BITS, Pilani – Dubai
Dubai International Academic City, Dubai

IV Year (ALL)
First Semester, 2009-2010

Comprehensive Examination

Course No: EA C482 | **Course Title: Fuzzy Logic and Applications**
Date: 22nd Dec 2009 | **Weightage: 40%**
Duration: 3 Hours | **Max. Marks. 80**

(Answer Parts A and B on separate answer books.)
(Answer the questions in the sequential order.)
(Answer all the parts of a question together.)

PART – A

1. a) Define the following in the context of fuzzy sets.
   i) Level Set   ii) Fuzzy Cardinality **[2 Marks]**

   b) Find the α-cut decomposition of the following fuzzy set, defined on the universe of discourse U = {a, b, c, d, e, f, g} by
   A = 0/a + 0.1/b + 0.3/c + 1/d + 0.4/e + 0.2/f + 0/g **[3 Marks]**

   c) Two fuzzy sets A, B are defined on the universe of discourse N, the set of natural numbers by
   A = 0/1 + 0.4/2 + 0.5/3 + 0.75/4 + 0.8/5 + 1/6
   B = 0.1/1 + 0.2/2 + 0.6/3 + 0.8/4 + 0.9/5 + 0.9/6
   Assuming that the non-standard operations for union, intersection and complement are defined as follows,
   $(A \cup B)(x) = \min(1, A(x)+B(x))$
   $(A \cap B)(x) = \max(0, A(x)+B(x)-1)$
   $A'(x) = 1 - A(x)$
   Find the $(A \cup B)$, $(A \cap B)$ and A'. **[2 + 2 + 1 Marks]**

2. A is a fuzzy set on the universe of discourse U = { a, b, c} defined by
   A = { 0/a + 0.1/b + 0.5/c }
   Find the fuzzification of A (i.e. F(A)) by fuzzy set K(x) on U defined by
   K(a) = { 0.1/a + 0.2/b }, K(b) = 0.3/b + 0.4/c } and K(c) = 0.6/a + 0.7/c} **[5 Marks]**

[P.T.O.]

3. a) Consider the ternary fuzzy relations R on U x V x W and S on V x W x Y, given by

$$R = \frac{0.1}{(a,c,x)} + \frac{0.3}{(a,d,x)} + \frac{0.5}{(b,c,x)} + \frac{0.9}{(b,d,x)}$$

$$S = \frac{0.2}{(c,x,y)} + \frac{0.4}{(c,x,z)} + \frac{0.6}{(d,x,y)} + \frac{0.7}{(d,x,z)}$$

where, U = {a, b} , V = {c, d}, W = {x}, and Y = {y, z}.

Find SoR of relations R and S, where o stands for composition. **[5 Marks]**

b) Let R be a BFR on U. Define another BFR on U denoted by Inv(R), as Inv(R)(x,y) = R(y,x) for all x, y in U. Show that R is symmetric if and only of Inv(R) is contained in R. **[5 Marks]**

4. Consider a temperature controller where the thermostat controls the heater fan. The input temperature(°C) has two linguistic values Cool and Warm, and the output fan speed(rpm) has two values, Low and High. Fuzzy set representation of different input and output values using triangular membership functions are as follows.

Cool(x) = 0 for x = 25°, 75°; Cool(x) = 1 for x = 50°
Warm(x) = 0 for x = 50°, 100°; Warm(x) = 1 for x = 75°
Low(x) = 0 for x = 0, 20; Low(x) = 1 for x = 10
High(x) = 0 for x = 10, 40; High(x) = 1 for x = 30

a) Draw the profile of the membership functions for temperature and fan speed, along with the equations representing them. **[2 Marks]**
b) Show the Fuzzy Rule Base for the above controller. **[3 Marks]**
c) For an input temperature value = 70°C, calculate the output fan speed. (Use Mean of Mean of Maxima method for defuzzification). **[5 Marks]**

5. A person indenting to buy a computer has five models M1, M2, M3, M4, M5 costing 20K, 27K, 36K, 42K, 45K, 51K respectively. His goal is G1: Good Performance. His constrains are C1: Affordability, C2: CPU Speed, C3: GPU Speed, C4: RAM and Disk Size. The goals and constraints are assumed to be fuzzy sets on these prices. Find out which model should he prefer based on the decision fuzzy set, computed from the data shown in the table below. Show the steps clearly.

Price	20K	27K	36K	42K	45K	51K
G1	0.1	0.2	0.5	0.6	0.8	1.0
C1	1.0	0.8	0.5	0.4	0.2	0.1
C2	0.3	0.5	0.8	1.0	0.4	0.2
C3	0.2	0.4	0.6	1.0	0.5	0.3
C4	0.1	0.3	0.5	0.6	0.8	0.7

[5 Marks]

[P.T.O.]

PART – B

6. (a) Prove the following by the truth table method (TTM).
   i) F ^ (G v H) = (F ^ G) v (F ^ H)
   ii) [F ^ (F→G)] → G **[4 Marks]**

   (b) Show that the following are invalid reasoning.
   i) [(F→G) ^ G] → F
   ii) [(F→G) ^ (¬F)] → ¬G **[4 Marks]**

   (c) Show, by derivation, that S => F, where
   $S = \{F_1 \rightarrow F_2;\ \neg F_2;\ \neg F_1 \rightarrow (F_3 \vee F_4);\ F_3 \rightarrow F_5;\ F_6 \rightarrow \neg F_5;\ F_6\}$
   $F = F_4$ **[3 Marks]**

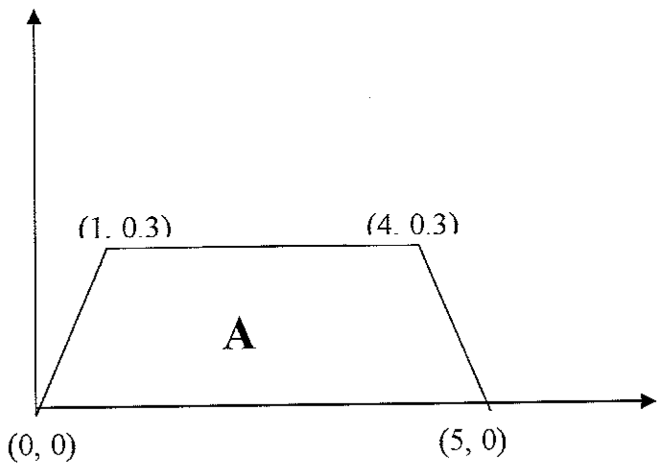
7. (a) Find the clausal form of:
   (Ex) [(Ey) {P(x, y) v R(x)} → (Ez) Q(z, x)] **[3 Marks]**

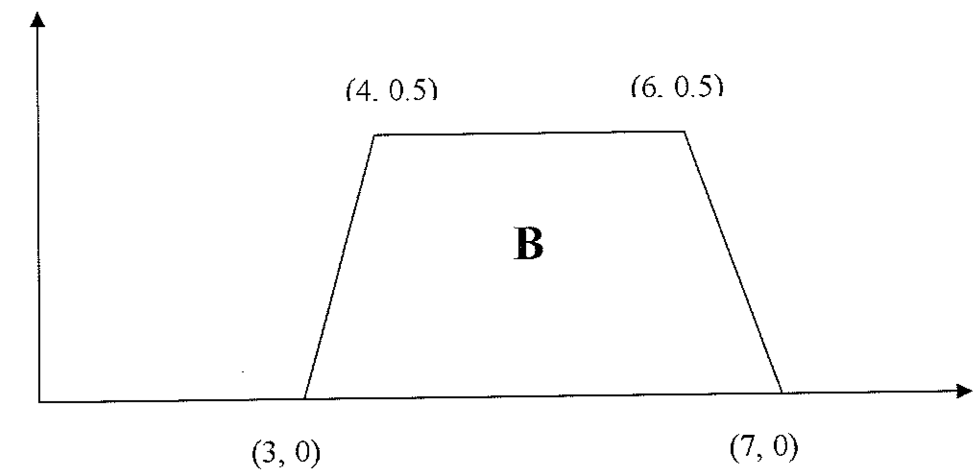
   (b) Identify and write down the free and bound variables in each of the following WFFs.
   i) [P(x, y) ^ (Ey) Q(y)] → (Ay) (Az) R(x, y, z)
   ii) (Ax) [P(z) → Q(u)]
   iii) P(x) → [Q(y) → (Ez) {R(u) → S(z)}]
   Also, in each case, write down all the assignments associated with an interpretation over a two element domain. **[6 Marks]**

8. (a) Apply the fuzzy Modus Ponens rule to deduce asuitable conclusion, given
   (i) If the temperature is high then the rotation is slow.
   (ii) The temperature is very high.
   The fuzzy sets High (H), and Very High are defined over the domain for temperature
   X = {30, 40, 50, 60, 70, 80, 90, 100} as follows.
   H = {(70, 1), (80, 1), (90, 0.3)}
   VH = {(90, 0.9), (100, 1)}.
   The fuzzy sets Slow (S), and Quite Slow (QS) are defined over the domain for rotation
   Y = {10, 20, 30, 40, 50, 60} as follows.
   S = {(30, 0.8), (40, 1), (50, 0.6)}
   QS = {(10, 1), (20, 0.8)}. **[6 Marks]**

[P.T.O.]

(b) Two fuzzy sets A and B are defined by their membership functions as shown below.

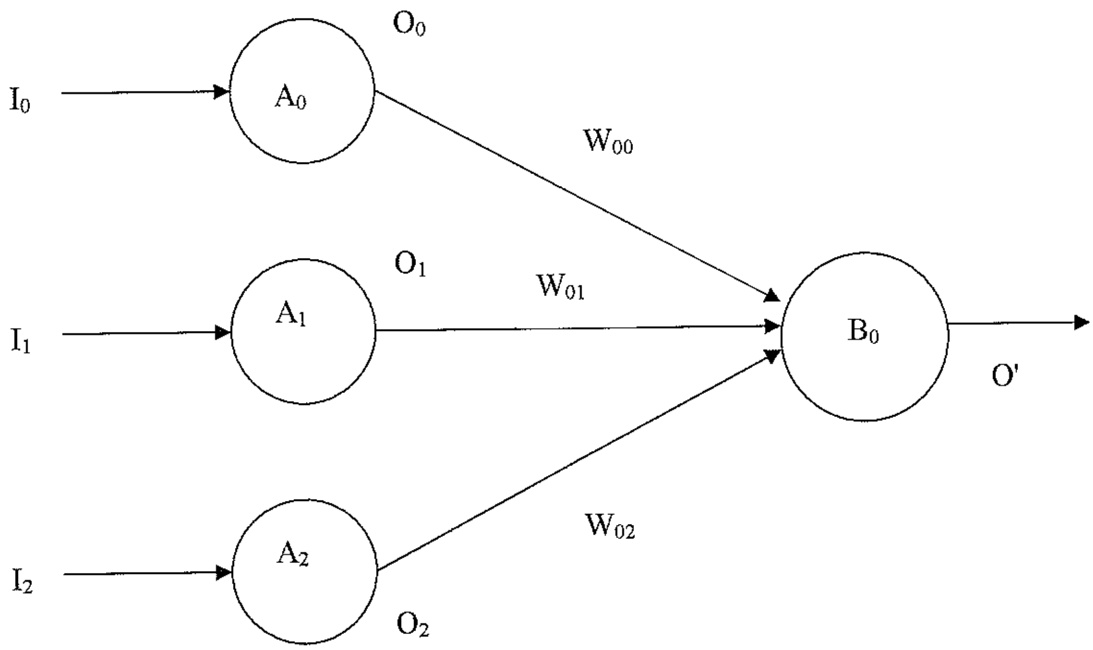




Defuzzify A U B using CoG method. **[6 Marks]**

[P.T.O.]

9. A fuzzy BP neural network with only input and output layers and having the following architecture is given.



The input layer neurons simply copy their input to the output. i.e. $O_i = I_i$, $i = 0, 1, 2$.
The output neuron works using the weights W_{00}, W_{01}, W_{02}, and the sigmoidal function f, the details of which are given below.

$W_{00} = (0.62, 0.505, 0.405)$, $W_{01} = (0.894, 0.634, 0.101)$, $W_{02} = (0.66, 0.567, 0.64)$,

$f(x) = 1 / (1+e^{-x})$

Find the output O' of the network for the following input.

$I_0 = (1, 0, 0)$, $I_1 = (1, 0.2, 0.3)$, $I_2 = (0, 0.1, 0.4)$. **[8 Marks]**

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BITS, Pilani – Dubai
Dubai International Academic City, Dubai

IV Year (ALL)
First Semester, 2009-2010

Test 2 (Open Book)
Text Book (printed or photocopy) or class notes/ppt slides may be used.

Course No: EA C482 | **Course Title: Fuzzy Logic and Applications**
Date: 15th Nov 2009 | **Weightage: 20%**
Duration: 50 minutes | **Max. Marks. 40**

(Answer the questions in the sequential order.)
(Answer all the parts of a question together.)
(All steps must be shown clearly.)

Note: The symbol ¬ stands for logical negation.

1. Using the Truth Table Method show that
   F v (G ^ H) = (F v G) ^ (F v H) **(4 marks)**

2. Show that F is a logical consequence of S by the method of Derivation.
   S = { $F_1 \rightarrow F_2$; ¬F_2 ; F_1 v F_2; $F_2 \rightarrow$ F3}
   F = F_3 **(6 marks)**

3. Given the definition of NAND(F, G) = ¬ (F ^ G), Show that
   a. F v G = NAND[NAND(F, F), NAND(G, G)]
   b. F → G = NAND[F, NAND(G, G)] **(3 +3 marks)**

4. For the WFF F given by F: (Ax)(Ey)P(x,y,c) , where x, y, are variables, c is a constant, Domain D={J,K,L} and I(c) =L. Find out whether F is true in the interpretation of P shown in the table.

x/y	J	K	L
J	1	0	0
K	0	1	0
L	0	0	1

(4 marks)

5. Show that (Ez) (Q(z) ^ R(z)) is not implied by the formulas (Ex) (P(x) ^ Q(x)) and (Ey) (P(y) ^ R(y)) by assuming a universe of discourse which has two elements.
   (4 marks)

[P.T.O]

6. (a) Prove that the following properties are satisfied in the three valued logic L_3.

   (i) $F \wedge (F \vee G) = F$

   (ii) $\neg (F \vee G) = (\neg F) \wedge (\neg G)$ **(2+2 marks)**

   (b) Show that LEM and LoC are not satisfied in L_3. **(1 mark)**

7. Prove that L_m is contained in L_n (i.e. L_n is a genuine generalisation of L_m) if and only if (m-1) divides (n-1). **(5 marks)**

8. Derive a suitable conclusion using Generalised Modus Ponens (GMP) on the following rule and fact.

   Rule: If x is A then y is B.

   Fact: x is A_1

   where $A = 1/a + 0.9/b + 0.1/c$

   $B = 1/p + 0.2/q$

   and $A_1 = 0.8/a + 0.9/b + 0.1/c$ **(6 marks)**

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BITS, Pilani – Dubai
Dubai International Academic City, Dubai

IV Year (ALL)
First Semester, 2009-2010

Test 1 (Closed Book)

Course No: EA C482 — **Course Title: Fuzzy Logic and Applications**
Date: 27th Sep 2009 — **Weightage: 25%**
Duration: 50 minutes — **Max. Marks. 50**

(Answer the questions in the sequential order.)
(Answer all the parts of a question together.)
(All steps must be shown clearly.)

1. For the fuzzy set A defined on U = {a, b, c, d } as
   $A= 0/a + 0.4/b + 0.6/c + 1/d$,
   find the following.
   a) Complement A' of A (2 marks)
   b) Core of A (1 mark)
   c) Support of A (1 mark)
   d) Scalar Cardinality of A (2 marks)
   e) Fuzzy Cardinality of A (4 marks)

2. a) All nonempty crisp sets are normal. Why? (2 marks)
   b) For the fuzzy set $A = 0/a + 0.3/b + 0.7/c$ defined on U = {a, b, c}
   find the normal form (A_N) of the fuzzy set A. (2 marks)
   c) Give an example of a fuzzy set over each of the following universal sets
   (i) U = N, the set of natural numbers {1, 2, 3 ...}
   (ii) U = R, the set of all real numbers. (4 marks)

3. For the fuzzy set A, B defined on U = {a, b, c} as
   $A= 0.1/a + 0.5/b + 1.0/c$, $B = 0.2/a + 0.4/b + 0.8c$
   a) Find RSM on fuzzy sets A, B, and A U B where $\alpha = 0.3$
   b) Prove that $\alpha A \cup \alpha B = \alpha(A \cup B)$ for all fuzzy sets A, B and $\alpha \in (0, 1]$
   (2+4=6 marks)

4. a) For any two fuzzy sets A and B on U, prove the following
   Con(A U B) = Con(A) U Con(B) (5 marks)
   supp(A U B) = supp(A) U supp(B) (5 marks)
   b) Find the α-cuts, corresponding to $\alpha = 0.5$, of the following fuzzy sets on
   U = {0, 1, 2, ..., 10}.
   (i) A defined by the membership function $A(x) = x / (x+2)$
   (ii) B defined by the membership function $B(x) = 2^{-x}$ (4 marks)

5. Consider three domains U, V, and W where U = {a, b, c}, V = {x, y, z}, and W = {p, q, r}. Consider the function f: U x V → W given by the following table.

f	x	y	z
a	p	q	r
b	r	p	q
c	q	r	p

Consider the fuzzy sets A and B on U and V respectively, where

A = 0.2/a + 0.7/b + 0.5/c

and B = 0.5/x + 0.3/y + 1.0/z

Find C = f(A x B) by Extension Principle. (6 marks)

6. If union, intersection, and complement are defined by the following non-standard operations, then show that the law of excluded middle and the law of contradiction are satisfied.

$(A \cup B)(x) = \min(1, A(x)+B(x))$

$(A \cap B)(x) = \max(0, A(x)+B(x)-1)$

$A'(x) = 1 - A(x)$ (6 marks)

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BITS, PILANI – DUBAI

Version A

FIRST SEMESTER 2009 – 2010
FOURTH YEAR

Course Code: EA 482
Course Title: Fuzzy Logic and Applications
Duration: 20 minutes

Date: 14.10.09
Max Marks: 16
Weightage: 8%

Name: . . . . **ID No:** . . . . **Sec / Prog:** . . . .

Instructions: Write your answers in the blank space provided after each question.
Use the margin to do rough work, if any.

1. R is a fuzzy relation on U x V and S on V x W, where U = {a, b, c}, V = {p, q, r}, and W = {x, y, z}, given in the matrix form by

$$R = \begin{matrix} 1.0 & 0.4 & 0.5 \\ 0.3 & 0.0 & 0.7 \\ 0.6 & 0.8 & 0.2 \end{matrix} \qquad S = \begin{matrix} 0.2 & 1.0 & 0.4 \\ 0.0 & 0.5 & 0.3 \\ 0.0 & 0.9 & 0.4 \end{matrix}$$

Find S o R. **[4 marks]**

2. Give an example of a fuzzy relation that is reflexive and symmetric. **[2 marks]**

[PTO]

3. T is a ternary fuzzy relation on U x V x W given by

$T = 0.3/(a, x, \&) + 0.4/(b, x, *) + 0.2/(a, y, \&) + 0.7/(a, y, *) + 1.0/(b, y, \&)$

where U = {a, b}, V = {x, y, z}, and W = {*, &}

Find T_{12}^{3}. **[4 marks]**

4. Give a counter example to show that the complement of the projection on U of R is not (in general) same as the projection on U of the complement of R.

[2 marks]

5. R is a BFR on U x V, where U = {a, b, c} and V = {x, y}. R_1 (the projection of R on U) is given by $R_1 = (0.2, 0.6, 1.0)$. R_2 (the projection of R on V) is given by $R_2 = (0.5, 0.3)$. Find the cylindrical closure $[R_1, R_2]$ of R_1 and R_2. **[4 marks]**

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BITS, PILANI – DUBAI

Version B

FIRST SEMESTER 2009 – 2010
FOURTH YEAR

Course Code: EA 482
Course Title: Fuzzy Logic and Applications
Duration: 20 minutes

Date: 14.10.09
Max Marks: 16
Weightage: 8%

Name: . . . . **ID No:** . . . . **Sec / Prog:** . . . .

Instructions: Write your answers in the blank space provided after each question.
Use the margin to do rough work, if any.

1. R is a fuzzy relation on U x V and S on V x W, where U = {a, b, c}, V = {p, q, r}, and W = {x, y, z}, given in the matrix form by

$$R = \begin{matrix} 0.2 & 0.6 & 0.5 \\ 0.1 & 0.7 & 0.4 \\ 0.8 & 0.2 & 1.0 \end{matrix} \qquad S = \begin{matrix} 0.7 & 0.3 & 0.5 \\ 0.6 & 0.9 & 1.0 \\ 0.2 & 0.4 & 0.8 \end{matrix}$$

Find S o R. **[4 marks]**

2. Give an example of a fuzzy relation that is reflexive and symmetric. **[2 marks]**

[PTO]

3. T is a ternary fuzzy relation on U x V x W given by
$T = 0.4/(a,x,*) + 0.3/(b,x,\&) + 0.2/(b,x,*) + 0.8/(a,y,*) + 0.1/(b,y,*) + 0.5/(b,y,\&)$
where $U = \{a, b\}$, $V = \{x, y, z\}$, and $W = \{*, \&\}$
Find T_{12}^{3}. **[4 marks]**

4. Give a counter example to show that the complement of the projection on U of R is not (in general) same as the projection on U of the complement of R.
[2 marks]

5. R is a BFR on U x V, where $U = \{a, b, c\}$ and $V = \{x, y\}$. R_1 (the projection of R on U) is given by $R_1 = (0.7, 0.4, 0.3)$. R_2 (the projection of R on V) is given by $R_2 = (0.2, 0.8)$. Find the cylindrical closure $[R_1, R_2]$ of R_1 and R_2. **[4 marks]**

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