

Name:

Id. No.:

BITS, PILANI - DUBAI
International Academic City, Dubai
IV Year (CS/EIE)
Semester I 2007 - 2008
SURPRISE QUIZ III (Closed Book)

Course No.: EA UC482

Course Title: Fuzzy Logic & Applications

Date: 26 November 2007

Time: 10 Minutes

M.M. = 10 (5 %)

-
1. Identify the following tautologies used in propositional logic:
 - (i) $((P \Rightarrow Q) \wedge (Q \Rightarrow R)) \Rightarrow (P \Rightarrow R)$
 - (ii) $(\bar{Q} \wedge (P \Rightarrow Q)) \Rightarrow \bar{P}$
 - (iii) $(P \wedge (P \Rightarrow Q)) \Rightarrow Q$
 2. Largest intersection and the smallest union operations in fuzzy set theory are:
 - (i) Algebraic product and algebraic sum operation respectively.
 - (ii) Min and max operations respectively.
 - (iii) Drastic intersection and drastic union operation respectively.
 - (iv) Bounded Difference and bounded sum operation respectively.
 3. A fuzzy relation is known as 'Pre-order', when it is ...
 - (i) Reflexive and Symmetric
 - (ii) Reflexive and max-min transitive
 - (iii) Symmetric and max-min transitive
 - (iv) Reflexive, Symmetric and max-min transitive
 4. Give the truth table for the $p \Leftrightarrow q$ (Double Implication) operation.
 5. Give the proof of law of absorption $p \wedge (p \vee q) = p$.

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Date: 26 November 2007

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1. Which one of the following is not true for aggregation operations in fuzzy set theory?
 - (i) All aggregation operations are necessarily symmetric functions.
 - (ii) OWA operations are aggregation operations that cover the entire interval between min and max operations.
 - (iii) Generalized means are aggregation operations that cover the entire interval between min and max operations.
 - (iv) All aggregation operations are monotonic increasing in all their arguments.

2. Which aggregation operation gives a meaningful result for normalized data?
 - (i) Arithmetic Mean
 - (ii) Harmonic Mean
 - (iii) Geometric Mean
 - (iv) All of the above

3. Triangular membership functions are preserved under:
 - (i) Multiplication
 - (ii) Division
 - (iii) Subtraction
 - (iv) All of the above

4. Lukasiewicz used only negation and implication as primitives in his n-valued logic. Write down his definition of other fuzzy operations i.e. \vee , \wedge , and \Leftrightarrow in terms of negation and implication.

5. Identify the tautology and contradiction in $(P \vee \bar{P})$ and $(P \wedge \bar{P})$.

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SURPRISE QUIZ II (Closed Book)

Course No.: EA UC482

Course Title: Fuzzy Logic & Applications

Date: 29 October, 2007

Time: 10 Minutes

M.M. = 10 (5 %)

NOTE: Choose the most appropriate one.

[10 × 1 = 10]

1. Which one of the following is not an essential condition for a fuzzy union operation?
 - (i) $b \leq d$ implies $u(a, b) \leq u[a, d]$
 - (ii) $u(a, b) = u(b, a)$
 - (iii) $u(a, a) > a$
 - (iv) $u(a, u(b, d)) = u(u(a, b), d)$

2. The standard fuzzy intersection is the only
 - (i) Sub idempotent t-norm.
 - (ii) Idempotent t-norm.
 - (iii) Super idempotent t-norm.
 - (iv) All of the above

3. The operation needed to be performed on the special fuzzy sets defined by ${}_{\alpha}A(x) = \alpha \cdot A(x)$, in order to obtain the original fuzzy set A is
 - (i) Standard Fuzzy Union
 - (ii) Standard Fuzzy intersection
 - (iii) Bounded Sum operation
 - (iv) Max-min composition

4. Which of the following properties does not apply to the standard Fuzzy complement, intersection and union operations?
 - (i) Satisfy the cutworthy and strong cutworthy properties.
 - (ii) Prevent the compounding of errors of the operands.
 - (iii) Context-dependent.
 - (iv) All of the above properties apply to standard fuzzy operations.

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M.M. = 10 (5 %)

NOTE: Choose the most appropriate one.

[10 × 1 = 10]

1. **Classical (standard) fuzzy complement is one which**
 - (i) satisfies the axiomatic skeleton for fuzzy complements.
 - (ii) is continuous fuzzy complement.
 - (iii) is involutive fuzzy complement.
 - (iv) satisfies all the above conditions

2. **Which one of the following is not an essential condition for a fuzzy union operation?**
 - (i) $b \leq d$ implies $u(a, b) \leq u[a, d]$
 - (ii) $u(a, b) = u(b, a)$
 - (iii) $u(a, a) > a$
 - (iv) $u(a, u(b, d)) = u(u(a, b), d)$

3. **Which one of the following is a correct relation?**
 - (i) $u_{\max}(a, b) \geq \min(1, a+b) \geq a+b-ab \geq \max(a, b)$
 - (ii) $\max(a, b) \leq \min(1, a+b) \leq a+b-ab \leq u_{\max}(a, b)$
 - (iii) $u_{\max}(a, b) \leq \min(1, a+b) \leq a+b-ab \leq \max(a, b)$
 - (iv) $a+b-ab \leq \min(1, a+b) \leq u_{\max}(a, b) \leq \max(a, b)$

4. **Which one of the following statements is incorrect?**
 - (i) i_{\min} and u_{\max} operations are dual of each other with respect to any fuzzy complement c .
 - (ii) \min and \max operations are dual of each other with respect to only standard fuzzy complement.
 - (iii) Given an involutive fuzzy complement c and an increasing generator g of c , the t -norm and t -conorm generated by g are dual with respect to c .
 - (iv) Let $\langle i, u, c \rangle$ be a dual triple that satisfy the law of excluded middle and the law of contradiction, then $\langle i, u, c \rangle$ does not satisfy the distributive laws.

BITS, PILANI - DUBAI
International Academic City, Dubai
IV Year (EEE/ME/CS/EIE)
Semester I 2007 - 2008
TEST I (Closed Book)

Course No.: EA UC482

Course Title: Fuzzy Logic & Applications

Date: September 30, 2007

Time: 50 Minutes

M.M. : 40 (20 %)

Question 1

[2 × 5 = 10]

- (a) Give the definition of "Contrast-Intensification" operation performed on linguistic fuzzy sets.
- (b) Obtain the max-product composition of sets $A = [0.5 \ 1 \ 0.6]$ and
- $$B = \begin{bmatrix} 0.02 & 0.2 & 0.16 \\ 0.04 & 0.4 & 0.32 \\ 0.1 & 1 & 0.8 \end{bmatrix}.$$
- (c) Cite any three reasons, which make fuzzy variables more attuned to reality than crisp variables.
- (d) If sets A and B are defined as follows then obtain the absolute complement of A and also complement of A with respect to B.
 $A = 0.3/0 + 0.4/1 + 0.6/2 + 0.7/3$ and $B = 0.4/0 + 0.6/1 + 0.8/2 + 0.8/3$
- (e) Let the universe of discourse be $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = 0.3/1 + 0.5/2 + 1/3 + 0.7/4 + 0.2/5$, then obtain $|A|$ (cardinality) and $\|A\|$ (relative cardinality).

Question 2

[5 + 5 = 10]

- (a) Let $X = \{0, 1, 2, \dots, 6\}$, and let two fuzzy subsets, A and B, of X be defined by:

x	0	1	2	3	4	5	6
$\mu_A(x)$	1	0.7	0	1	0.5	0	0.4
$\mu_B(x)$	0.9	0.7	1	0.2	0.8	0.3	0

Find

- (i) \bar{B}
 - (ii) $\overline{A \cup B}$
 - (iii) $\overline{A \cap B}$
 - (iv) What is the Core of A?
 - (v) What is the height of fuzzy set B?
- (b) Suppose we have a universe of integers, $Y = \{1, 2, 3, 4, 5\}$. We define the following terms as a mapping onto Y:

$$\text{"Small"} = \left\{ \frac{1}{1} + \frac{0.8}{2} + \frac{0.6}{3} + \frac{0.4}{4} + \frac{0.2}{5} \right\}$$

$$\text{"Large"} = \left\{ \frac{0.2}{1} + \frac{0.4}{2} + \frac{0.6}{3} + \frac{0.8}{4} + \frac{1}{5} \right\}$$

Then obtain the fuzzy set representation of "not very small and not very, very large"

Question 3

[5 + 5 = 10]

- (a) Show the graphical representation of two types of π membership functions and write down the mathematical definitions of these functions, clearly mentioning the number of parameters needed to describe them completely.
- (b) What is the use of 'Hedges' in the fuzzy set theory? Give example of any two commonly used hedges and define them clearly with the help of mathematical expression and graphical representation.

Question 4

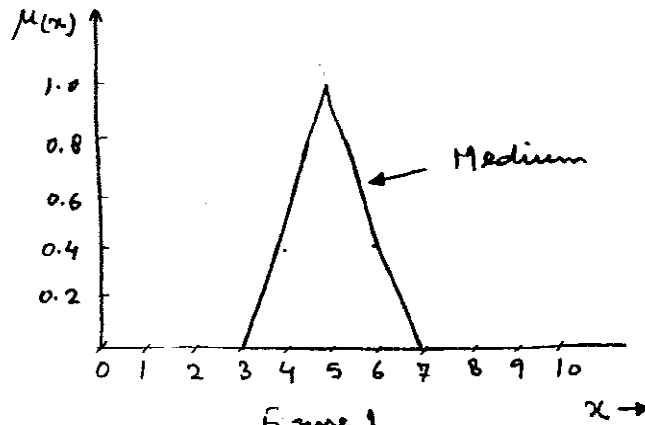
[5 + 5 = 10]

- (a) Obtain the max-min composition of fuzzy relations R and S (i.e. $R \circ S$), if

S	α	β	γ
a	0.9	0.0	0.3
b	0.2	1.0	0.8
c	0.8	0.0	0.7
d	0.4	0.2	0.3

R	a	b	c	d
1	0.1	0.2	0.0	1.0
2	0.3	0.3	0.0	0.2
3	0.8	0.9	1.0	0.4

- (b) Suppose expression "medium" for a fuzzy variable is defined as given in figure 1, then draw the graphs for expressions "more medium" and "less medium", if they are defined by the relations $\text{more medium} = \text{CON}(\text{medium})$; and $\text{less medium} = \text{DIL}(\text{medium})$



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IV Year (EEE/ME/CS/EIE)
Semester I 2007 - 2008
TEST II (Open Book)

Course No.: EA UC482

Course Title: Fuzzy Logic & Applications

Date: 18th November 2007

Time: 50 Minutes

M.M. = 40 (20 %)

NOTE: Text Book, Reference Books, as well as Class Notes can be used for answering.

1. A binary relation between $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4\}$ is defined with the help of following relation matrix: (05)

	y_1	y_2	y_3	y_4
x_1	0.1	0.2	0.3	0.2
x_2	0.2	0.1	0.5	0.7
x_3	0.5	1.0	0.8	0.6

Obtain

- (i) Projections $[R \downarrow X]$ and $[R \downarrow Y]$
 - (ii) Cylindrical Extensions $[R \uparrow X]$ and $[R \uparrow Y]$
 - (i) Join and Meet of the Cylindrical Extensions obtained in (ii).
2. There is a fuzzy rule in the following:

x and y are approximately equal

This rule can be represented by following relation $R(x, y)$.

$x \backslash y$	1	2	3	4
1	1	0.5	0	0
2	0.5	1	0.5	0
3	0	0.5	1	0.5
4	0	0	0.5	1

Assume that the variables x and y are positive integers in $[1, 4]$. If one of the variable is given as a singleton, $x = 2$, then apply the modus ponens type of reasoning using standard max-min composition to infer the value of y . (08)

3. What are the criteria for choosing a combining operator in a Fuzzy System? (05)

4. There is a fuzzy rule in the following:

If temperature is high, then humidity is fairly high

If T and H are universe of discourse of temperature and humidity, respectively, variables t and h are defined as $t \in T$ and $h \in H$. Fuzzy terms "high" and "fairly high" are represented as A and B respectively and defined below:

$$\begin{aligned} A &= \text{"high"}, & A &\subseteq T \\ B &= \text{"fairly - high"}, & B &\subseteq H \end{aligned}$$

Membership of A in T (temperature):

t	20	30	40
$\mu_A(t)$	0.1	0.5	0.9

Membership of B in H (humidity):

h	20	50	70	90
$\mu_B(h)$	0.2	0.6	0.7	1

(a) Use Mamdani's implication rule $R = A \rightarrow B = A \times B = \int_{X \times Y} (\mu_A(t) \wedge \mu_B(h)) / (t, h)$ and obtain the membership function of the above fuzzy rule. (07)

(b) Obtain the information about the humidity when there is the following premise about the temperature:

"Temperature is fairly high"

Where fuzzy term 'fairly high (A')' $\subseteq T$, has the following membership function in T (temperature):

t	20	30	40
$\mu_{A'}(t)$	0.01	0.25	0.81

(05)

5. Sally is nearsighted and colorblind. When she goes to a local grocery where fruits are placed on high shelves, she cannot see them very well. She can only recognize the size and blurred shape of the fruits. Her knowledge about the fruits can be represented by the following fuzzy relation.

	<i>tan gerine</i>	<i>apple</i>	<i>pineapple</i>	<i>watermelon</i>	<i>strawberry</i>
<i>long</i>	0	0	0.3	0	0.8
<i>round</i>	0.9	1.0	0.3	1.0	0.2
<i>large</i>	0.2	0.4	0.7	1.0	0.1

Guess a fruit that Sally describes as $\begin{matrix} \textit{long} & \textit{round} & \textit{large} \\ [0 & 0.7 & 1.0] \end{matrix}$ (05)

6. HYPOTHESES: Either Arlen is lying or Brewster was in Mexico in April or Crawford was not a blackmailer. If Brewster was not in Mexico in April, then either Arlen is telling the truth or Crawford was a blackmailer.

CONCLUSION: Brewster must have been in Mexico in April.

Using the above mentioned hypotheses, establish the veracity of the given conclusion. (05)

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Semester I 2007 - 2008
IV Year (EEE/ME/CS/EIE)

COMPREHENSIVE EXAMINATION (Closed Book)

Course No.: EA UC482

Course Title: Fuzzy Logic & Applications

Date: December 30, 2007

Time: 3 Hours

M.M. = 80 (40 %)

- NOTE:**
- (i) Answer all questions. Assume missing data, if any and make a note about it.
 - (i) Answer all parts of a question in continuation.
 - (ii) Do not leave any blank page(s) in between the answers.

QUESTION 1 GIVE THE MOST APPROPRIATE ANSWER. (10 × 1 = 10)

- (1) Which one of the following is not true for aggregation operations in fuzzy set theory?
 - (i) All aggregation operations are necessarily symmetric functions.
 - (ii) OWA operations are aggregation operations that cover the entire interval between min and max operations.
 - (iii) Generalized means are aggregation operations that cover the entire interval between min and max operations.
 - (iv) All aggregation operations are monotonic increasing in all their arguments.

- (2) Largest intersection and the smallest union operations in fuzzy set theory are:
 - (i) Algebraic product and algebraic sum operation respectively.
 - (ii) Min and max operations respectively.
 - (iii) Drastic intersection and drastic union operation respectively.
 - (iv) Bounded Difference and bounded sum operation respectively.

- (3) The basic difference between the probabilistic reliability theories and possibilistic reliability theories is
 - (i) Assumption of dichotomous states
 - (ii) Assumption of fuzzy states
 - (iii) Characterization of the behavior with respect to the two critical states.
 - (iv) All of the above

- (4) Ordering of fuzzy numbers can be done using
 - (i) Defining Hamming distance on the set of all fuzzy numbers
 - (ii) Based on α -cuts
 - (iii) Defining Euclidean distance on the set of all fuzzy numbers
 - (iv) All of the above

- (5) If A is a Fuzzy set then ${}^{\alpha}A$ and ${}_{\alpha}A$ are respectively:
- Fuzzy set and Crisp set
 - Crisp set and Fuzzy set
 - Both Fuzzy sets
 - Both Crisp sets
- (6) Give one example of a fuzzy complement function that is continuous but not involutive.
- (7) What is the general expression of 'Sugeno' class of involutive fuzzy complements?
-
- (8) The Yager t -conorm for which $\omega = 1$, is very weak and indicates perfect interchangeability between the two arguments, identify it.
- (9) Which t -norm is referred as strict Archimedean t -norm?
- (10) A fuzzy set can be visualized as a point in an n -dimensional cube in n -space. Inside this cube the 'Cardinality' of any fuzzy set is defined as

QUESTION 2

(5 × 2 = 10)

- (a) If sets A and B are defined as follows then obtain the absolute complement of A and also complement of A with respect to B .
- $$A = 0.3/0 + 0.4/1 + 0.6/2 + 0.7/3 \text{ and } B = 0.4/0 + 0.6/1 + 0.8/2 + 0.8/3$$
- (b) Which are the two laws of crisp set theory that are violated in fuzzy set theory? Justify your answers.
- (c) If α -cuts of a fuzzy set A are as follows then obtain the fuzzy set A .
- $${}^2A = 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$
- $${}^4A = 0/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$
- $${}^6A = 0/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$
- $${}^8A = 0/x_1 + 0/x_2 + 0/x_3 + 1/x_4 + 1/x_5$$
- $${}^1A = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$
- (d) What are the axioms included in the axiomatic skeleton for fuzzy intersections? Write down their mathematical definitions.
- (e) Let fuzzy sets $A = 0.2/1 + 0.4/3 + 0.6/5$ and $B = 0.1/2 + 0.3/4 + 0.5/6$; then find $A \oplus B$.

QUESTION 3

(5 + 5 = 10)

- (a)
- (i) In a class of 10 students (the universal set), 3 students speaks German to some degree, namely Alice to degree 0.7, Bob to degree 1.0, Cathrine to degree 0.4. What is the size of the subset A of German speaking students in the class?
 - (ii) What is the support of the fuzzy set A in part (a)?
 - (iii) Is A normal?
 - (iv) What is $^{0.5}A$ (the 0.5-cut of the fuzzy set A)?
 - (v) What is $^{0.4+}A$ (the strong 0.4-cut of the fuzzy set A)?
- (b) Describe a 'Type-n Fuzzy Set' and a 'Level-k Fuzzy Set'. Give graphical representation of one example of each.

QUESTION 4

(3 + 7 = 10)

- (a) Define 'Concentration (CON)', 'Dilation (DIL)', and 'Contrast Intensification (INT)' operations. If $A = 0.2/1 + 0.4/2 + 0.6/4 + 1/5$, then obtain , CON(A), DIL(A), and INT(A).
- (b) Suppose you have a collection (universe) of five data points,
- $$X = (x_1, x_2, x_3, x_4, x_5)$$
- and these data points show similarity to one another according to the following relation.

$$R_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Check whether it is an equivalent relation or not? If it is not then convert it into an equivalent relation and classify the data points into different classes.

QUESTION 5

(4 + 6 = 10)

- (a) How many GUI tools are available in Fuzzy Logic Toolbox of MATLAB for building, editing, and observing any fuzzy inference system? Describe their functions briefly.
- (b) Fuzzy Numbers 3 (FN3), Fuzzy Numbers 6 (FN6), and Fuzzy Numbers 7 (FN7) are defined as $FN3 = 0.3/1 + 0.7/2 + 1.0/3 + 0.7/4 + 0.3/5$, $FN6 = 0.2/4 + 0.6/5 + 1.0/6 + 0.6/7 + 0.2/8$ and $FN7 = 0.2/5 + 0.6/6 + 1.0/7 + 0.6/8 + 0.2/9$ respectively on an universe of discourse $X = 1 + 2 + \dots + 60$. Obtain
- (i) $FN3 + FN7$,
 - (ii) $FN7 - FN3$,
 - (iii) $FN7 * FN3$
 - and (iv) $FN6 / FN3$

QUESTION 6

(5 + 5 = 10)

- (a) Suppose we want to measure the value of a microprocessor to a potential client. In conducting this evaluation, the client suggests that certain criteria are important. They can include performance (MIPS), cost (AED), availability (AV), and software

(SW). A particular microprocessor (CPU) has been introduced into the market. It is measured against these criteria and given ratings categorized as excellent (e), superior(s), adequate (a), and inferior (i). Suppose the microprocessor just introduced has been assigned the following relation based on the consensus of the design team:

$$R = \begin{matrix} & & e & s & a & i \\ \begin{matrix} MIPS \\ AED \\ AV \\ SW \end{matrix} & & \begin{bmatrix} 0.1 & 0.3 & 0.4 & 0.2 \\ 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0.6 & 0.2 & 0.1 \\ 0.1 & 0.4 & 0.3 & 0.2 \end{bmatrix} \end{matrix}$$

If the evaluation team applies a scoring factor of 0.4 for performance, 0.3 for cost, 0.2 for availability, and 0.1 for software then obtain the evaluation vector of the new microprocessor.

- (b) Suppose that an individual needs to decide which of four possible jobs (described below) to choose. His or her goal is to choose a job that offers a high salary under the constraints that the job is interesting and within close driving distance. Apply the fuzzy logic to arrive at an appropriate conclusion.

Job	Salary	Nature of job	Distance from home
I	\$40,000	Less interesting	27 miles
II	\$45,000	Interesting	7.5 miles
III	\$50,000	Least interesting	12 miles
IV	\$60,000	Least interesting	2.5 miles

QUESTION 7

(5 + 5 = 10)

Write down the comprehensive notes on the relevance of fuzzy set theory to any two of the following:

- (i) Civil engineering
- (ii) Databases and Information Retrieval Systems
- (iii) Reliability Theory

QUESTION 8

(10)

Let A and B be two universe representing numeric ratings (from 1 to 10) of quality of service given in a restaurant and amount of tip offered respectively. Let quality of service be a fuzzy subset of A defined as

$$\text{Quality of service is good} = .25/6 + 0.5/7 + 0.75/8 + 1/9 + 1/10$$

and amount of tip be a fuzzy subset of B defined as

$$\text{Amount of tip is high} = 0.25/7 + 0.5/8 + 0.75/9 + 1/10$$

We are also given the fuzzy implication:

$$\text{Quality of service is good} \rightarrow \text{Amount of tip is high}$$

Using standard sequence implication, defined as to be true whenever the consequent is truer than the antecedent, i.e., $t(p \rightarrow q) = 1$ whenever $t(p) < t(q)$, generate the fuzzy implication relation matrix.

Suppose the quality of service is very good, apply the max-min composition to the standard sequence implication relation to guess about the amount of tip to be offered.