BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

Year IV – Semester II 2003 – 2004

COMPREHENSIVE EXAMINATION (Closed Book)

Course No.: EA UC482 **Course Title:** Fuzzy Logic & Applications

Date: May 31, 2004 **Time:** 3 Hours **M.M.** = 80 (40 %)

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1. (a) **Let fuzzy sets A = 0.2/1 + 0.4/3 + 0.6/5 and B = 0.1/2 + 0.3/4 + 0.5/6; then find the following** (5)

   (i) **A × B**

   (ii) **A B**

   (iii) **CON (A + B)**

   (iv) **A ⊕ B**

   (v) **A ⊖ B**

   (b) **If α-cuts of a fuzzy set A are as follows then obtain the fuzzy set A.** (2)

   $^{0.1}A = {}^{0.2}A = \{1, 2, 3, 4, 5, 6, 7\}$

   $^{0.3}A = {}^{0.4}A = \{2, 3, 4, 5, 6\}$

   $^{0.5}A = \{2, 3, 4, 5\}$

   $^{0.6}A = {}^{0.7}A = \{3, 4, 5\}$

   $^{0.8}A = \{4, 5\}$

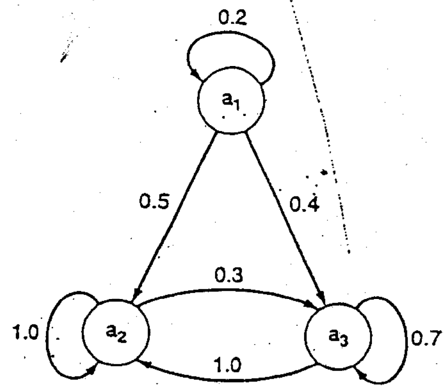
   $^{0.9}A = {}^{1.0}A = \{4\}$

   (c) **If R is a binary relation which gives a correspondence from A to A then define the following properties and give one example of each.** (3)

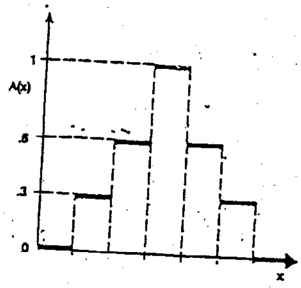
   (i) Reflexive (ii) Antireflexive (iii) Symmetric (iv) Antisymmetric

2. (a) **Prove the identity** $\mu_a(x) \wedge \mu_B(x) = \mu_B(x) \Theta (\mu_B(x) \Theta \mu_A(x))$. **Here symbols carry their usual meanings.** (5)

(b) (i) Give the fuzzy matrix representation of the following graph. (1)

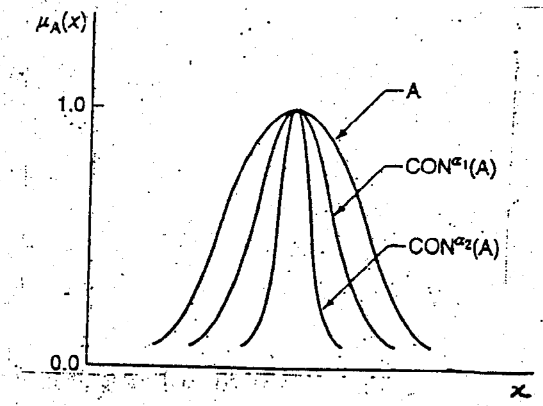


(ii) If the fuzzy set A is given by the following diagram then draw the graph of ${}_{.6}A$. (1)



(iii) Define the convex combination of n fuzzy sets $A_1, A_2, \ldots\ldots\ A_n$. (1)

(iv) If following graph represents the results of CON operation on fuzzy set A for two different values of α, then the relation between these two values will be (1)



(I) $\alpha_1 = \alpha_2$

(II) $\alpha_1 > \alpha_2$

(III) $\alpha_1 < \alpha_2$

(IV) can not be told with this much information.

(v) What is the effect of contrast intensifier (INT) operation on the membership values of a fuzzy set? (1)

3. **Describe in brief, affects of the new methodological possibilities opened by fuzzy sets and fuzzy measures on any two of the following fields of study:**
(i) Civil Engineering (ii) Decision Making (iii) Databases and Information Retrieval Systems (10)

4. **Consider the following rules**

Rule 1: IF x is A_1 THEN y is B_1

Rule 2: IF x is A_2 THEN y is B_2

Where $X = \{x_1, x_2, x_3\}$ **and** $A_1, A_2 \subset X$; $Y = \{y_1, y_2, y_3\}$ **and** $B_1, B_2 \subset Y$. **Fuzzy sets** A_1, A_2, B_1, **and** B_2 **are given as follows:**

$A_1 = 0.2/x_1 + 0.4/x_2 + 0.6/x_3$ $\quad$ $A_2 = 0.8/x_1 + 0.6/x_2 + 0.4/x_3$

$B_1 = 0.1/y_1 + 0.3/y_2 + 0.5/y_3$ $\quad$ $B_2 = 0.9/y_1 + 0.7/y_2 + 0.5/y_3$

Mamdani proposed the following method to compose fuzzy relations R:

$$R = A \rightarrow B = A \times B = \int_{X \times Y} (\mu_A(x) \wedge \mu_B(y))/(x, \ y)$$

Use Mamdani's method to convert the Rule 1 into fuzzy relation matrix R_1 **and Rule 2 into fuzzy relation matrix** R_2 **and obtain** $R = R_1 \cup R_2$. (10)

5. **Show the inter-relation among the basic components of a general pattern recognition system with the help of a diagram and explain them in brief. What is the relevance of fuzzy set theory in pattern recognition field?** (10)

6. **Consider the case of a subway train approaching a station. Describe a fuzzy logic based control strategy to obtain the amount of brakes power (in terms of linguistic values Very_Heavy, Heavy, Light, Very_Light) used to halt the train if the linguistic values of distance from the station (Very_Close, Close, Far, and Very_Far), and the speed of the train (Very_Slow, Slow, Fast, Very_Fast) are known. Define these linguistic values in the range of 0 – 500 meters (distance); 0 – 100 km/h (speed); and 0 – 100 % (of maximm brake power). What will be the control action if the distance is 100m and the speed is 24.6 km/h?** (10)

7. (a) **Write down a brief note about the use of fuzzy logic in medical diagnosis assistance.** (5)

(b) **Suppose that patient x displays the symptoms s_1, s_2, s_3, and s_4 at the levels of severity given by the fuzzy set $A_x = .1/s_1 + .7/s_2 + .4/s_3 + .6/s_4$.** (5)

Perform the diagnosis of this patient among three possible diseases d_1, d_2, and d_3 using fuzzy logic technique. Each of these diseases is described by the following matrices respectively, giving the upper and lower bounds of normal range of severity of each of the four symptoms that can be expected in a patient with the disease.

$$B_1 = \begin{matrix} lower \\ upper \end{matrix}\begin{bmatrix} 0 & .6 & .5 & 0 \\ .2 & 1 & .7 & 0 \end{bmatrix}$$

$$B_2 = \begin{matrix} lower \\ upper \end{matrix}\begin{bmatrix} 0 & .9 & .3 & .2 \\ .0 & 1 & 1 & .4 \end{bmatrix}$$

$$B_3 = \begin{matrix} lower \\ upper \end{matrix}\begin{bmatrix} 0 & 0 & .7 & 0 \\ .3 & 0 & .9 & 0 \end{bmatrix}$$

Fuzzy relation W on the set of symptoms and diseases that specifies the pertinence or importance of each symptom s_i in the diagnosis of the matrix of each disease d_j is given as:

$$W = \begin{matrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{matrix}\begin{bmatrix} .4 & .8 & 1 \\ .5 & .6 & .3 \\ .7 & .1 & .9 \\ .9 & .6 & .3 \end{bmatrix}$$

<u>NOTE</u>: you may use the clustering technique using Minkowski distance given by

$$D_p(d_j, x) = \left[\sum_{i \in I_l} \left|W(s_i, d_j)(B_{jl}(s_i) - A_x(s_i))\right|^p + \sum_{i \in I_u} \left|W(s_i, d_j)(B_{ju}(s_i) - A_x(s_i))\right|^p\right]^{1/p}$$

Where $I_l = \{i \in N_m | A_x(s_i) < B_{jl}(s_i)\}$ $I_u = \{i \in N_m | A_x(s_i) > B_{jl}(s_i)\}$ **and m denotes the total number of symptoms.**

8. (a) Sally is nearsighted and colorblind. When she goes to a local grocery where fruits are placed on high shelves, she cannot see them very well. She can only recognize the size and blurred shape of the fruits. Her knowledge about the fruits can be represented by the following fuzzy relation. (5)

	tangerine	apple	pineapple	watermelon	strawberry
long	0	0	0.3	0	0.8
round	0.9	1.0	0.3	1.0	0.2
large	0.2	0.4	0.7	1.0	0.1

Guess a fruit that Sally describes as

long	round	large
0	0.7	1.0

(b) Suppose the universe X and Y are given as $X = \{x_1,\ x_2,\ x_3\}$ and $Y = \{y_1,\ y_2,\ y_3\}$. The fuzzy relation R on X × Y is given as follows: (5)

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.1 & 0.4 & 0.6 \\ 0.5 & 0.2 & 0.8 \\ 0.7 & 0.9 & 0.3 \end{bmatrix} \end{array}$$

Obtain (i) Projection of R on X

(ii) Projection of R on Y

(iii) Cylindrical extension of A (fuzzy set projected on X) on the Cartesian product X × Y and

(iv) Cylindrical extension of B (fuzzy set projected on Y) on the Cartesian product X × Y

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Year IV – Semester II 2003 – 2004

TEST II (Open Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: May 2, 2004 Time: 50 Minutes M.M. = 40 (20 %)

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1. Let A = $\{a_1, a_2\}$, B = $\{b_1, b_2, b_3\}$, and C = $\{c_1, c_2\}$. Let R be a relation from A to B defined by the matrix

$$\begin{array}{c} \\ a_1 \\ a_2 \end{array}\begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.5 & 0 \\ 0.2 & 0.8 & 0.2 \end{bmatrix} \end{array}$$

Let S be a relation from B to C defined by the matrix

$$\begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \end{array}\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.7 \\ 0.3 & 0.8 \\ 1 & 0 \end{bmatrix} \end{array}$$

Then obtain the max-av composition of R and S. (4)

2. What are the main reasoning patterns used in the propositional logic? Explain them with the help of one example each. (3)

3. Let $U = \{a, \; b, \; c\}$ and $V = \{x, \; y\}$. If R be a FUR on U × V, defined by (3)

$$\begin{array}{c} \\ a \\ b \\ c \end{array}\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} 0.3 & 1.0 \\ 0.6 & 0.2 \\ 0.4 & 0.5 \end{bmatrix} \end{array}$$

Then obtain Dom (R), Ran (R), and h (R).

4. Suppose the universe X and Y are given as:

$$X = \{x_1, \; x_2, \; x_3\} \quad \text{and} \quad Y = \{y_1, \; y_2, \; y_3\}$$

If fuzzy relations R and S on X × Y are given as follows, then prove that the De Morgan's laws hold good: (8)

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 1.0 & 0.4 & 0.6 \\ 0.5 & 0.8 & 0.7 \\ 0.9 & 0.6 & 1.0 \end{bmatrix} \end{array} \quad \text{and} \quad S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.3 & 0.7 & 0.9 \\ 0.6 & 1.0 & 0.4 \\ 0.2 & 0.9 & 1.0 \end{bmatrix} \end{array}$$

5. The fuzzy relation R is defined on sets U = {&, *}, V = {x, y}, and W = {a, b, c} as follows

   R = 0.9/(a, x, &) + 0.4/(b, x, &) + 1/(a, y, &) + 0.7/(a, y, *) + 0.8/(b, y, *)

   Obtain the cylindrical extensions $cyl\langle R_{12},\ R_{13},\ R_{23}\rangle$ and $cyl\langle R_{23},\ R_{2}\rangle$. (6)

6. Draw the correct conclusion using the following premises.
   (i) No student that studies hard is not having good CGPA.
   (ii) No student not residing in hostel play in hostel team.
   (iii) Students who are good players always study hard
   (iv) No Good CGPA students are black listed.
   (v) No student resides in hostel unless he is a good player. (2)

7. Consider the following rules

   Rule 1: IF x is A_1 THEN y is B_1

   Rule 2: IF x is A_2 THEN y is B_2

   Where $X = \{x_1,\ x_2,\ x_3\}$ and A_1, $A_2 \subset$ X

   $Y = \{y_1,\ y_2,\ y_3\}$ and B_1, $B_2 \subset$ Y

   Fuzzy sets A_1, A_2, B_1, and B_2 are given as follows:

   $$A_1 = {}^{1.0}/_{x_1} + {}^{0.6}/_{x_2}$$

   $$A_2 = {}^{0.8}/_{x_2} + {}^{1.0}/_{x_3}$$

   $$B_1 = {}^{1.0}/_{y_1} + {}^{0.6}/_{y_2} + {}^{0.1}/_{y_3}$$

   $$B_2 = {}^{0.2}/_{y_1} + {}^{0.8}/_{y_2} + {}^{0.9}/_{y_3}$$

Mamdani proposed the following method to compose fuzzy relations R:

$$R = A \rightarrow B = A \times B = \int_{X \times Y} (\mu_A(x) \wedge \mu_B(y))/(x, \quad y)$$

Use Mamdani's method to convert the Rule 1 into fuzzy relation matrix R_1 and Rule 2 into fuzzy relation matrix R_2 and obtain $R = R_1 \cup R_2$. (5)

8. (a) Simplify the switching circuit shown in figure 1. (3)

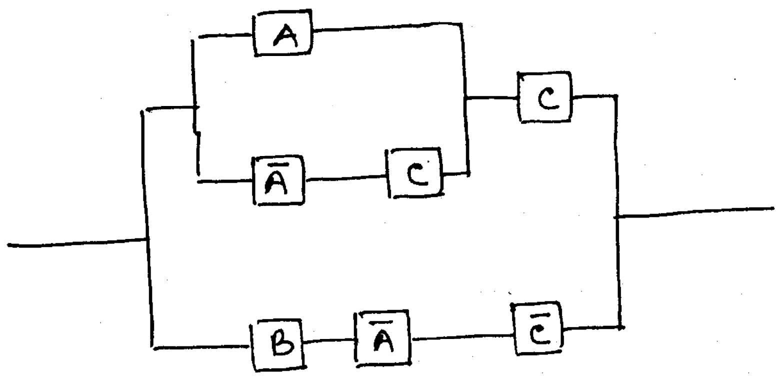

Figure 1

(b) Suppose the universe X and Y are given as $X = \{x_1, \quad x_2, \quad x_3\}$ and $Y = \{y_1, \quad y_2, \quad y_3\}$. The fuzzy relation R on X × Y is given as follows: (6)

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.6 & 1 & 0.3 \\ 0.5 & 0.2 & 0.8 \\ 0.1 & 0.4 & 0.7 \end{bmatrix} \end{array}$$

Obtain (i) Projection of R on X

(ii) Projection of R on Y

(iii) Cylindrical extension of A (fuzzy set projected on X) on the Cartesian product X × Y and

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Name: Id. No.:

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

Year IV (EEE/CSE) – Semester II 2003 – 2004

QUIZ I (Closed Book)

Course No.: EA UC482 **Course Title: Fuzzy Logic & Applications**

Date: April 1, 2004 **Time: 40 Minutes** **M.M. = 20 (10 %)**

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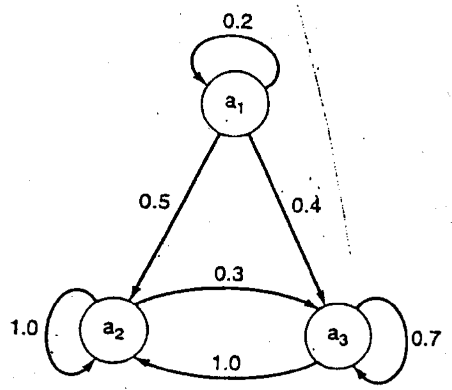
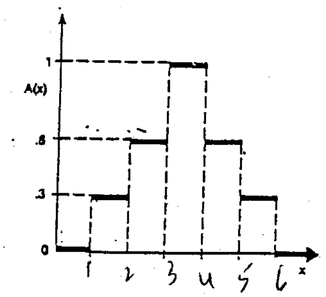
1. **A Fuzzy Relation is said to be .... relation, if it is reflexive and symmetric.**

2. **max-product composition of two fuzzy relations R on A × B and S on B × C can be defined by the equation ....**

3. **Fuzzy matrix representation of the following graph will be ....**



4. **If the fuzzy set A is given by the following diagram then the graph of ${}_{.6}A$ will be**



5. A Fuzzy Relation given as $M_R = \begin{bmatrix} 0 & .7 & .4 \\ .7 & 0 & .1 \\ .4 & .1 & 0 \end{bmatrix}$ is

(i) Reflexive and symmetric

(ii) Antireflexive and symmetric

(iii) Reflexive and antisymmetric

(iv) Antireflexive and antisymmetric

6. Which of the following properties does not apply to the standard Fuzzy complement, intersection and union operations.

(i) Satisfy the cutworthy and strong cutworthy properties.

(ii) Prevent the compounding of errors of the operands.

(iii) Context-dependent.

(iv) All of the above properties apply to standard fuzzy operations.

7. Which one of the following is incorrect relation

(i) $A = \bigcup_{\alpha \in [0,\ 1]} {}_{\alpha}A$

(ii) ${}_{\alpha+}A(x) = \alpha \cdot {}^{\alpha+}A(x)$

(iii) $A = \bigcup_{\alpha \in [0,\ 1]} {}_{\alpha+}A$

(iv) All of the above relations are correct.

8. Which of the following property is not included in the axiomatic skeleton for fuzzy complements.

(i) Monotonicity

(ii) Boundary conditions

(iii) Involutive

(iv) All of the above properties are included.

9. Which one of the following is not correct

(i) $\mu_a(x) \wedge \mu_B(x) = \mu_B(x) \Theta (\mu_B(x) \Theta \mu_A(x))$

(ii) $\mu_A(x) \vee \mu_B(x) = \mu_A(x) + (\mu_B(x) \Theta \mu_A(x))$

(iii) $\mu_A(X) \rightarrow \mu_B(X) = (X \Theta Y)$

(iv) $\mu_A(x) \leftrightarrows \mu_B(x) = 1 \Theta (\mu_A(x) \Theta \mu_B(x) + (\mu_B(x) \Theta \mu_A(x))$.

10. The convex combination of n fuzzy sets $A_1, A_2, \ldots\ldots\ldots A_n$, is defined as ....

11. Which one of the following is not correct

(i) ${}^{\alpha+}A \subseteq {}^{\alpha}A$;

(ii) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$

(iii) ${}^{\alpha+}(A \cup B) = {}^{\alpha+}A \cup {}^{\alpha+}B$;

(iv) All of them are correct

12. If A = 0.5/4 + 1/5 + 0.6/7 and B = 0.3/4 + 0.6/5, Then A $\oplus$ B will be ....

13. What will be the effect of contrast intensifier (INT) operation on the membership values of fuzzy set A?

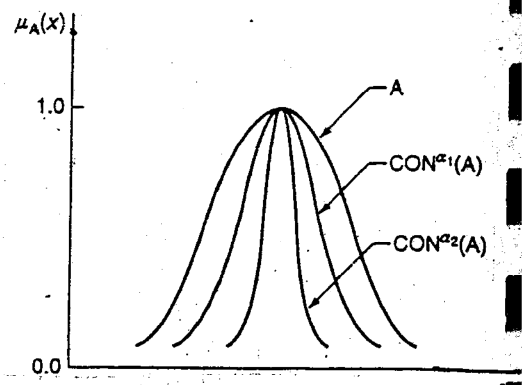
14. If following graph represents the results of CON operation on fuzzy set A for two different values of α, then the relation between these two values will be

(i) $\alpha_1 = \alpha_2$

(ii) $\alpha_1 > \alpha_2$

(iii) $\alpha_1 < \alpha_2$

(iv) can not be told with this much information.



15. What will be the circuit diagram representation of fuzzy inverter operation?

16. Fuzzy sets, by definition, violate two properties of the complement of crisp sets, these properties are ....

17. Classical (standard) fuzzy complement is one which

(i) satisfies the axiomatic skeleton for fuzzy complements.

(ii) is continuous fuzzy complement.

(iii) is involutive fuzzy complement.

(iv) satisfies all the above conditions

18. **The equilibrium of a complement c is that degree of membership in a fuzzy set A which equals the degree of membership**

(i) in the complement of A

(ii) in the complement of P(A)

(iii) in the Complement of universe of discourse.

(iv) None of the above

19. **The standard fuzzy intersection is the only**

(i) Subidempotent t-norm.

(ii) Idempotent t-norm.

(iii) Super idempotent t-norm.

(iv) All of the above

20. **Interval-valued fuzzy sets can further be generalized by allowing their intervals to be fuzzy, the sets so obtained are referred to as ....**

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Knowledge Village, Dubai

Year IV – Semester II 2003 – 2004

TEST I (Closed Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: March 28, 2004 Time: 50 Minutes M.M. = 40 (20 %)

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1. (a) Mention clearly any three advantages of using fuzzy logic systems in the area of information systems. (6)

   (b) What are the axioms that must be satisfied by any binary operation, on the unit interval, to be considered as the fuzzy intersection? (4)

2. (a) Explain the concept of 'support fuzzification' and 'grade fuzzification' with the help of one example. (5)

   (b) What is the difference between $\alpha(\bar{A})$ and $\alpha\bar{A}$? (1)

   (c) Define the following terms in context to fuzzy set theory: (4)

   (i) Height of a fuzzy set (ii) Core of a fuzzy set

   (iii) Cutworthy Property (iv) Equilibrium of a fuzzy complement

3. Let fuzzy sets A = {(1, 0.1), (3, 0.3), (5, 0.6)} and B = {(2, 0.3), (4, 0.6), (6, 1)}; then find the following (10)

   (i) $A \times B$

   (ii) A B

   (iii) CON (A + B)

   (iv) $A \oplus B$

   (v) $A \ominus B$

5. (a) If α-cuts of a fuzzy set A are as follows then obtain the fuzzy set A. (2)

$$^{.2}A = 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$^{.4}A = 0/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$^{.6}A = 0/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$^{.8}A = 0/x_1 + 0/x_2 + 0/x_3 + 1/x_4 + 1/x_5$$

$$^{1}A = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$

(b) Suppose expression "medium" for a fuzzy variable is defined as given in figure 1, then draw the graphs for expressions "more medium" and "less medium", if they are defined by the relations *more medium* = *CON(medium)*; and *less medium* = *DIL(medium)* (4)

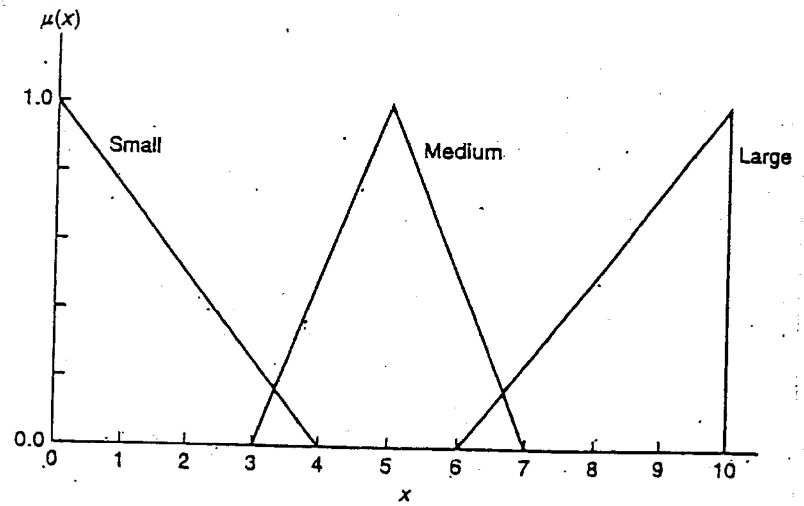

Figure 1

(c) If A, B ∈ ℑ (X), then prove (mathematically) that the property (4)

$^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ holds good for all α, β ∈ [0, 1].