BITS, PILANI - DUBAI CAMPUS
Knowledge Village, Dubai

Semester I 2006 - 2007
IV Year (EEE/EIE)

COMPREHENSIVE EXAMINATION (Closed Book)

Course No.: EA UC482 **Course Title: Fuzzy Logic & Applications**

Date: December 26, 2006 **Time: 3 Hours** **M.M. = 80 (40 %)**

NOTE: (i) Answer all the questions.
(i) Answer all parts of a question in continuation.
(ii) Do not leave any blank page(s) in between the answers.

QUESTION 1 **Choose the most appropriate one.** **(10 × 1 = 10)**

(1) Which one of the following is not true for aggregation operations in fuzzy set theory?
(i) All aggregation operations are necessarily symmetric functions.
(ii) OWA operations are aggregation operations that cover the entire interval between min and max operations.
(iii) Generalized means are aggregation operations that cover the entire interval between min and max operations.
(iv) All aggregation operations are monotonic increasing in all their arguments.

(2) The equilibrium of a complement c is that degree of membership in a fuzzy set A which equals the degree of membership
(i) in the complement of A
(ii) in the complement of P(A)
(iii) in the Complement of universe of discourse.
(iv) None of the above

(3) Sugeno class of fuzzy complements is defined by
(i) $c_\lambda(a)=\frac{1-a}{1+\lambda a}$, where $\lambda \in (-1, \infty)$
(ii) $c(a)=\frac{1}{2}(1+\cos \pi a)$,
(iii) $c(a)=\begin{cases}1 & for \quad a \le t \\ 0 & for \quad a > t\end{cases}$
(iv) All of the above

(4) **Largest intersection and the smallest union operations in fuzzy set theory are:**

(i) Algebraic product and algebraic sum operation respectively.
(ii) Min and max operations respectively.
(iii) Drastic intersection and drastic union operation respectively.
(iv) Bounded Difference and bounded sum operation respectively.

(5) **If A and B are crisp sets, then operation D = A ⊕ B will be equivalent to a**

(i) Union Operation
(ii) Intersection Operation
(iii) A - B Operation
(iv) A/B Operation

(6) **Which one of the following is not an essential condition for a fuzzy union operation?**

(i) $b \le d$ implies $u(a, \ b) \le u[a, \ d]$
(ii) $u(a, \ b) = u(b, \ a)$
(iii) $u(a, \ a) > a$
(iv) $u(a, \ u(b, \ d)) = u(u(a, \ b), \ d)$

(7) **A Fuzzy Relation given as** $M_R = \begin{bmatrix} 0 & .7 & .4 \\ .7 & 0 & .1 \\ .4 & .1 & 0 \end{bmatrix}$ **is**

(i) Reflexive and symmetric
(ii) Antireflexive and symmetric
(iii) Reflexive and antisymmetric
(iv) Antireflexive and antisymmetric

(8) **Which aggregation operation gives a meaningful result for normalized data?**

(i) Arithmetic Mean
(ii) Harmonic Mean
(iii) Geometric Mean
(iv) All of the above

(9) **Triangular membership functions are preserved under:**

(i) Multiplication
(ii) Division
(iii) Subtraction
(iv) All of the above

(10) **A fuzzy relation is known as 'Pre-order', when it is ...**

(i) Reflexive and Symmetric
(ii) Reflexive and max-min transitive
(iii) Symmetric and max-min transitive
(iv) Reflexive, Symmetric and max-min transitive

QUESTION 2

(5 × 2 = 10)

Let fuzzy sets $A = 0.2/1 + 0.4/3 + 0.6/5$ and $B = 0.1/2 + 0.3/4 + 0.5/6$; then find the following:

(i) A × B
(ii) A B
(iii) CON (A + B)
(iv) A ⊕ B
(v) A ⊖ B

Question 3

(5 + 5 = 10)

(a) Suppose we have a universe of integers, Y = {1, 2, 3, 4, 5}. We define the following terms as a mapping onto Y:

"Small" = $\{1/1 + 0.8/2 + 0.6/3 + 0.4/4 + 0.2/5\}$

"Large" = $\{0.2/1 + 0.4/2 + 0.6/3 + 0.8/4 + 1/5\}$

Then obtain the fuzzy set representation of:

(i) "not very small and not very, very large" and

(ii) "Intensely small"

(b) Show the graphical representation of two types of π membership functions and write down the mathematical definitions of these functions, clearly mentioning the number of parameters needed to describe them completely.

Question 4

(10)

Suppose you have a collection (universe) of five data points,

$$X = (x_1,\ x_2,\ x_3,\ x_4,\ x_5)$$

and these data points show similarity to one another according to the following relation.

$$R_1 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Check whether it is an equivalent relation or not? If it is not then convert it into an equivalent relation and classify the data points into different classes.

Question 5

(10)

Categorize the classical pattern recognition approaches into different categories. What are the limitations of classical pattern recognition systems? Describe the relevance of fuzzy logic to the pattern recognition systems to overcome these problems.

Question 6

(5 + 5 = 10)

(a) What is the subject of decision making theory? Explain the possible applications of fuzzy sets within the field of decision making.

(b) Suppose that an individual needs to decide which of four possible jobs (described below) to choose. His or her goal is to choose a job that offers a high salary under the constraints that the job is interesting and within close driving distance. Apply the fuzzy logic to arrive at an appropriate conclusion.

Job	Salary	Nature of job	Distance from home
I	$40,000	Less interesting	27 miles
II	$45,000	Interesting	7.5 miles
III	$50,000	Least interesting	12 miles
IV	$60,000	Least interesting	2.5 miles

Question 7

(7 + 3 = 10)

(a) **HYPOTHESES:** If the budget is not cut, then a necessary and sufficient condition for prices to remain stable is that taxes will be raised. Texas will be raised only if the budget is not cut. If prices remain stable, then taxes will not be raised.

CONCLUSION: Taxes will not be raised.

Using the above mentioned hypotheses, establish the veracity of the given conclusion.

(b) Write down the DNF and CNF of $((P \wedge (P \Rightarrow Q)) \Rightarrow Q)$.

Question 8

(5 + 5 = 10)

(a) Name and give mathematical definition of any two aggregation operations that cover the entire range between the largest intersection and the smallest union.

(b) For three given intervals, $S_1 = [-1, \quad 0]$, $S_2 = [-1, \quad 2]$, and $S_3 = [2, \quad 10]$, obtain

(i) $S_2 \cap S_3$

(ii) $S_2 \cup S_3$

(iii) $w\{S_1\}$

(iv) $|S_2|$; and

(v) $m\{S_3\}$

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, Dubai

IV Year (EEE/EIE)

Semester I 2006 - 2007

TEST II (Open Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: 10th December 2006 Time: 50 Minutes M.M. = 40 (20 %)

NOTE: Text Book, Reference Books, as well as Class Notes can be used for answering.

Question 1

There is a fuzzy rule in the following: **[5 + 5 = 10]**

If temperature is high, then humidity is fairly high

If **T** and **H** are universe of discourse of temperature and humidity, respectively, variables **t** and **h** are defined as $t \in T$ and $h \in H$. Fuzzy terms "**high**" and "**fairly high**" are represented as **A** and **B** respectively and defined below:

$A =$"$high$", $A \subseteq T$

$B =$"$fairly-high$", $B \subseteq H$

Membership of **A** in **T** (temperature):

t	20	30	40
$\mu_A(t)$	0.1	0.5	0.9

Membership of B in H (humidity):

h	20	50	70	90
$\mu_B(h)$	0.2	0.6	0.7	1

(a) Mamdani proposed the following definition of fuzzy implication

$$R = A \rightarrow B = A \times B = \int_{T \times H} (\mu_A(t) \wedge \mu_B(h))/(t, \quad h)$$

Use Mamdani's implication rule and obtain the membership function of the above fuzzy rule.

(b) Obtain the information about the humidity when there is the following premise about the temperature:

"Temperature is fairly high"

Where fuzzy term 'fairly high $(A') \subseteq T$, has the following membership function in T (temperature):

t	20	30	40
$\mu_{A'}(t)$	0.01	0.25	0.81

Question 2

[7]

HYPOTHESES: Either Arlen is lying or Brewster was in Mexico in April or Crawford was not a blackmailer. If Brewster was not in Mexico in April, then either Arlen is telling the truth or Crawford was a blackmailer.

CONCLUSION: Brewster must have been in Mexico in April.

Using the above mentioned hypotheses, establish the veracity of the given conclusion.

Question 3

[8]

A binary relation between $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3, y_4\}$ is defined with the help of following relation matrix:

	y_1	y_2	y_3	y_4
x_1	0.1	0.2	0.3	0.2
x_2	0.2	0.1	0.5	0.7
x_3	0.5	1.0	0.8	0.6

Obtain

(i) Projections $[R \downarrow X]$ and $[R \downarrow Y]$

(ii) Cylindrical Extensions $[R \uparrow X]$ and $[R \uparrow Y]$

(i) Join and Meet of the Cylindrical Extensions obtained in (ii).

Question 4

[3 + 7 = 10]

(a) What do you mean by 'Fuzzy Numbers'? List out their characteristics.

(b) Fuzzy Numbers 3 (FN3), Fuzzy Numbers 6 (FN6), and Fuzzy Numbers 7 (FN7) are defined as:

X = 1 + 2 + ... + 60

FN3 = 0.3/1 + 0.7/2 + 1.0/3 + 0.7/4 + 0.3/5

FN6 = 0.2/4 + 0.6/5 + 1.0/6 + 0.6/7 + 0.2/8

FN7 = 0.2/5 + 0.6/6 + 1.0/7 + 0.6/8 + 0.2/9

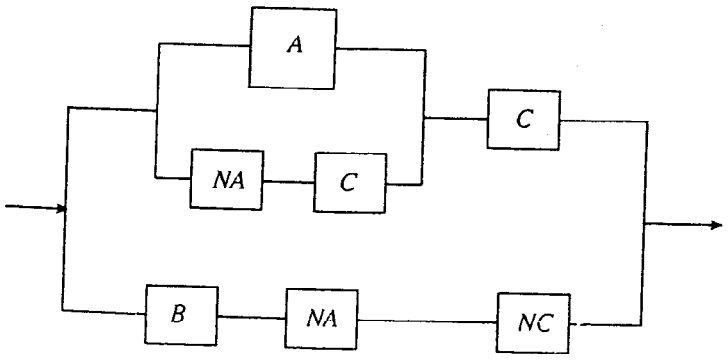
Obtain the following

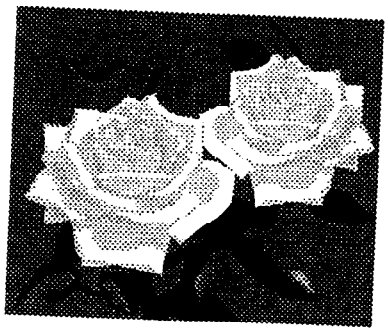
(i) FN3 + FN7 (ii) FN3 × FN7, and (iii) FN6 / FN3

Question 5

[5]

(a) Perform the simplification of the following switching circuit.





BITS, PILANI - DUBAI CAMPUS

Knowledge Village, Dubai

IV Year (EEE/EIE)

Semester I 2006 - 2007

TEST I (Closed Book)

Course No.: EA UC482 **Course Title:** Fuzzy Logic & Applications

Date: October 30, 2006 **Time:** 50 Minutes **M.M. :** 40 (20 %)

Question 1 **[2 × 5 = 10]**

(a) Let fuzzy sets $A = 0.2/1 + 0.4/3 + 0.6/5$ and $B = 0.1/1 + 0.3/3 + 0.5/5$ then find the following

(i) $A \ominus B$ (Bounded Difference)

(ii) $A \oplus B$ (Bounded Sum)

(b) Give the mathematical definition of "Contrast-Intensification" operation performed on linguistic fuzzy sets.

(c) Obtain the max-product composition of sets $A = [.5 \quad 1 \quad .6]$ and $B = \begin{bmatrix} .02 & .2 & .16 \\ .04 & .4 & .32 \\ .1 & 1 & .8 \end{bmatrix}$.

(d) If α-cuts of a fuzzy set A are as follows then obtain the fuzzy set A.

$${}^{.2}A = 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$${}^{.4}A = 0/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$${}^{.6}A = 0/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$${}^{.8}A = 0/x_1 + 0/x_2 + 0/x_3 + 1/x_4 + 1/x_5$$

$${}^{1}A = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$

Question 2

[5 + 5 = 10]

(a) Describe a 'Type-n Fuzzy Set' and a 'Level-k Fuzzy Set'. Give graphical representation of one example of each.

(b) If $A = 0.3/x_1 + 0.6/x_2 + 1/x_3 + 1/x_4 + 0.6/x_5 + 0.3/x_6$ is a fuzzy set defined in $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ then show that ${}^{(1-\alpha)+}\overline{A} = {}^{(1-\alpha)+}(\overline{A})$.

Question 3

[5 + 5 = 10]

(a) Show the graphical representation of (i) Trapezoidal (ii) Gaussian membership functions and write down the mathematical definitions of these functions, clearly mentioning the number of parameters needed to describe them completely.

(b) What is the use of 'Hedges' in the fuzzy set theory? Give example of any two commonly used hedges and define them clearly with the help of mathematical expression and graphical representation.

Question 4

[5 + 5 = 10]

(a) If A_1 = 0.2/3 + 0.5/4 + 0.8/5 and A_2 = 0.4/3 + 0.7/4 + 1/5, then obtain the following expression:

$$C \wedge \{\mu_{A_1}(x) + (\mu_{A_2}(x) \Theta \mu_{A_1}(x))\}$$

where 'C' stands for convex combination of A_1 and A_2 with ω_1 = 0.3 and ω_2 = 0.7. Symbol '+' and symbol Θ represent arithmetic sum and bounded difference respectively.

(b) Describe the concept of α-cut and strong α-cut used in the fuzzy set theory. In a triangular membership function of a fuzzy set, show its α-cuts and special α-cuts at various levels.

****

*****

Name:

Id. No.:

VERSION - A

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, Dubai

IV Year (EEE/EIE)

Semester I 2006 - 2007

QUIZ I (Closed Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: November 15, 2006 Time: 30 Minutes M.M. = 20 (10 %)

---

1. Give one example of a fuzzy complement function that is continuous but not involutive.

2. What is the general expression of 'Sugeno' class of involutive fuzzy complements?

3. If g is a continuous function from $[0,\ 1]$ to $\Re$ such that $g(0) = 0$, g is strictly increasing, then write down the expressing for obtaining associated fuzzy complement (involutive) for all $a \in [0,\ 1]$

4. Let class of two-parameter increasing generators is given by $g_{\lambda,\ \omega}(a) = \frac{1}{\lambda}\ln\left(1 + \lambda a^{\omega}\right)$ for $\lambda > -1$ and $\omega > 0$, then what will be the general expression for corresponding class of fuzzy complements?

5. Give the mathematical definition of 'Drastic intersection' operation.

6. What are the axioms included in the axiomatic skeleton for fuzzy intersections? Write down their mathematical definitions.

7. What is the definition of pseudo-inverse of decreasing generator $f_s(a) = -\ln \frac{s^a - 1}{s - 1} \quad (s > 0, \quad s \neq 1)$?

8. The number of edges traversed to get from one node to another node in a n-dimensional cube in n-space is referred as .....

9. Let the Universe of discourse be U=1+2+3+4, and let A = 0.8/1 + 0.5/2. Assume K(1) = 1/1 +0.3/2 and K(2) = 1/2 + 0.3/1 + 0.2/3, then obtain SF(A; K).

10. Let A_1 = 0.7/2 + 1/4 + 0.7/5 and A_2 = 0.3/2 + 1/3 + 0.4/5. Suppose ω_1 = 0.8 and ω_2 = 0.2, then what will be the convex combination of A_1 and A_2?

11. Write down the equivalence operation in terms of bounded-difference operation (and arithmetic sum).

12. Let fuzzy sets A = 0.5/1 + 0.4/2 + 0.2/3; then find NORM (A).

13. A fuzzy set can be visualized as a point in an n-dimensional cube in n-space. Inside this cube the 'Cardinality' of any fuzzy set is defined as its distance from the .....

14. The Yager t-conorm for which ω = 1, is very weak and indicates perfect interchangeability between the two arguments, identify it.

15. A Fuzzy Relation given as $M_R = \begin{bmatrix} 0 & .7 & .4 \\ .7 & 0 & .1 \\ .4 & .1 & 0 \end{bmatrix}$ is

(i) Reflexive and symmetric
(ii) Antireflexive and symmetric
(iii) Reflexive and antisymmetric
(iv) Antireflexive and antisymmetric

16. **Which one of the following is a correct relation?**

(i) $u_{max}(a, \ b) \geq \min(1, \ a+b) \geq a+b-ab \geq \max(a, \ b)$

(ii) $\max(a, \ b) \leq \min(1, \ a+b) \leq a+b-ab \leq u_{max}(a, \ b)$

(iii) $u_{max}(a, \ b) \leq \min(1, \ a+b) \leq a+b-ab \leq \max(a, \ b)$

(iv) $a+b-ab \leq \min(1, \ a+b) \leq u_{max}(a, \ b) \leq \max(a, \ b)$

17. **State the characterization theorem of t-conorm.**

18. **Which one of the following is not an essential condition for a fuzzy union operation?**

(i) $b \leq d$ **implies** $u(a, \ b) \leq u[a, \ d]$

(ii) $u(a, \ b) = u(b, \ a)$

(iii) $u(a, \ a) > a$

(iv) $u(a, \ u(b, \ d)) = u(u(a, \ b), \ d)$

19. **Relate the Items from list-A to list-B:**

List-A	List-B
(i) Pre-order Fuzzy Relation	(a) Reflexive and Symmetric
(ii) Similarity Fuzzy Relation	(b) Reflexive and max-min transitive
(iii) Resemblance Fuzzy Relation	(c) Reflexive, Symmetric, and max-min transitive

20. **Show graphically the approximate shape of the resultant Fuzzy set when 'Contrast Intensification' operation is performed on a triangular shaped Fuzzy set.**