{56}{85}$

$\frac{1}{2}$

$\frac{56}{85}$ and $\frac{27}{41}$

$56 \times 41 - 85 \times 27 = (-1)^{5-1} = 1$ $\frac{1}{2}$

$56(41 \times 25) + 85(-27 \times 25) = 25$

$x_0 = 1025$

$y_0 = -675$ $\frac{1}{2}$

$x = 1025 + 85t$

$y = -675 + 56t$ $t = 0, \pm 1, \pm 2$ $\frac{1}{2}$

Q9) Let there be finite number of prime of the form $6n+5$

Let $m = 6(2 \cdot 3 \cdot 5 \cdots p) - 1$

Also let $q = (2 \cdot 3 \cdot 5 \cdots p) - 1$

Then $m = 6q + 5$ (1)

Any prime except 2 and 3 is of the form $6n+1$ or $6n+5$

But product of two $6n+1$ type of numbers is again of the type $6n+1$. (1)

m will be either prime or divisible by a prime. If it's a prime we get one more prime of the form $6n+5$. If not then it is divisible by primes, m has at least one prime factor of the form $6n+5$ (1)

Otherwise ~~product~~ all factors if are $6n+1$ type the product will also be of $6n+1$ type. So let that prime be p $p_r | m$, $p_r | 6(2 \cdot 3 \cdot 5 \cdots p)$ so $p | 1$ contradiction

Q10). If no of Euros is e
of " " francs is f

Then

$$111e + 83f = 4626 \quad (1)$$

$$g.c.d\ (111, 83) = 1$$

Euclidean Algorithm gives

$$111(3) + 83(-4) = 1 \quad (\tfrac{1}{2})$$

So

$$111(13878) + 83(-18504) = 4626$$

$$x = 13878 + 83t \quad (\tfrac{1}{2})$$

$$y = -18504 - 111t$$

$$e = 17, \quad f = 33 \quad (1)$$

11 a). If $g.c.d(a,b) = 1$

Let $g.c.d(a+b, a-b) = d$.

$d \mid [(a+b \pm (a-b)] = d \mid 2b$ or $d \mid 2a$

Case 1. If a, b are odd

$a+b$ will be odd

$a-b$ will be odd

Hence d will have to be odd so

d cannot divide 2 and a

$d = 1$ (1)

Case 2. If both a and b are odd

$a+b$ will be even

$a-b$ will be even

$d \mid 2a$ and $d \mid 2b \Rightarrow d \mid 2$ (d even)

Let $d = 2e$

$2e \mid 2a \Rightarrow e \mid a$ and $e \mid b$ (1)

So $e = 1$ or 2.

b) $r = 68$ (2).

BITS Pilani, Dubai Campus
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Fourth Year – Second Semester 2012 – 2013

MATH C231 – Number Theory
Test 2 (Open Book)

Date: 12.05.2013 **Max. Marks: 20**
Time: 50 Minutes **Weightage: 20%**

Q1. For n= 206, show σ(n+1) =σ(n). [2]

Q2. Write [0; 1, 2, 3, 4, 3, 2] as a simple continued fraction and give the rational number. [2]

Q3.) What is the value of the Legendre symbol $(\frac{-72}{131})$ [2]

Q4. Find $\sum \mu(n)d(n)$ if n=15015 , where n= $p_1^{k_1} p_2^{k_2} .....p_r^{k_r}$ [2]

Q5. Show for an odd prime 7, any divisor of Mersenne Number M_7 is of the form 2kp+1. [2]

Q6. Knowing that 2 is the primitive root of 19, find all the quadratic residues of 19. [2]

Q7. Does the following Quadratic have a solution?
$(2x^2 + 5x - 9) \equiv 0 (\text{mod} 101)$. [2]

Q8. Prove that if n is even φ(3n) = 3φ(n) [3]

Q9. Solve the following equation using continued fractions
364x + 227y =1. [3]

dt 12/5/13

Marking Scheme
Number Theory
Test 2

Q1).

$n = 206$

$n+1 = 207$ (2)

$\sigma(206) = \sigma(2 \times 103) = \sigma(2) \times \sigma(103) = 3 \times 104$

$= 312$

$\sigma(207) = \sigma(3^2 \times 23) = \sigma(3^2)\,\sigma(23) = 13 \times 24$

$= 312$

Q2) $[0; 1, 2, 3, 4, 3, 2] = \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{2}}}}}}$ (2)

$= \dfrac{321}{460}$

Q3) $\left(\dfrac{-72}{131}\right) = \left(\dfrac{-1}{131}\right)\left(\dfrac{72}{131}\right) = \left(\dfrac{-1}{131}\right)\left(\dfrac{2^3}{131}\right)\left(\dfrac{3^2}{131}\right)$ (2)

$= \left(\dfrac{-1}{131}\right)\left(\dfrac{2}{131}\right)$

$= (-1) \cdot (-1)$

$= 1$

(4). $15015 = 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

$n = 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

$\mu(n) = (-1)^5 = -1$

$d(3) = 2 = d(5) = d(7) = d(11) = d(13)$

$\mu(3) = -1 = \mu(5) = \mu(7) = \mu(11) = \mu(13$

So $\sum \mu(n) d(n) = 2(-1)^5 \times 5 = -10$. (2)

(5)

$M_7 = 127$

127 is prime

$d = 127$

$127 = 2 \cdot 9 \cdot 7 + 1$ (2)

$n = 9$.

(6) 1, 4, 5, 6, 7, 9, 11, 16, 17 (2)

(7)

$$(2x^2+5x-9) \equiv 0 \pmod{101}$$

$$4\cdot 2(2x^2+5x-9) = (25+4\cdot 2\cdot 9)$$

$$y^2 \equiv 97 \pmod{101}$$

$$\left(\frac{97}{101}\right) = 1$$ so $f(x)$ exists (2)

(8) for $n = 3^k m$, $\gcd(3,m)=1$

for $k=0$, $\phi(3n) = 2\phi(n)$

for $k \geq 1$ $\phi(3n) = \phi(3^{k+1}m) =$

$$= (3^{k+1}-3^k)\phi(m)$$

$$= 3(3^k-3^{k-1})\phi(m)$$

$$= 3\phi(3^k m)$$

$$= 3\phi(n)$$

(3)

$\phi(3n) = 3\phi(n)$ iff $3 \mid n$

9).

$$364x + 227y = 1$$

$$\gcd(364, 227) = 1$$

3

$$x = 58 + 227t$$

$$y = -93 - 364t.$$

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Fourth Year – Second Semester 2012 – 2013

MATH C231 – Number Theory
Test 1 (Closed Book)

Date: 24.03.2013 **Max. Marks: 25**
Time: 50 Minutes **Weightage: 25%**

Q1. Use the Euclidean Algorithm to find the g.c.d (981,1234).Use the gcd to find the l.c.m (981,1234). [2]

Q2. Find the g.c.d and l.c.m of (280,330,405,490) using prime factorization. [2]

Q3. a)Find all positive integer m for which the following is true. [2]
$1000 \equiv 1 \pmod{m}$
b) Find out whether $3 \mid a(a+1)(a+2)$? Where a is some positive integer.

Q4. What time does a clock read 50 hours before it reads 6 o'clock? [2]

Q5. What is the inverse modulo 17 of 16? [2]

Q6. Find the g.c.d (a, a+2). [2]

Q7.Yen Kung a Chinese mathematician posed the following puzzle: We have an unknown number of coins. If you make 77 strings of them, you are 50 coins short, but if we make 78 strings, it is exact. How many coins are there? Solve Yen Kung's puzzle. [3]

Q8. Show that if n is an odd positive integer, then
$1+2+3+\ldots\ldots\ldots(n-1) \equiv 0 \pmod{n}$
Is the statement true when n is even? [3]

Q9. Find the least positive residue when 2^{200} is divisible by 47. [3]

Q10.Three children in a family have feet that are 5 inch, 7inch and 9inch long. When they measure the length of dining room of their house using their feet they each find that there are 3inch left over .How long is their dining room? [4]

dt 24/3/13

Marking Scheme
Number Theory
Test 1
4th year
2nd Semester 2012-13

Q1.

$1234 = 1 \cdot 981 + 253$

$981 = 3 \cdot 253 + 222$

$253 = 1 \cdot 222 + 31$ (1½)

$222 = 7 \cdot 31 + 5$

$31 = 6 \cdot 5 + 1$

$5 = 5 \cdot 1 + 0$

$g.c.d = 1$

$l.c.m = \frac{1234 \cdot 981}{1} = 1210554$ (½)

Q2.

$280 = 2^3 \cdot 5 \cdot 7$

$330 = 2 \cdot 3 \cdot 5 \cdot 11$ (1)

$405 = 3^4 \cdot 5$

$490 = 2 \cdot 5 \cdot 7^2$

$g.c.d = 2^0 \cdot 3^0 \cdot 5^1 \cdot 7^0 \cdot 11^0 = 5$

$l.c.m = 2^3 \cdot 3^4 \cdot 5^1 \cdot 7^2 \cdot 11^1 = 158760$ (1)

Q3. a) $1000 - 1 = 999$ (1)

for $m = 1, 3, 9, 27, 37, 111, 333, 999$

b) as $a + a+1 + a+2 = 3a+3$ if 3 divides $a, a+1, a+2$ it should divide the sum also, so as it divides $3a+3$ it divides $a \times (a+1)$ ~~and~~ $(a+3)$. (1)

Q4. $6 - 50 = -44 \equiv 4 \pmod{12}$ (1)

the 12 hour clock reads (1)

4 o'clock 50 hours before it reads 6 o'clock.

Q5. $1 = 16(-1) - 17(-1)$

$x_0 = -1$

$x = -1 + \frac{17}{1} t \quad = t = 0, \pm 1, \pm 2 \ldots$

inverse of $16 \pmod{17}$ is 16 (2)

Q6. g.c.d $(a, a+2)$

The common divisor of $a, a+2$ divides.

$(a+2) - a$ also

$a + 2 - a = 2$.

only divisor 1 and 2 (2)

If a is even g.c.divisor is 2

If a is odd g.c.d $(a, a+2) = 1$

Q7. Solve $77x + 27 = 78y$ (1)

$77x - 78y = -27$

$77(27) - 77(27) = -27$

$x_0 = 27, \quad y_0 = 27$ (1)

solutions

$x = 27 + 78t$

$y = 27 + 77t$

Number of coins are required. 8112, 14118 ... etc. (1)

Q8 $1+2+3+\cdots+n-1 = \frac{(n-1)n}{2}$

(I) If n is odd $(n-1)$ will be even so $n(n-1)$ will be divisible by 2 and. (1½)

$1+2+3+\cdots+(n-1) \equiv 0 \pmod{n}$

(II) If n is even let $n = 2k$

$1+2+3+\cdots+(n-1) = \frac{(n-1)n}{2} = k(n-1)$ (1½)

so $n \mid (n-1)k$ [as g.c.d $(n, n-1) = 1$ and $k<n$]

so $1+2+3+\cdots(n-1) \not\equiv 0 \pmod{n}$ if n is odd.

Q9. $2^{200} = (2^{128})(2^{64})(2^{8})$ (2)

$\equiv 14 \cdot 25 \cdot 21 \pmod{47}$

$\equiv 350 \cdot 21 \pmod{47}$

$\equiv 21 \cdot 21 \pmod{47}$

$\equiv 18 \pmod{47}$ (1)

Q10. $x \equiv 3 \pmod{5}$, $x \equiv 3 \pmod{7}$, $x \equiv 3 \pmod{9}$ (1)

$x = 3, 318, 633, 948$ (2)

3 inch not possible

318 is the ans (1)

26 ft 6 inches

Name :..... ID:.....

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Fourth Year – Second Semester 2012 – 2013

MATH C231– Number Theory
Quiz 1 (Closed Book)

Date: 17.04.2013 **Max. Marks: 7**
Time: 20 Minutes **Weightage: 07%**

Q1. Find the Least positive residue of 12! (mod13). [1]

Q2. Cristopher Columbus arrived in the New World on 12th October 1492. What day of the week was that? [1]

Q3. If $\Phi(n) \mid (n-1)$, Prove n is square free number. [1.5]

Q4. Find a positive n such that . [2]

a) $\sigma(n) = 84$
b) d(n)=100

Q5. Solve $42x \equiv 90 \pmod{156}$. How many incongruent solutions are there? list them. [1.5]

Name ID

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MATH C231– Number Theory
Quiz 1 (Closed Book)

Date: 27.02.2013 **Max. Marks: 8**
Time: 20 Minutes **Weightage: 08%**

Q1. Find the g.c.d of (1001,289) using Euclids division algorithm. [2]

Q2. Find the l.c.m of (105,140,350) using prime factorization. [2]

Q3. Show $8a+3$ and $5a+2$ are relatively prime. [2]

Q4. Find the solution of $20x+50y = 510$. [2]

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MATH C231– Number Theory
Quiz 1 (Closed Book)

Date: 27.02.2013 Max. Marks: 8
Time: 20 Minutes Weightage: 08%

Q1. Find the g.c.d of (1001,289) using Euclids division algorithm. [2]

$1001 = 289 \times 3 + 134$
$289 = 134 \times 2 + 21$
$134 = 21 \times 6 + 8$
$21 = 8 \times 2 + 5$
$8 = 5 \times 1 + 3$
$5 = 3 \times 1 + 2$
$3 = 2 \times 1 + 1$
$2 = 1 \times 2 + 0$

$g.c.d(1001, 289) = 1$

Q2. Find the l.c.m of (105,140,350) using prime factorization. [2]

$105 = 3 \times 5 \times 7$
$140 = 2^2 \times 5 \times 7$
$350 = 2 \times 5^2 \times 7$

$l.c.m = 2^2 \times 3 \times 5^2 \times 7 = 2100$

Q3. Show 8a+3 and 5a+2 are relatively prime. [2]

$$8a+3 = (5a+2)\times 1 + 3a+1$$
$$5a+2 = (3a+1)\times 1 + 2a+1$$
$$3a+1 = (2a+1)\times 1 + a$$
$$2a+1 = a\times 2 + 1$$
$$a = 1\times a + 0$$

Q4. Find the solution of 20x+50y = 510. [2]

$$20x + 50y = 510$$
$$g.c.d\ (20, 50) = 10$$
$$50 = 20\times 2 + 10$$
$$20 = 10\times 2 + 0$$

$$50 - 20\times 2 = 10$$

$$50\cdot(1) + 20(-2) = 10$$

$$50(51\times 1) + 20(-2\times 51) = 510$$
$$x_0 = -102$$
$$y_0 = 51$$

$$\begin{cases} x = -102 + \frac{50}{10}t \\ y = 51 - \frac{20}{10}t \end{cases}$$

$$x = -102 + 5t \qquad t = 0, \pm 1, \ldots$$
$$y = 51 - 2t$$

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MATH C231– Number Theory
Quiz 1 (Closed Book)

Date: 17.04.2013 **Max. Marks: 7**
Time: 20 Minutes **Weightage: 07%**

Q1. Find the Least positive residue of 12! (mod13). [1]

$1\cdot(2\cdot3\cdot4\cdot5\cdot6\cdot7\cdot8\cdot9\cdot10\cdot11)12 \equiv ?\ (\text{mod } 13)$

$\equiv 1\cdot 12 \equiv 12\ (\text{mod } 13)$

As
$2\cdot7 \equiv 1\ (\text{mod } 13)$
$3\cdot9 \equiv 1\ (\text{mod } 13)$
$6\cdot11 \equiv 1\ (\text{mod } 13)$
$4\cdot10 \equiv 1\ (\text{mod } 13)$
$5\cdot8 \equiv 1\ (\text{mod } 13)$

Q2. Cristopher Columbus arrived in the New World on 12th October 1492. What day of the week was that? [1]

yr code $= y + \left[\frac{y}{4}\right]$
$= 92 + \left[\frac{92}{4}\right] = 92+23 = 115 \equiv 3\ (\text{mod } 7)$

Month code $= 1$
date $= 12$
Correction: $18 - 14 = 4$

$3 + 1 + 12 + 4 \equiv 6\ (\text{mod } 7)$

It was a Friday.

Q3. If $\Phi(n) \mid (n-1)$, Prove n is square free ~~number~~. integer [1.5]

$n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_r^{\alpha_r}$, where $\alpha_1 \geq 2$

$p_1 \mid \phi(n)$ but $\phi(n) \mid n-1$ so $p_1 \mid \phi(n-1) \mid n-1$

$\Rightarrow p_1 \mid n-1$ Contradiction

So $\alpha_1 \neq 2$ (square free)

Q4. Find a positive n such that [2]

a) $\sigma(n) = 84$

b) d(n)=100

a) $n = 83$ or $n = 4 \cdot 11$ or $5 \cdot 13$

then $\sigma(83) = 83+1 = 84$

$\sigma(4 \cdot 11) = \sigma(4) \cdot \sigma(11) = 7 \cdot 12 = 84$

$\sigma(5 \cdot 13) = \sigma(5) \cdot \sigma(13) = 6 \cdot 14 = 84$

So $n = 83$ or 44 or 65

b). $n = ?$ $d(n) = 100$

$n = pq^{49}$, $n = p^3 q^{24}$ $n = p^4 q^{19}$, $n = p^9 q^9$ or $pq r^{24}$ etc

or $p^3 r^4 q^4$ or pqr^{24} or $pqr^4 s^4$

$n = 2^4 \cdot 3^4 \cdot 5 \cdot 7 = 45360$

Q5. Solve $42x \equiv 90 \pmod{156}$. How many incongruent [1.5]
solutions are there?

$d = \gcd(42, 156) = 6$

$7x \equiv 15 \pmod{26}$

replace 7 by 33

$33x \equiv 15 \pmod{26}$

$11x \equiv 5 \pmod{26}$

$-15x \equiv 5 \pmod{26}$

$-3x \equiv 1 \pmod{26}$

$\equiv 27 \pmod{26}$

$-x \equiv 9 \pmod{26}$

So $x \equiv -9 \pmod{26}$ soln

$\equiv 17$

6 solutions

17, 43, 69, 95, 121, 147