BITS, PILANI - DUBAI CAMPUS
Knowledge Village, Dubai

Semester II 2006 - 2007
IV Year (EEE/CS/EIE)

COMPREHENSIVE EXAMINATION (Closed Book)

Course No.: EA UC482 **Course Title: Fuzzy Logic & Applications**

Date: May 24, 2007 **Time: 3 Hours** **M.M. = 80 (40 %)**

NOTE:
(i) Answer all the questions.
(i) Answer all parts of a question in continuation.
(ii) Do not leave any blank page(s) in between the answers.

QUESTION 1 GIVE THE MOST APPROPRIATE ANSWER. (10 × 1 = 10)

(1) Let fuzzy sets $A = 0.1/1 + 0.3/3 + 0.6/5$ and $B = 0.3/2 + 0.6/4 + 1.0/6$; then the value of $A \cup B$ will be
(i) $0.1/1 + 0.3/2 + 0.3/3 + 0.6/4 + 0.6/5 + 1.0/6$
(ii) $0.1/1 + 0.3/3 + 0.6/5$
(iii) $0.3/2 + 0.6/4 + 1.0/6$
(iv) None of the above

(2) The basic difference between the probabilistic reliability theories and possibilistic reliability theories is
(i) Assumption of dichotomous states
(ii) Assumption of fuzzy states
(iii) Characterization of the behavior with respect to the two critical states.
(iv) All of the above

(3) If X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and $A = 0/1 + 0.1/2 + 0.2/3 + 0.5/4 + 0.3/5 + 0.1/6 + 0/7 + 0/8 + 0/9 + 0/10$; then the support of the fuzzy set A will be:
(i) {2, 3, 4, 5, 6}
(ii) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
(iii) {1, 7, 8, 9, 10}
(iv) None of the above

(4) Ordering of fuzzy numbers can be done using
   (i) Defining Hamming distance on the set of all fuzzy numbers
   (ii) Based on α-cuts
   (iii) Defining Euclidean distance on the set of all fuzzy numbers
   (iv) All of the above

(5) Let the universe of discourse be X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and $A = 0.3/1 + 0.5/2 + 1.0/3 + 0.7/4 + 0.2/5$; then the value of $\|A\|$ will be:
   (i) 0.27
   (ii) 10
   (iii) 2.7
   (iv) None of the above

(6) Give the proof of law of absorption $p \wedge (p \vee q) = p$.

(7) Interval-valued fuzzy sets can further be generalized by allowing their intervals to be fuzzy, the sets so obtained are referred to as ....

(8) Let fuzzy sets A = 0.5/1 + 0.4/2 + 0.2/3; then find NORM (A).

(9) Give the truth table for the $p \Leftrightarrow q$ (Double Implication) operation.

(10) Lukasiewicz used only negation and implication as primitives in his n-valued logic. Write down his definition of other fuzzy operations i.e. $\vee$, $\wedge$, and $\Leftrightarrow$ in terms of negation and implication.

<u>QUESTION 2</u> **(5 × 2 = 10)**

(a) Which are the two laws of crisp set theory that are violated in fuzzy set theory? Justify your answers.

(b) Find Hamming distance between the two fuzzy sets given by $A = \frac{1}{3}\Big/x_1 + \frac{2}{3}\Big/x_2$ and $B = \frac{3}{4}\Big/x_1 + \frac{1}{2}\Big/x_2$.

(c) If R is a binary relation which gives a correspondence from A to A then define the following properties and give one example of each.

   (i) Reflexive   (ii) Symmetric

(d) Compute the simple difference $A - B$, if

$$A = 0.2/x_1 + 0.7/x_2 + 1.0/x_3 + 0/x_4$$
$$B = 0.5/x_1 + 0.3/x_2 + 1.0/x_3 + 0.1/x_4$$

(e) Obtain the $R \underset{av}{\circ} S$ composition of fuzzy relations R and S, if

$$R = \begin{bmatrix} 0.4 & 0.5 & 0 \\ 0.2 & 0.8 & 0.2 \end{bmatrix} \text{ and } S = \begin{bmatrix} 0.2 & 0.7 \\ 0.3 & 0.8 \\ 1 & 0 \end{bmatrix}$$

QUESTION 3 **(5 + 5 = 10)**

(a) What is the special significance of standard fuzzy operations among the various fuzzy complements, intersections and unions?

(b) Define the 'Yager' class of decreasing and increasing generators and write down their pseudo inverses. Use the characterization theorems of t-norms and t-conorms respectively to obtain the corresponding class of fuzzy intersections and unions.

QUESTION 4 **(3 + 7 = 10)**

(a) Sally is nearsighted and colorblind. When she goes to a local grocery where fruits are placed on high shelves, she cannot see them very well. She can only recognize the size and blurred shape of the fruits. Her knowledge about the fruits can be represented by the following fuzzy relation.

$$\begin{array}{c} \\ long \\ round \\ large \end{array} \begin{array}{c} \begin{array}{ccccc} tangerine & apple & pineapple & watermelon & strawberry \end{array} \\ \begin{bmatrix} 0 & 0 & 0.3 & 0 & 0.8 \\ 0.9 & 1.0 & 0.3 & 1.0 & 0.2 \\ 0.2 & 0.4 & 0.7 & 1.0 & 0.1 \end{bmatrix} \end{array}$$

Guess a fruit that Sally describes as $\begin{array}{ccc} long & round & large \\ [0 & 0.7 & 1.0] \end{array}$.

(b) A small data set X consists of the following five points:

k	1	2	3	4	5
x_{k1}	0	1	2	3	4
x_{k2}	0	1	3	1	0

Let a fuzzy compatibility relation R, on X be defined in terms of an appropriate distance function of the Minkowski class by the formula:

$$R(x_i, \; x_k) = 1 - \delta\left(\sum_{j=1}^{p} |x_{ij} - x_{kj}|^q\right)^{\frac{1}{q}}$$

for all pairs $\langle x_i, \; x_k \rangle \in X$ and δ is the inverse value of the largest distance in X. Use the Euclidean distance (q = 2) to perform the partitioning of X using its α-cuts.

QUESTION 5 (10)

Define 'Domain', 'Range', and 'Height' of a fuzzy relation. Suppose the universe X and Y are given as $X = \{x_1, \ x_2, \ x_3\}$ and $Y = \{y_1, \ y_2, \ y_3\}$. The fuzzy relation R on X × Y is given as follows:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.6 & 1 & 0.3 \\ 0.5 & 0.2 & 0.8 \\ 0.1 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain domain, range and height of R.

QUESTION 6 (5 + 5 = 10)

(a) The fuzzy relation R is defined on sets U = {&, *}, V = {x, y}, and W = {a, b, c} as follows:

R = 0.9/(a, x, &) + 0.4/(b, x, &) + 1/(a, y, &) + 0.7/(a, y, *) + 0.8/(b, y, *)

Obtain the Cylindrical Extensions $cy\langle R_{12}, \ R_{13}, \ R_{23}\rangle$ and $cy\langle R_{23}, \ R_2\rangle$.

(b) Suppose the universe X and Y are given as $X = \{x_1, \ x_2, \ x_3\}$ and $Y = \{y_1, \ y_2, \ y_3\}$. If fuzzy relations R and S on $X \times Y$ are given as follows, then prove that the De Morgan's laws hold good.

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1.0 & 0.4 & 0.6 \\ 0.5 & 0.8 & 0.7 \\ 0.9 & 0.6 & 1.0 \end{bmatrix} \end{array} \quad \text{and} \quad S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.3 & 0.7 & 0.9 \\ 0.6 & 1.0 & 0.4 \\ 0.2 & 0.9 & 1.0 \end{bmatrix} \end{array}$$

QUESTION 7 (5 + 5 = 10)

Write down the comprehensive notes on the relevance of fuzzy logic to any two of the following:

(i) Civil Engineering
(ii) Industrial Engineering
(iii) Robotics

QUESTION 8 (10)

Fuzzy Numbers 3 (FN3), Fuzzy Numbers 6 (FN6), and Fuzzy Numbers 7 (FN7) are defined as FN3 = 0.3/1 + 0.7/2 + 1.0/3 + 0.7/4 + 0.3/5, FN6 = 0.2/4 + 0.6/5 + 1.0/6 + 0.6/7 + 0.2/8 and FN7 = 0.2/5 + 0.6/6 + 1.0/7 + 0.6/8 + 0.2/9 respectively on an universe of discourse X = 1 + 2 + ... + 60. Obtain

(i) FN3 + FN7, (ii) FN3 - FN7, (iii) FN7× FN3 and (iv) FN6 / FN3

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BITS, PILANI - DUBAI CAMPUS

Knowledge Village, Dubai

IV Year (EEE/CS/EIE)

Semester II 2006 - 2007

TEST II (Open Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: 10^{th} May 2007 Time: 50 Minutes M.M. = 40 (20 %)

NOTE: Text Book, Reference Books, as well as Class Notes can be used for answering.

Question 1 **[8]**

There is a fuzzy rule in the following:

x and y are approximately equal

This rule can be represented by following relation R(x, y).

x \ y	1	2	3	4
1	1	0.5	0	0
2	0.5	1	0.5	0
3	0	0.5	1	0.5
4	0	0	0.5	1

Assume that the variables x and y are positive integers in $[1, \ 4]$. If one of the variable is given as a singleton, x = 2, then apply the modus ponens type of reasoning using standard max-min composition to infer the value of y.

Question 2 **[7]**

In BPDC, one student among the graduating students is awarded an 'All Rounder Student' trophy on the basis of his/her performance in academics, cultural activities and sports. This year there are five candidates for this trophy; there relative performance in the three categories is described with the help of following fuzzy relation matrix:

$$\begin{array}{c} \\ Academics \\ Cultural \\ Sports \end{array} \begin{array}{c} \begin{array}{ccccc} Malpani & Vijetha & Amol & Gayatri & Kavitha \end{array} \\ \begin{bmatrix} 0.3 & 0.7 & 0.5 & 0.4 & 0.4 \\ 0.6 & 0.4 & 0.7 & 0.8 & 0.6 \\ 0.2 & 0.2 & 0.4 & 0.2 & 0.2 \end{bmatrix} \end{array}$$

The weightage given to each of these three category in selecting the winner is described as $\begin{matrix} Academics & Cultural & Sports \\ [0.4 & 0.3 & 0.3] \end{matrix}$, guess the winner of the 'All rounder Student' trophy for this year.

Question 3 **[5]**

Draw the correct conclusion using the following premises.

1. No student that studies hard is having low CGPA.
2. No student not residing in hostel will play in hostel team.
3. Students who are good players always study hard
4. Good CGPA students are awarded scholarships.
5. No student resides in hostel unless he is a good player.

Question 4 **[10]**

Suppose temperature T_1 can have five values, 11, 12, 13, 14 15, and temperature T_2 can have three values, 10, 12, 14. Obtain

(i) Membership-function for the statement "Temperature T_1 is about Temperature T_2 **and** Temperature T_1 seems to be larger than Temperature T_2".

(ii) Projections of relation "Temperature T_1 is about Temperature T_2 **and** Temperature T_1 seems to be larger than Temperature T_2" on T_1 and on T_2.

(iii) Cylindrical Extensions of two projections obtained in part (ii).

(iv) Join and Meet of the Cylindrical Extensions obtained in (iii).

Question 5 **[3 + 7 = 10]**

(a) What do you mean by 'Fuzzy Numbers'? List out their characteristics.

(b) How many GUI tools are available in Fuzzy Logic Toolbox of MATLAB for building, editing, and observing any fuzzy inference system? Describe their functions briefly.

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Name:

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BITS, PILANI - DUBAI CAMPUS

Knowledge Village, Dubai

IV Year (EEE/CS/EIE)

Semester II 2006 - 2007

QUIZ I (Closed Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

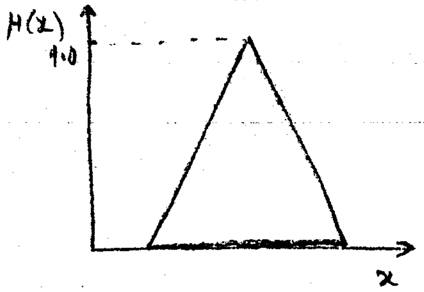
Date: April 24, 2007 Time: 30 Minutes M.M. = 20 (10 %)

QUESTION NO 1: [10 × 1 = 10]

(I) For fuzzy relations P(X, Y) and Q(Y, Z), the standard max-min join $S = P * Q$ is given as follows, convert this into the corresponding composition $R = P \circ Q$.

x	y	z	$\mu_S(x, y, z)$
1	a	α	0.6
1	a	β	0.7
1	b	β	0.5
2	a	α	0.6
2	a	β	0.8
3	b	β	1.0
4	b	β	0.4
4	c	β	0.3

(II) Show ${}^{0.4}A$ and ${}_{0.4}A$ in the following diagram representing Fuzzy set A.



(III) Obtain the Fuzzy Graph of the relational matrix

$$\begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccc} a_1 & a_2 & a_3 \end{array} \\ \begin{bmatrix} .2 & .5 & .4 \\ 0 & 1 & .3 \\ 0 & 1 & .7 \end{bmatrix} \end{array}.$$

(IV) Obtain the negation of circuit C drawn in Figure 1.

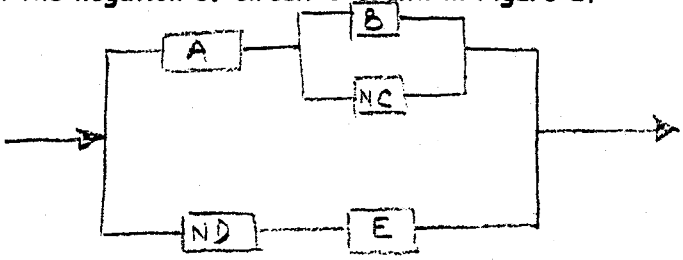


Figure 1

(V) If R(X, Y) represents a fuzzy relation then define its domain and range.

(VI) Identify the following tautologies used in propositional logic:

(i) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$

(ii) $(\overline{Q} \wedge (P \Rightarrow Q)) \Rightarrow \overline{P}$

(iii) $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$

(VII) Give the mathematical definition of 'Drastic intersection' operation.

(VIII) Let A_1 = 0.7/2 + 1/4 + 0.7/5 and A_2 = 0.3/2 + 1/3 + 0.4/5. Suppose ω_1 = 0.8 and ω_2 = 0.2, then what will be the convex combination of A_1 and A_2?

(IX) A fuzzy set can be visualized as a point in an n-dimensional cube in n-space. Inside this cube the 'Cardinality' of any fuzzy set is defined as ....

(X) Show graphically the approximate shape of the resultant Fuzzy set when 'Contrast Intensification' operation is performed on a triangular shaped Fuzzy set.

QUESTION NO 2: Choose the most appropriate one. **[10 × 1 = 10]**

(I) **Which one of the following is not an essential condition for a fuzzy union operation?**

(i) $b \le d$ implies $u(a, \ b) \le u[a, \ d]$

(ii) $u(a, \ b) = u(b, \ a)$

(iii) $u(a, \ a) > a$

(iv) $u(a, \ u(b, \ d)) = u(u(a, \ b), \ d)$

(II) **Classical (standard) fuzzy complement is one which**

(i) satisfies the axiomatic skeleton for fuzzy complements.

(ii) is continuous fuzzy complement.

(iii) is involutive fuzzy complement.

(iv) satisfies all the above conditions

(III) **Relate the Items from list-A to list-B:**

List-A	List-B
(i) Pre-order Fuzzy Relation	(a) Reflexive and Symmetric
(ii) Similarity Fuzzy Relation	(b) Reflexive and max-min transitive
(iii) Resemblance Fuzzy Relation	(c) Reflexive, Symmetric, and max-min transitive

(IV) **The standard fuzzy intersection is the only**

(i) Sub idempotent t-norm.

(ii) Idempotent t-norm.

(iii) Super idempotent t-norm.

(iv) All of the above

(V) **Which one of the following is a correct relation?**

(i) $u_{\max}(a, \ b) \ge \min(1, \ a+b) \ge a+b-ab \ge \max(a, \ b)$

(ii) $\max(a, \ b) \le \min(1, \ a+b) \le a+b-ab \le u_{\max}(a, \ b)$

(iii) $u_{\max}(a, \ b) \le \min(1, \ a+b) \le a+b-ab \le \max(a, \ b)$

(iv) $a+b-ab \le \min(1, \ a+b) \le u_{\max}(a, \ b) \le \max(a, \ b)$

(VI) **The operation needed to be performed on the special fuzzy sets defined by $_{\alpha}A(x) = \alpha \cdot {}^{\alpha}A(x)$, in order to obtain the original fuzzy set A is**

(i) Standard Fuzzy Union

(ii) Standard Fuzzy intersection

(iii) Bounded Sum operation

(iv) Max-min composition

(VII) Which one of the following statements is incorrect?

(i) i_{min} and u_{max} operations are dual of each other with respect to any fuzzy complement c.

(ii) min and max operations are dual of each other with respect to only standard fuzzy complement.

(iii) Given an involutive fuzzy complement c and an increasing generator g of c, the t-norm and t-conorm generated by g are dual with respect to c.

(iv) Let $\langle i, \ u, \ c \rangle$ be a dual triple that satisfy the law of excluded middle and the law of contradiction, then $\langle i, \ u, \ c \rangle$ does not satisfy the distributive laws.

(VIII) Which of the following properties does not apply to the standard Fuzzy complement, intersection and union operations?

(i) Satisfy the cutworthy and strong cutworthy properties.

(ii) Prevent the compounding of errors of the operands.

(iii) Context-dependent.

(iv) All of the above properties apply to standard fuzzy operations.

(IX) The equilibrium of a complement c is that degree of membership in a fuzzy set A which equals the degree of membership

(i) in the complement of A

(ii) in the complement of P(A)

(iii) in the Complement of universe of discourse.

(iv) None of the above

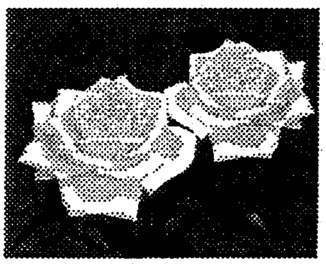
(X) 'Sugeno' class of fuzzy complements is defined by

(i) $c_\lambda(a) = \dfrac{1-a}{1+\lambda a}$, where $\lambda \in (-1, \infty)$

(ii) $c(a) = \dfrac{1}{2}(1 + \cos \pi a)$,

(iii) $c(a) = \begin{cases} 1 & for \quad a \le t \\ 0 & for \quad a > t \end{cases}$

(iv) All of the above



BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

IV Year (EEE/EIE/CS)

Semester II   2006 – 2007

TEST I (Closed Book)

Course No.: EA UC482   **Course Title:** Fuzzy Logic & Applications

Date: March 25, 2007   **Time:** 50 Minutes   **M.M.** : 40 (20 %)

Question 1   **[1 × 5 = 05]**

(a) In a class of 10 students (the universal set), 3 students speaks German to some degree, namely Alice to degree 0.7, Bob to degree 1.0, Cathrine to degree 0.4. What is the size of the subset A of German speaking students in the class?

(b) What is the support of the fuzzy set A in part (a)?

(c) Is A normal?

(d) What is ${}^{0.5}A$ (the 0.5-cut of the fuzzy set A)?

(e) What is ${}^{0.4+}A$ (the strong 0.4-cut of the fuzzy set A)?

Question 2   **[1 × 5 = 05]**

Let $X = \{0, \ 1, \ 2, \ \ldots, \ 6\}$, and let two fuzzy subsets, A and B, of X be defined by:

X	0	1	2	3	4	5	6
$\mu_A(x)$	1	0.7	0	1	0.5	0	0.4
$\mu_B(x)$	0.9	0.7	1	0.2	0.8	0.3	0

Find

(a) $\overline{B}$

(b) $\overline{A \cup B}$

(c) $\overline{A} \cap \overline{B}$

(d) What is the Core of A?

(e) What is the algebraic sum of fuzzy sets A and B?

Question 3 **[5 + 5 = 10]**

(a) Describe a 'Type-n Fuzzy Set' and a 'Level-k Fuzzy Set'. Give graphical representation of one example of each.

(b) Let A and B be fuzzy subsets of a universal set X. Show that:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

Question 4 **[5 + 5 = 10]**

(a) How many parameters are needed to describe following types of membership functions? Write down their formulae and give the graphical representations also.

(i) Triangular

(ii) Trapezoidal

(b) Suppose we have a universe of integers, Y = {1, 2, 3, 4, 5}. We define the following terms as a mapping onto Y:

"Small" = $\left\{\frac{1}{1} + \frac{0.8}{2} + \frac{0.6}{3} + \frac{0.4}{4} + \frac{0.2}{5}\right\}$

"Large" = $\left\{\frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5}\right\}$

Then obtain the fuzzy set representation of:

(i) "not very small and not very, very large" and (ii) "Intensely small"

Question 5 **[5 + 5 = 10]**

(a) Find Hamming distance and Euclidian distance between the two fuzzy sets given by $A = \frac{1}{3}\Big/x_1 + \frac{2}{3}\Big/x_2$ and $B = \frac{3}{4}\Big/x_1 + \frac{1}{2}\Big/x_2$.

(b) Let the universe of discourse be X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = 0.3/1 + 0.5/2 + 1/3 + 0.7/4 + 0.2/5, then obtain (i) Scalar Cardinality of A, (ii) Relative Cardinality of A, and (iii) Fuzzy Cardinality of A.