**BITS Pilani, Dubai Campus**
**Dubai International Academic City, Dubai**
**Fourth Year – First Semester 2011 – 2012**
**MATH C231 – Number Theory**
**Comprehensive Examination**

**Date: 12.01.2012** **Max. Marks: 40**
**Time: 3 hours** **Weightage: 40%**

Q1 (a). Find the g.c.d of (24, 138). Use the Euclidean Algorithm to obtain integers x and y satisfying,
g.c.d (24, 138) = 24x+138y
b). Find the l.c.m (39, 102, 75) using prime factorization. [2+2]

Q2 (a). Show 1, 5, 7,11,13,17 is a reduced residue system modulo 18.
b). Prove that if $bd \equiv b\ d'$ (mod p), where p is a prime and p does not divide b then $d \equiv d'$ (mod p) [2+2]

Q3 (a). Determine whether 2093 is a prime or not and how many primes are there before 2093?
b). A Theatre charges $1.80 for adult admissions and $0.75 for children. On a particular evening the total receipts were $90. Assuming that more adults were present, find whether the problem has a solution and if it has find how many adults and children ~~people~~ attended? [2+2]

Q4 (a) Find the residue when $41^{65}$ is divided by 7
b) How many faro shuffles are required to return the cards to their original position in a deck of 12 cards? [2+2]

Q5 a).Find the value of the Legendre symbol $(-\frac{219}{383})$
b). Prove that for n=30 "The sum of positive integers less than n and relatively prime to n is equal to $\frac{1}{2}n\phi(n)$". [2+2]

Q6.a) For n=206 show
$\sigma(n)=\sigma(n+1)$
b)Show $\mu(n)$ is a multiplicative function. [2+2]

Q7 . Find a complete set of mutually incongruent solution for the congruence $9x \equiv 12 \pmod{15}$. [4]

Q8 . Solve the quadratic congruence [4]
$x^2 = 11 \pmod{35}$

Q9.Solve 158x - 57y = 1 using continued fractions. Give the general Solution. [4]

Q10.Prove There are infinitely many primes of the form 4n+3. [4]

**BITS Pilani, Dubai Campus**
**Dubai International Academic City, Dubai**
**Fourth Year – First Semester 2011 – 2012**

**MATH C231 – Number Theory**
**Test 2 (Open Book)**

**Date: 22.12.2011** **Max. Marks: 20**
**Time: 50 Minutes** **Weightage: 20%**

Q1. Find the value of the following Legendre symbol

$(\frac{-46}{17})$ [2]

Q2. Find $\phi(42)$ and confirm it by finding a reduced set of residues mod(42) [2]

Q3. Show

$\sum_{d|n} d\mu(n) = 4$ for n=12 [2]

Q4. (i) Find $\sigma(496)$.

(ii) Goldbach conjecture states for each even number 2m there exist $m_1$ and $m_2$ such that $\sigma(m_1) + \sigma(m_2) = 2m$,

Find $m_1$ and $m_2$ for m=248 [3]

Q5. Write

$\frac{712}{1027}$ as a continued fraction [3]

Q6. Solve 56x +72y = 40 using continued fractions. Give the general Solution. [4]

Q7. Find out whether the quadratic congruence

$x^2 \equiv -1 (\mathrm{mod}\, 5^3)$ has a solution or not? If yes find it. [4]

**BITS Pilani, Dubai Campus**
**Dubai International Academic City, Dubai**
**Fourth Year – First Semester 2011 – 2012**

**MATH C231 – Number Theory**
**Test 1 (Closed Book)**

**Date: 3.11.2011** **Max. Marks: 25**
**Time: 50 Minutes** **Weightage: 25%**

Q1. For an arbitrary integer a show that

$$2 \mid a(a+1)$$ [2]

Q2. Find the greatest common divisor d of (56, 72) using Euclids division Lemma. Then find integers x and y such that

$$56x+72y=d$$ [2]

Q3.If m=13, then write the reduced residue system modulo m, and pair the numbers from the above system with their inverses. How many such pairs do you get? [2]

Q4. Find the general solution of $17x + 19y = 23$. [2]

Q5. Find the l.c.m of (111,1221) using prime factorization(by expressing the numbers 111 and 1221 as product of powers of primes).Use the lcm to find the gcd. [2]

Q6. Prove that if $r_1, r_2, \ldots\ldots\ldots r_{\varphi(m)}$ is a reduced residue system modulo m ,and m is odd , then

$$r_1 + r_2 + \ldots\ldots\ldots + r_{\varphi(m)} \equiv 0 (\text{mod } m)$$ [3]

Q7.Find a complete set of mutually incongruent solution for the following congruence $8x \equiv 10 (\text{mod } 30)$. [3]

Q8. What is the remainder when $5^{38}$ is divided by 11. [3]

Q9. Solve the simultaneous congruences
$x \equiv 1 \pmod 2$, $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$. [3]

Q10. Find the solution to $x^2 \equiv -1 (\text{mod } 29)$ [3]

**BITS Pilani, Dubai Campus**
**Dubai International Academic City, Dubai**
**Fourth Year – First Semester 2011 – 2012**

**MATH C231 – Number Theory**
**Quiz 2 (Closed Book)**

**Date: 12.12.2011** **Max. Marks: 7**
**Time: 20 Minutes** **Weightage: 07%**

Q1. Name 4 prime triplets of the form (p, p+2, p+6). [1]

Q2. Write $\frac{118}{303}$ as a finite continued fraction. [2]

Q3. Approximately how many primes are there before the number 645? [1]

Q4. Write the continued fraction [0; 1, 2, 3, 4, 3, 2, 1] as a rational number. [2]

Q5. Is 127 a Mersenne Number? Justify your answer. [1]

**BITS Pilani, Dubai Campus**
**Dubai International Academic City, Dubai**
**Fourth Year – First Semester 2011 – 2012**

**MATH C231 – Number Theory**
**Quiz 1 (Closed Book)**

**Date: 3.10.2011** **Max. Marks: 8**
**Time: 20 Minutes** **Weightage: 08%**

Q1. Show if a and b are odd integers then $a^2 - b^2$ is divisible by 8. [2]

Q2. Find the g.c.d of (306,657) using division algorithm. [2]

Q3.Find the l.c.m of (39,102,75) using prime factorization. [2]

Q4. Find the solution of $18x+5y = 48$ if it exists. [2]