BITS, PILANI - DUBAI CAMPUS
Knowledge Village, Dubai

Semester II 2005 - 2006
IV Year (EEE/EIE/CS)

COMPREHENSIVE EXAMINATION (Closed Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: May 30, 2006 Time: 3 Hours M.M. = 80 (40 %)

Question 1 [10]

(a) The equilibrium of a complement c is that degree of membership in a fuzzy set A which equals the degree of membership
- (i) in the complement of A
- (ii) in the complement of P(A)
- (iii) in the Complement of universe of discourse.
- (iv) None of the above

(b) Classical (standard) fuzzy complement is one which
- (i) satisfies the axiomatic skeleton for fuzzy complements.
- (ii) is continuous fuzzy complement.
- (iii) is involutive fuzzy complement.
- (iv) satisfies all the above conditions

(c) In principle, fuzzy computer hardware allows all inference rules of a complex fuzzy inference engine to be processed
- (i) in parallel
- (ii) as per their associated weights
- (iii) in sequence
- (iv) All of the above

(d) The operation needed to be performed on the special fuzzy sets defined by ${}_{\alpha}A(x)=\alpha\cdot{}^{\alpha}A(x)$, in order to obtain the original fuzzy set A is
- (i) Standard Fuzzy Union
- (ii) Standard Fuzzy intersection
- (iii) Bounded Sum operation
- (iv) Max-min composition

(e) The standard fuzzy intersection is the only
   (i) Sub idempotent t-norm.
   (ii) Idempotent t-norm.
   (iii) Super idempotent t-norm.
   (iv) All of the above

(f) Obtain the max-product composition of sets $A=[.5 \quad 1 \quad .6]$ and $B=\begin{bmatrix} .02 & .2 & .16 \\ .04 & .4 & .32 \\ .1 & 1 & .8 \end{bmatrix}$.

(g) What is the formula for removing the implication operation from a proposition?

(h) Identify the tautology and contradiction in $(P \vee \overline{P})$ and $(P \wedge \overline{P})$.

(i) Give the mathematical definition of "Intensification" operation performed on linguistic fuzzy sets.

(j) Write down the truth tables for primitives $\wedge$, $\vee$, $\Rightarrow$, and $\Leftrightarrow$ used in Lukasiewicz 3-valued logic.

Question 2 [5×2=10]

(a) Let fuzzy sets A and B are given as $A=\frac{0.3}{x_1}+\frac{0.4}{x_2}+\frac{0.6}{x_3}+\frac{0.7}{x_4}$ and $B=\frac{0.4}{x_1}+\frac{0.6}{x_2}+\frac{0.8}{x_3}+\frac{0.8}{x_4}$. Determine:

   (i) A × B

   (ii) Height of A and B

   (iii) CON (A + B)

(b) Let fuzzy sets $A=\frac{0.2}{1}+\frac{0.4}{3}+\frac{0.6}{5}$ and $B=\frac{0.1}{1}+\frac{0.3}{3}+\frac{0.5}{5}$ then find the following

   (i) A Θ B (Bounded Difference)

   (ii) A ⊕ B (Bounded Sum)

Question 3 [2×5=10]

(a) Suppose we have a universe of integers, Y = {1, 2, 3, 4, 5}. We define the following terms as a mapping onto Y:

$$\text{"Small"} = \left\{\frac{1}{1}+\frac{0.8}{2}+\frac{0.6}{3}+\frac{0.4}{4}+\frac{0.2}{5}\right\}$$

$$\text{"Large"} = \left\{\frac{0.2}{1}+\frac{0.4}{2}+\frac{0.6}{3}+\frac{0.8}{4}+\frac{1}{5}\right\}$$

Then obtain the fuzzy set representation of:

(i) "not very small and not very, very large" and

(ii) "Intensely small"

(b) In BPDC, one student among the graduating students is awarded an 'All rounder Student' trophy on the basis of his/her performance in academics, cultural activities and sports. This year there are five candidates for this trophy, there relative performance in the three categories is described with the help of following fuzzy relation matrix:

	Bokka	*Astrid*	*Meet*	*Ankit*	*Nashrah*
Academics	0.3	0.7	0.5	0.4	0.4
Cultural	0.6	0.4	0.7	0.8	0.6
Sports	0.2	0.2	0.4	0.2	0.2

The weightage given to each of these three category in selecting the winner is described as $\begin{matrix} Academics & Cultural & Sports \end{matrix}$ $[0.4 \quad 0.3 \quad 0.3]$, guess the winner of the 'All rounder Student' trophy for this year.

Question 4 [10]

Let A and B be two universe representing numeric ratings (from 1 to 10) of quality of service given in a restaurant and amount of tip offered respectively. Let quality of service be a fuzzy subset of A defined as

Quality of service is good = .25/6 + 0.5/7 + 0.75/8 + 1/9 + 1/10

and amount of tip be a fuzzy subset of B defined as

Amount of tip is high = 0.25/7 + 0.5/8 + 0.75/9 + 1/10

We are also given the fuzzy implication:

Quality of service is good $\rightarrow$ Amount of tip is high

Using standard sequence implication, defined as to be true whenever the consequent is truer than the antecedent, i.e., $t(p \rightarrow q) = 1$ whenever $t(p) < t(q)$, generate the fuzzy implication relation matrix.

Suppose the quality of service is very good, apply the max-min composition to the standard sequence implication relation to guess about the amount of tip to be offered.

Question 5 [10]

Show the inter-relation among the basic components of a general pattern recognition system with the help of a diagram and explain them in brief. What is the relevance of fuzzy set theory in pattern recognition field?

Question 6 [2×5=10]

(a) Define 'Domain', 'Range', and 'Height' of a fuzzy relation. Suppose the universe X and Y are given as $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$. The fuzzy relation R on X × Y is given as follows:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.6 & 1 & 0.3 \\ 0.5 & 0.2 & 0.8 \\ 0.1 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain domain, range and height of R.

(b) Name and give mathematical definition of any two aggregation operations that cover the entire range between the largest intersection and the smallest union.

Question 7

[2×5=10]

Write down the comprehensive notes on the relevance of fuzzy logic in (any) two of the following:

(i) Civil engineering

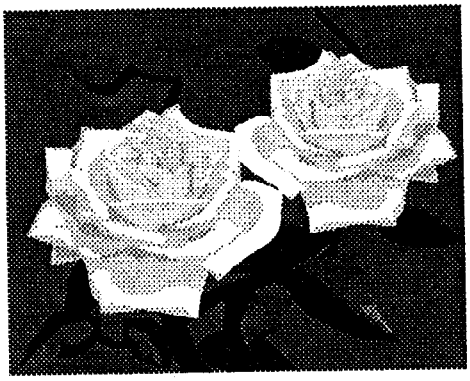
(ii) Databases and Information Retrieval Systems

(iii) Reliability Theory

Question 8

[10]

Consider the case of a subway train approaching a station. Describe a fuzzy logic based control strategy to obtain the amount of brakes power (in terms of linguistic values Very_Heavy, Heavy, Light, Very_Light) used to halt the train if the linguistic values of distance from the station (Very_Close, Close, Far, and Very_Far), and the speed of the train (Very_Slow, Slow, Fast, Very_Fast) are known. Define these linguistic values in the range of 0 - 500 meters (distance); 0 - 100 km/h (speed); and 0 - 100 % (of maximum brake power). What will be the control action if the distance is 100m and the speed is 24.6 km/h?



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Knowledge Village, Dubai

IV Year (EEE/EIE/CS)

Semester II 2005 - 2006

TEST II (Open Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: 14th May 2006 Time: 50 Minutes M.M. = 40 (20 %)

NOTE: Text Book, Reference Books, as well as Class Notes can be used for answering.

Question 1 [05]

Let fuzzy set A be "the set of people with an infectious disease" and the crisp set B be "the set of people having been in contact with the infected people". The contact relation is given by R in Figure (1).

$A = \{(a_1, 0.4), (a_2, 0.5), (a_3, 0.9), (a_4, 0.6)\}$; $B = \{b_1, b_2, b_3\}$

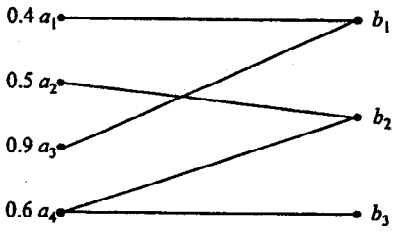

Figure (1)

Obtain the infectious set B' in B.

Question 2 [05]

What do you mean by 'Fuzzy Numbers'? List out their characteristics.

Question 3 [05]

Define the 'Yager' class of increasing generators and write down its pseudo inverse. Use the characterization theorem of t-conorms to obtain the corresponding class of fuzzy unions.

Question 4 **[05]**

HYPOTHESES: An Englishman, a Frenchman, and an Italian interviewed for a position as head of a United Nations agency. If the Italian got the job the agency would serve pasta for every meal. If the Frenchman got the job the agency would be moved to Paris. One of the three people must be hired. The Englishman is not hired and the headquarters did not move to Paris.

CONCLUSION: Pasta is served at every meal.

Using the above mentioned hypotheses, establish the veracity of the given conclusion.

Question 5 **[05]**

What are the criteria for choosing a combining operator in a Fuzzy System?

Question 6 **[05]**

Define 'Domain', 'Range', and 'Height' of a fuzzy relation. Suppose the universe X and Y are given as $X=\{x_1,\ x_2,\ x_3\}$ and $Y=\{y_1,\ y_2,\ y_3\}$. The fuzzy relation R on X × Y is given as follows:

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.6 & 1 & 0.3 \\ 0.5 & 0.2 & 0.8 \\ 0.1 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain domain, range and height of R.

Question 7 **[10]**

For three given intervals, $S_1=[-1,\ 0]$, $S_2=[-1,\ 2]$, and $S_3=[2,\ 10]$, obtain

(i) $S_1 \cap S_2$, $S_1 \cap S_3$, $S_2 \cap S_3$

(ii) $S_1 \cup S_2$, $S_1 \cup S_3$, $S_2 \cup S_3$

(iii) $w\{S_1\},\ w\{S_2\},\ w\{S_3\}$

(iv) $|S_1|,\ |S_2|,\ |S_3|$ **: and**

(v) $m\{S_1\},\ m\{S_2\},\ m\{S_3\}$

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IV Year (EEE/EIE/CS)

Semester II 2005 - 2006

MAKEUP TEST I (Closed Book)

Course No.: EA UC482 **Course Title:** Fuzzy Logic & Applications

Date: **Time:** 50 Minutes **M.M. :** 40 (20 %)

Question 1 [2 x 5 = 10]

(a) Let the universe of discourse be X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = { (1, 0.3), (2, 0.5), (3, 1), (4, 0.7), (5, 0.2) }, then obtain $|A|$ and $\|A\|$.

(b) Define 'Support' of a fuzzy set. Obtain the support of the fuzzy set A described below:
X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and
A = { (1, 0), (2, 0.1), (3, 0.2), (4, 0.5), (5, 0.3), (6, 0.1), (7, 0), (8, 0), (9, 0), (10, 0) }

(c) Cite any three reasons, which make fuzzy variables more attuned to reality than crisp variables.

(d) If α-cuts of a fuzzy set A are as follows then obtain the fuzzy set A.

$$^{.2}A = 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$
$$^{.4}A = 0/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$
$$^{.6}A = 0/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$
$$^{.8}A = 0/x_1 + 0/x_2 + 0/x_3 + 1/x_4 + 1/x_5$$
$$^{1}A = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$

(e) If sets A and B are defined as follows then obtain the absolute complement of A and also complement of A with respect to B.
A = { (0, 0.3), (1, 0.4), (2, 0.6), (3, 0.7) } and
B = { (0, 0.4), (1, 0.6), (2, 0.8), (3, 0.8) }

Question 2 **[5 + 5 = 10]**

(a) Suppose we have a universe of integers, Y = {1, 2, 3, 4, 5}. We define the following terms as a mapping onto Y:

$$\text{"Small"} = \left\{\frac{1}{1}+\frac{0.8}{2}+\frac{0.6}{3}+\frac{0.4}{4}+\frac{0.2}{5}\right\}$$

$$\text{"Large"} = \left\{\frac{0.2}{1}+\frac{0.4}{2}+\frac{0.6}{3}+\frac{0.8}{4}+\frac{1}{5}\right\}$$

Then obtain the fuzzy set representation of:
(i) "not very small and not very, very large" and
(ii) "Intensely small"

(b) If A_1 = 0.2/3 + 0.5/4 + 0.8/5 and A_2 = 0.4/3 + 0.7/4 + 1/5, then obtain the following expression:

$$C \wedge \{\mu_{A_1}(x)+(\mu_{A_2}(x)\Theta\mu_{A_1}(x))\}$$

where 'C' stands for convex combination of A_1 and A_2 with ω_1 = 0.3 and ω_2 = 0.7. Symbol '+' and symbol Θ represent arithmetic sum and bounded difference respectively.

Question 3 **[5 + 5 = 10]**

(a) What is axiomatic skeleton of fuzzy intersection? Give one example of the functions which satisfy the same.

(b) What is the special significance of standard fuzzy operations among the various fuzzy complements, intersections and unions?

Question 4 **[5 + 5 = 10]**

(a) How many parameters are needed to describe following types of membership functions? Write down their formulae and give the graphical representations also.
(i) Triangular
(ii) Trapezoidal

(b) If $A = 0.1/x_1 + 0.3/x_2 + 0.6/x_3 + 1/x_4$ then show that the equality of first decomposition theorem i.e. $A = \bigcup_{\alpha\in[0,\ 1]} {}^{\alpha}A$ is hold good, if it is not so then why?

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IV Year (EEE/EIE/CS)

Semester II 2005 - 2006

TEST I (Closed Book)

Course No.: EA UC482 **Course Title:** Fuzzy Logic & Applications

Date: March 26, 2006 **Time:** 50 Minutes **M.M. :** 40 (20 %)

Question 1 **[2 × 5 = 10]**

(a) Find the Hamming distance between fuzzy sets A and B, if

$A = \{(x_1, 0.4), (x_2, 0.8), (x_3, 1), (x_4, 0)\}$

$B = \{(x_1, 0.4), (x_2, 0.3), (x_3, 0), (x_4, 0)\}$

(b) Compute the simple difference $A - B$, if

$A = \{(x_1, 0.2), (x_2, 0.7), (x_3, 1), (x_4, 0)\}$

$B = \{(x_1, 0.5), (x_2, 0.3), (x_3, 1), (x_4, 0.1)\}$

(c) What do you mean by partition of a set?

(d) Obtain the $R \underset{av}{\circ} S$ composition of fuzzy relations R and S, if

$$R = \begin{bmatrix} 0.4 & 0.5 & 0 \\ 0.2 & 0.8 & 0.2 \end{bmatrix} \text{ and } S = \begin{bmatrix} 0.2 & 0.7 \\ 0.3 & 0.8 \\ 1 & 0 \end{bmatrix}$$

(e) Obtain $A \cup B$, if

$A = \{x | 100K \le x \le 200K, \quad x \in U\}$ and $B = \{x | 50K \le x \le 120K, \quad x \in U\}$

where U is the universe of discourse [0, 1000K].

Question 2 **[5 + 5 = 10]**

(a) Describe a 'Type-n Fuzzy Set' and a 'Level-k Fuzzy Set'. Give graphical representation of one example of each.

(b) If R is a relation defined on a set A i.e. $R \subseteq A \times A$ then define the following properties for relation R:

(i) Reflexive Relation

(ii) Symmetric Relation

(iii) Transitive elation

(iv) Equivalence Relation

Question 3 **[5 + 5 = 10]**

(a) Show the graphical representation of two types of π membership functions and write down the mathematical definitions of these functions, clearly mentioning the number of parameters needed to describe them completely.

(b) What is the special significance of standard fuzzy operations among the various fuzzy complements, intersections and unions?

Question 4 **[5 + 5 = 10]**

(a) How many GUI tools are available in Fuzzy Logic Toolbox of MATLAB for building, editing, and observing any fuzzy inference system? Describe their functions briefly.

(b) Describe the concept of α-cut and strong α-cut used in the fuzzy set theory. In a triangular membership function of a fuzzy set, show its α-cuts and special α-cuts at various levels.