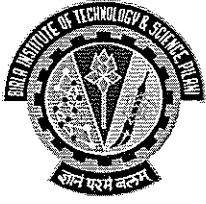


Dubai International Academic City, Dubai
Fourth Year – First Semester 2012 – 2013
MATH C231 – Number Theory
Comprehensive Examination

Date: 2.01.2013
Time: 3 hours

Max. Marks: 40
Weightage: 40%

- Q1 (a). Find the g.c.d of (119, 272). Use the Euclidean Algorithm to obtain integers x and y satisfying, $\text{g.c.d of } (119, 272) = 119x + 272y$
b). Is $\{-3, 34, 8, 12, -1, -11\}$ a complete residue system modulus 6? Verify your answer. [2+2]
- Q2 (a). Find two integers n such that $\phi(n) = 18$.
b). Check whether 1949 and 1951 are primes. If they are prime what type of prime are they? [2+2]
- Q3 (a) Use Eulers criterion to find out whether 83 divides $2^{41} - 1$ or not?
b). Battle of Hastings was fought on October 14, 1066. What day of the week was this? [2+2]
- Q4 a). Show if $d|n$ then $\phi(d) | \phi(n)$ for $n=12$
b) Find the value of $d(5112)$ and $\sigma(5112)$. [2+2]
- Q5 a). Which positive integers less than 15 have inverses modulus 15? Find their inverses.
b). What is the remainder when $16!$ is divided by 19? [3+3]
- Q6. An astronomer knows that a satellite orbits the earth in a period that is an exact multiple of 1 hour that is less than a day. If the astronomer notes that the satellite completes 11 orbits in an interval that starts when a 24 hour clock reads 0 hours and ends when the clock reads 17 hours. How long is the orbital period of the satellite? [3]
- Q7. Find the integer that leaves remainder of 1 when divided by either 2 or 5 but that is divisible by 3. [3]
- Q8. Solve $377x - 120y = -3$ using continued fractions. Give the general Solution. [4]
- Q9. Prove there are infinitely many primes of the form $6n+5$. [4]
- Q10. Does the following quadratic congruence have a solution? If yes solve it.
 $x^2 \equiv 7 \pmod{3^3}$ [4]



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Dubai Campus

First Semester 2012-2013

Test 2, (OB)

Course: NUMBER THEORY

Max marks: 20

Date: 4-11-2012

Year: IVth Year

Course No. MATH C231

Weightage: 20%

Time: 50 Minutes

Q1. Find the value of d ($13!$). [2]

Q2. Carl Frederic was born on April 30, 1977, on what day of the week was he born? [2]

Q3. . Find the value of $\phi(30)$. Also write a reduced set of residues mod (30) Will a value of "a" satisfy the following? Justify your answer.

$$1+a+a^2+a^3+\dots+a^{\phi(m)-1} \equiv 0 \pmod{m} \quad [3]$$

Q4. Find the solution x , which satisfies of the following congruence's simultaneously

$$x \equiv 1 \pmod{4}, 2x \equiv 3 \pmod{5}, 4x \equiv 5 \pmod{7} \quad [3]$$

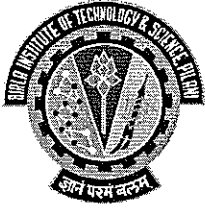
Q5. Find n such that $\sigma(n) = 91$ [3]

Q6. Show 229 divides $13^{2k} + 17^{2k}$ only if k is odd, for k is even it does not divide. [3]

Q7. Show that for $n=15$ [4]

$$(a) \quad n / \phi(n) = \sum_{d|n} \mu^2(d) / \phi(d)$$

$$(b) \quad \phi(n) = \sum_{d|n} \frac{n}{d} \mu(d) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$$



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Dubai Campus

1/2 May 2012

**First Semester 2012-2013
TEST -I (CB)**

Course: NUMBER THEORY

Course No. MATH C231

Max marks: 25

Date: 07-10-2012

Year: IV

Weightage: 25%

Time: 50 Minutes

Q1. Find the prime factorization of 132, 400 and 1995 and using that find the $l, c, m(132, 400, 1995)$ and $g.c.d(132, 400, 1995)$. [2]

Q2. Is 1,5,7,11,13,17 a reduced residue system mod 18? Justify your answer. [2]

Q3. Find the g.c.d of 210 and 495 using Euclid's division algorithm. Find integers x and y such that

$$g.c.d(210, 495) = 210x + 495y \quad [3]$$

Q4. Show that $(6k+5)$ and $(7k+6)$ are relatively prime. [3]

Q5. If $a \mid c$, $b \mid c$ and $g.c.d(a, b) = 1$ then prove that $ab \mid c$. [3]

Q6. Solve the linear Diophantine equation $11x + 7y = 200$ and find the general solution. [4]

Q7. Solve the Euler's problem of dividing 100 into two summands such that one is divisible by 7 and other by 11. Form the Diophantine equation and solve it. [4]

Q8. If $a \equiv b \pmod{n}$, then $a+c \equiv b+c \pmod{n}$ and $ac \equiv bc \pmod{n}$ [4]



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First Semester 2012-2013
Quiz-2, (CB)

Course: NUMBER THEORY
Max marks: 7
Date: 28-11-2012
Year: IVth Year

Course No. MATH C231
Weightage: 7%
Time: 20 Minutes

Q1. How many primes are there before 1949?

[2]

Q2. Find the Mersenne Number M_{13} . Is it a Mersenne prime?

[2]

Q3. Write 81 as a sum of primes.

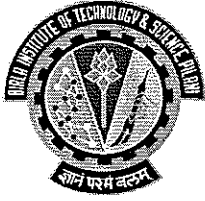
[1]

Q4. Find a prime divisor of $4(3 \cdot 7 \cdot 11) - 1$, which is of the form $4n + 3$

[2]

Name:

ID:



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First Semester 2012-2013

Quiz-I, (CB)

Course: NUMBER THEORY

Max marks: 8

Date: 21-10-2012

Year: IVth Year

Course No. MATH C231

Weightage: 8%

Time: 20 Minutes

Q1. Find a complete set of mutually incongruent solution for the following congruence

$$11x \equiv 28 \pmod{1943}$$

[2]

Q2. Formulate the problem of finding all positive integers that leave remainders 2,4,8 when divided by 9,10,11 respectively. Is it true that $74 + 990t$ is the general solution? [2]

Q3. Reason why any integer satisfies at least one of the following congruence's [2]
 $x \equiv 0 \pmod{2}$, $x \equiv 0 \pmod{3}$, $x \equiv 1 \pmod{4}$, $x \equiv 3 \pmod{8}$, $x \equiv 7 \pmod{12}$, $x \equiv 23 \pmod{24}$,

Q4. Use the fact that $640 = 5 \times 2^7$ to prove Fermat's number $2^{32} + 1$ is divisible by 641. [2]