|Dubai International Academic City, Dubai
Fourth Year – Second Semester 2011 – 2012
MATH C231 – Number Theory
Comprehensive Examination

Date: 10.06.2012 **Max. Marks: 40**
Time: 3 hours **Weightage: 40%**

Q1.(a). Find the g.c.d of (963, 657). Use the Euclidean Algorithm to obtain integers x and y satisfying, g.c.d of (963, 657) = 963x+657y
b). Prove for every integer n that $n^3 - n$ is divisible by 6. [2+2]

Q2.(a). Write a reduced residue modulo system mod (30) and find $\phi(30)$ by factorizing 30 into prime factors.
b). Determine whether 1009 is a prime or not? [2+2]

Q3 (a) Find the remainder when 2^{68} is divided by 19.
b) Show that (5a +2) and (7a + 3) are relatively prime. [2+2]

Q4 a).For which prime p, $x^2 = 13 (\mathrm{mod}\, p)$ has a solution?

b).Find the value of the Legendre symbol $(-\frac{42}{61})$. [2+2]

Q5.a) Show $\sum_{d|36} (-1)^{\frac{36}{d}} \phi(d) = 0$

b) If n= $p_1^{k_1} \cdot p_2^{k_2} \ldots\ldots p_r^{k_r}$ show that for n=20

$\sum_{d|n} \mu(d)\sigma(d) = (-1)^r p_1 \cdot p_2 \ldots\ldots p_r$ [2+2]

Q6. Find all the integers that give remainder 1, 2, 3 when divided by 3, 4, 5 respectively. [4]

Q7. Does the following quadratic congruence have a solution? If yes solve it. [4]

$x^2 = -1 (\mathrm{mod}\, 5^3)$

Q8. Solve 18x + 5y = 24 using continued fractions. Give the general Solution. [4]

Q9.Prove that there are infinitely many primes of the form 6n+5. [4]

Q10.When Mr Smith cashed a check at his bank; the Teller mistook the number of cents for the number of dollars and vice versa. Unaware of this, Mr Smith spent 68 cents and then noticed to his surprise that he had twice the amount of the original check. Determine the smallest value for which the check was written? [4]

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MATH C231 – Number Theory
Test 2 (Open Book)

Date: 8.05.2012 **Max. Marks: 20**
Time: 50 Minutes **Weightage: 20%**

Q1. Is 221 a prime number? Justify your answer [1]

Q2. Prove that the only prime of the form $n^3 - 1$ is 7 [2]

Q3.Given that n is a positive integer such that $2^n - 1$ is a prime; show that n itself is a prime. [3]

Q4.Find the least positive integer n such that
$2^{44} \equiv n(\text{mod}\,89)$ [3]

Q5. Show that $\phi(n)\sigma(n)$ is a perfect square when n= 63457. [3]

Q6. Find the solution of the following congruence
$17x \equiv 9(\text{mod}\ 276)$. [4]

Q7.Find the value of the following for n=12,
$\sum_{d|n} \mu^2(d)/\phi(d)$
and [4]
$\sum_{d|n} \mu^2(d)/\sigma(d)$

BITS PILANI, DUBAI CAMPUS

DUBAI INTERNATIONAL ACADEMIC CITY
FOURTH YEAR SECOND SEMESTER 2011-2012

Test – 1 (Closed Book)

COURSE TITLE: NUMBER THEORY **COURSE CODE: MATH C231**
MAX MARKS: 25 **WEIGHTAGE: 25%**
DATE: 20/03/2012

Q1.Prove that if g.c.d (a, b) = d then a/d and b/d are relatively prime [2]

Q2.Use Euclidean algorithm to find the g.c.d of (2187, 999) and use it to find l.c.m of (2187,999) [2]

Q3. Is 1 , 5, 25, 125, 625, 3125 a reduced residue system mod 18?
Support your answer. [2]

Q4.Let the vertices of a triangle be O: (0, 0), B: (b, a) and C: (x, y) , show the area is $|(b\,y - a\,x)/2|$ [4]

Q5.Solve the Linear Diophantine equation 37x- 107y =25 and give the
General solution. [4]

Q6.If a, b, c, d are any integers ($c \neq 0$), will the following assertion hold? [4]

If $a \equiv b \pmod c$ and $b \equiv d \pmod c$ then $a \equiv d \pmod c$

Q7. Find such an x which satisfies the congruence $12x \equiv 9 \pmod 6$ if it exists? [3]

Q8. if m is a non-negative integer, then $\gcd(m \cdot a, m \cdot b) = m \cdot \gcd(a, b)$. [4]

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MATH C231 – Number Theory
Quiz 2 (Closed Book)

Date: 17.05.2012 **Max. Marks: 7**
Time: 20 Minutes **Weightage: 07%**

Q1. Write the simple continued fraction of $\frac{71}{55}$ [1]

Q2. Solve the Legendre symbol $(-\frac{72}{131})$ without using the Eulers criterion. [2]

Q3. Show that 3 is a quadratic residue of 23 but a non residue of 31. [2]

Q4. Is the congruence $x^2 \equiv 219 (\text{mod } 419)$ solvable? [2]

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MATH C231 – Number Theory
Quiz 1 (Closed Book)

Date: 13.03.2012
Time: 20 Minutes

Max. Marks: 8
Weightage: 08%

Q1. Show if a is an integer g.c.d $(2a+1,9a+4)=1$. [2]

Q2. Find d the g.c.d of $(121,66)$ using division algorithm. Find x and y such that $121x+66y = d$

Q3.Find the l.c.m of (1092, 1155, 2002) using prime power factorization. [2]

Q4. Find the general solution of $7x+18y=208$, if it exists. [2]