**|Dubai International Academic City, Dubai**
**Third & Fourth Year – First Semester 2013 – 2014**
**MATH F231/MATH C231 – Number Theory**
**Comprehensive Examination**

**Date: 02.01.2014** **Max. Marks: 40**
**Time: 3 hours** **Weightage: 40%**

Q1a). Find the l.c.m and gcd of 3675, 4374, 18375, 74088.
b).How many modified perfect faro shuffles are needed to return the cards to their original position in a deck of 62 cards? [2+2]

Q2a). When n is divided by 8, the remainder is 5. What is the remainder of the division of $n^3$ + 5n by 8?
b). How many incongruent solutions does $7x \equiv 5 \pmod{12}$ have? Find them. [2+2]

Q3a). Find the last two digits of the number $37^{1992}$
b). Show that Mersenne number $M_{13}$ is a prime number? [2+2]

Q4a). It is conjectured that every even integer can be written as a difference of two consecutive primes in infinite ways. Write 6 as a difference of two consecutive primes in two ways.
b) if gcd(a, c) = gcd(b, c) = 1, then show gcd(ab, c) = 1 [2+2]

Q5.a) Determine whether or not 35 is a quadratic residue modulo 323.
b) Show that for n=24

$$\sum_{d|n} \sigma(d)\phi(\frac{n}{d}) = nd(n)$$ [2+2]

Q6. A student returning from Europe changes his euros and swiss francs in US money. If she receives $46.26 and $ 1.11 for each euro and 83 cents for each franc. How much of each type of currency did she exchange? [4]

Q7. Does the following quadratic congruence have a solution? If yes solve it. [4]

$$x^2 + 5x \equiv 12 (\mathrm{mod}\, 31)$$

Q8. Solve $858x + 253y = 33$ using continued fractions. Give the general Solution. [4]

Q9.Prove There is infinitely many primes of the form 4n+1. [4]

Q10.When a class of number theory was divided into groups of 3 two students were left out; when into groups of 4 one was left out. When it was divided into groups of 5 the students found that if the professor was added to one of the group no one was left out .How many students were there in the class? [4]

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**MATH C231 – Number Theory**
**Test 2 (Open Book)**

**Date: 20.11.2013** **Max. Marks: 20**
**Time: 50 Minutes** **Weightage: 20%**

Q1.Find whether 1387 is a prime number or not? [1]

Q2.If n and n+2 are twin primes then show [1]
$\sigma(n+2) = \sigma(n) + 2$.

Q3. Find $\phi(21600)$ and d (21600). [2]

Q4. What day of the week was 10th December 1547? [2]

Q5. Prove that the following congruence has a solution. Find all the incongruent solutions of the congruence.
$18x \equiv 30 (\text{mod}\, 42)$ [2]

Q6.What is the last two digits of $7^{1000}$ [2]

Q7. Find n such that $\sigma(n) = 217$ [2]

Q8.Find a positive integer n such that $\mu(n) + \mu(n+1) + \mu(n+2) = 3$. [2]

Q9. Solve the following congruence using Chinese remainder theorem.
$17x \equiv 9 (\text{mod}\, 276)$ [3]

Q10. Prove $2^{4n} - 1$ is divisible by 15. [3]

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**Fourth & Third Year – First Semester 2013 – 2014**

**MATH C231 – Number Theory**
**Test 1 (Closed Book)**

**Date: 25.09.2013** **Max. Marks: 25**
**Time: 50 Minutes** **Weightage: 25%**

Q1. a)Use the Euclidean Algorithm to find the g.c.d (1769,2378).Use the g.c.d of (1769,2378) to find the l.c.m (1769,2378).
b)Find integers x and y such that
g.c.d(1769,2378) = 1769x + 2378y [2]

Q2. Find the g.c.d and l.c.m of (910,780,286,195) using prime factorization. [2]

Q3. Prove if n is odd then n and n-2 are relatively prime. [2]

Q4. Show if $(x_o, y_o)$ is a solution of ax-by =1, then the area of triangle whose vertices are (0,0),(b,a) and $(x_o, y_o)$ is $\frac{1}{2}$. [2]

Q5. For any odd integers a and b ,show $a^2-b^2$ is divisible by 8. [2]

Q6. Find the g.c.d (2a + 3,4a + 5) [3]

Q7. Prove if $a \mid c$ and $b \mid c$ with g.c.d(a,b) =1, then $ab \mid c$ [3]

Q8. Determine the general solution of 158x - 57y=7 [3]

Q9.A man has $4.55 in change composed entirely of dimes and quarters .What are the maximum and minimum number of coins that he can have ? Is it possible for the number of dimes to equal the number of quarters? Given a dime is 10 cents and a quarter is 25 cents. [3]

Q10. Do there exist integers a and b which add to 500 and have a g.c.d as 7? [3]

**SET B**

**Name:.....ID.No.....**

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**MATH C231/MATH F231 – Number Theory**
**Quiz 2 (Closed Book)**

**Date: 05.12.2013** **Max. Marks: 07**
**Time: 20 Minutes** **Weightage: 07%**

Q1. Find the value of $d(20)$. [1]

Q2. Find the value of $\sigma(12!)$ [1]

Q3. Find the smallest n, such that $\phi$ (n) =6 [1]

Q4. Find a prime q such that $(\frac{13}{q}) = 1$. [2]

Q5. Does the quadratic $x^2 \equiv 5 (\mathrm{mod}\, 229)$ have a solution? [2]

SETA

Name:.....ID.No.....

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**MATH C231/MATH F231 – Number Theory**
**Quiz 2 (Closed Book)**

**Date: 05.12.2013** **Max. Marks: 07**
**Time: 20 Minutes** **Weightage: 07%**

Q1. Find the value of $\phi(20)$. [1]

Q2. Find the value of $\sigma(10!)$ [1]

Q3. . Find the smallest n, such that d(n)=6 [1]

Q4. Find a prime q such that $(\frac{5}{q}) = -1$. [2]

Q5. Does the quadratic $x^2 \equiv 5 (\mathrm{mod}\, 227)$ have a solution? [2]

Name: ..... ID No: .....

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**MATH C231– Number Theory**
**Quiz 1 (Closed Book)**

**Date: 08.10.2013** **Max. Marks: 8**
**Time: 20 Minutes** **Weightage: 08%**

Q1. For what values of m is $1331 \equiv 0 (\mathrm{mod}\, m)$ [1]

Q2. Find the nonnegative residue mod 13 of -100. [1]

Q3. Are there integers a, b, c such that a | bc, but a ┼b and a┼c .Justify your answer. [1]

Q4.Find the prime factorization of 111111. [1]

Q5. Which pair a, b have g.c.d 18 and l.c.m 540? [2]

Q6. Give a reduced residue system mod 7 composed entirely of powers of 3. [2]