**Dubai International Academic City, Dubai**
**Fourth Year – First Semester 2012 – 2013**
**MATH C231 – Number Theory**
**Comprehensive Examination**

**Date: 2.01.2013** | **Max. Marks: 40**
**Time: 3 hours** | **Weightage: 40%**

Q1 (a). Find the g.c.d of (119, 272). Use the Euclidean Algorithm to obtain integers x and y satisfying, g.c.d of (119, 272) = 119x+272y
b). Is {-3, 34, 8, 12,-1,-11} a complete residue system modulus 6? Verify your answer. [2+2]

Q2 (a). Find two integers n such that $\phi(n) = 18$.
b). Check whether 1949 and 1951 are primes. If they are prime what type of prime are they? . [2+2]

Q3 (a) Use Eulers criterion to find out whether 83 divides $2^{41} - 1$ or not?
b). Battle of Hastings was fought on October 14, 1066. What day of the week was this? [2+2]

Q4 a). Show if d|n then $\phi(d) \mid \phi(\text{n})$ for n=12
b) Find the value of d (5112) and $\sigma(5112)$. [2+2]

Q5 a). Which positive integers less than 15 have inverses modulus 15? Find their inverses.
b).What is the remainder when 16! Is divided by 19? [3+3]

Q6.An astronomer knows that a satellite orbits the earth in a period that is an exact multiple of 1 hour that is less than a day. If the astronomer notes that the satellite completes 11 orbits in an interval that starts when a 24 hour clock reads 0 hours and ends when the clock reads 17 hours .How long is the orbital period of the satellite? [3]

Q7.Find the integer that leaves remainder of 1when divided by either 2 or 5 but that is divisible by 3. [3]

Q8. Solve 377x – 120y = -3 using continued fractions. Give the general Solution. [4]

Q9.Prove there are infinitely many primes of the form 6n+5. [4]

Q10.Does the following quadratic congruence have a solution? If yes solve it.
$x^2 \equiv 7 (\bmod 3^3)$ [4]

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## First Semester 2012-2013
### Test 2, (OB)

**Course: NUMBER THEORY** **Course No. MATH C231**
**Max marks: 20** **Weightage: 20%**
**Date: 4-11-2012** **Time: 50 Minutes**
**Year: IVth Year**

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Q1. Find the value of d (13!). [2]

Q2. Carl Frederic was born on April 30, 1977, on what day of the week was he born? [2]

Q3. . Find the value of $\phi(30)$.Also write a reduced set of residues mod (30) Will a value of "a" satisfy the following? Justify your answer.

$1+a+a^2+a^3+\dots a^{\phi(m)-1} \equiv 0 (\bmod m)$ [3]

Q4.Find the solution x, which satisfies of the following congruence's simultaneously

$x \equiv 1(\bmod 4), 2x \equiv 3(\bmod 5), 4x \equiv 5(\bmod 7)$ [3]

Q5. Find n such that $\sigma(n) = 91$ [3]

Q6. Show 229 divides $13^{2k}+17^{2k}$ only if k is odd, for k is even it does not divide. [3]

Q7. Show that for n=15 [4]

(a) $n/\phi(n) = \sum_{d|n} \mu^2(d)/\phi(d)$

(b) $\phi(n) = \sum_{d|n} \frac{n}{d}\mu(d) = n\prod_{p|n}(1-\frac{1}{p})$

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**First Semester 2012-2013**
**TEST -I (CB)**

**Course: NUMBER THEORY**
**Course No. MATH C231**
**Max marks: 25**
**Date: 07-10-2012**
**Year: IV**

**Weightage: 25%**
**Time: 50 Minutes**

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Q1. Find the prime factorization of 132, 400 and 1995 and using that find the l,c,m(132,400 ,1995) and g.c.d(132,400 ,1995). [2]

Q2. Is 1,5,7,11,13,17 a reduced residue system mod 18? Justify your answer. [2]

Q3. Find the g.c.d of 210 and 495 using Euclid's division algorithm. Find integers x and y such that

g.c.d(210,495)= 210x+495y [3]

Q4. Show that (6k+5) and (7k+6 ) are relatively prime. [3]

Q5. If a| c, b |c and g.c.d(a,b)=1 then prove that ab |c. [3]

Q6. Solve the linear Diophantine equation 11x+7y=200 and find the general solution. [4]

Q7. Solve the Euler's problem of dividing 100 into two summands such that one is divisible by 7 and other by 11.Form the Diophantine equation and solve it. [4]

Q8. If $a \equiv b \pmod{n}$, then $a+c \equiv b+c \pmod{n}$ and $ac \equiv bc \pmod{n}$ [4]

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## First Semester 2012-2013
**Quiz–2, (CB)**

**Course: NUMBER THEORY**
**Max marks: 7**
**Date: 28-11-2012**
**Year: IVth Year**

**Course No. MATH C231**
**Weightage: 7%**
**Time: 20 Minutes**

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Q1.How many primes are there before 1949? [2]

Q2. Find the Mersenne Number $M_{13}$. Is it a Mersenne prime? [2]

Q3. Write 81 as a sum of primes. [1]

Q4. Find a prime divisor of 4(3.7.11)-1 , which is of the form 4n+3 [2]

Name:

ID:

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## First Semester 2012-2013
**Quiz–I, (CB)**

**Course: NUMBER THEORY**
**Max marks: 8**
**Date: 21-10-2012**
**Year: IVth Year**

**Course No. MATH C231**
**Weightage: 8%**
**Time: 20 Minutes**

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Q1. Find a complete set of mutually incongruent solution for the following congruence
$11x \equiv 28 \pmod{1943}$ [2]

Q2. Formulate the problem of finding all positive integers that leave remainders 2,4,8 when divided by 9,10,11 respectively. Is it true that 74 +990t is the general solution? [2]

Q3. Reason why any integer satisfies at least one of the following congruence's [2]

$x \equiv 0 \pmod{2}$, $x \equiv 0 \pmod{3}$, $x \equiv 1 \pmod{4}$, $x \equiv 3 \pmod{8}$, $x \equiv 7 \pmod{12}$, $x \equiv 23 \pmod{24}$,

Q4.Use the fact that $640=5 \times 2^7$ to prove Fermat's number $2^{32}+1$ is divisible by 641. [2]