BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

Year IV – Semester I 2003 – 2004

COMPREHENSIVE EXAMINATION (Closed Book)

Course No.: EA UC482 **Course Title: Fuzzy Logic & Applications**

Date: January 6, 2004 **Time: 3 Hours** **M.M. = 80 (40 %)**

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1. (a) Let fuzzy sets A = {(1, 0.1), (3, 0.3), (5, 0.6)} and B = {(2, 0.3), (4, 0.6), (6, 1)}; then the value of $A \cup B$ will be 2

   (b) The basic difference between the probabilistic reliability theories and possibilistic reliability theories is 2

   (c) If X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = { (1, 0), (2, 0.1), (3, 0.2), (4, 0.5), (5, 0.3), (6, 0.1), (7, 0), (8, 0), (9, 0), (10, 0) }; then the support of the fuzzy set A will be 2

   (d) Ordering of fuzzy numbers can be done using 2

   (e) Let the universe of discourse be X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and A = { (1, 0.3), (2, 0.5), (3, 1), (4, 0.7), (5, 0.2) }; then the value of $\|A\|$ will be 2

2. Describe, how the new methodological possibilities opened by fuzzy sets and fuzzy measures have affected any two of the following engineering disciplines: 10

   (i) Mechanical Engineering (ii) Industrial Engineering (iii) Robotics

3. Describe an strategy, using fuzzy set theory, for assessing the physical condition of existing bridges based on examining the three components, *deck*, *beams*, and *piers* evaluated in linguistic terms *poor*, *fair*, and *good*. Define

the fuzzy sets for structural importance associated with given condition assessments and interpret the result in terms of one of the three linguistic terms used, with the help of Euclidean distance. 10

4. Show the inter-relation among the basic components of a general pattern recognition system with the help of a diagram and explain them in brief. What is the relevance of fuzzy set theory in pattern recognition field? 10

5. The fuzzy relation R is defined on sets U = {a, b, c}, V = {x, y}, and W = {&, *} as R = 0.9/(a, x, &) + 0.4/(b, x, &) + 1/(a, y, &) + 0.7/(a, y, *) + 0.8/(b, y, *). How many different projections of the relation can be taken? Determine all one-dimensional projections. 10

6. Suppose that an individual needs to decide which of four possible jobs (described below) to choose. His or her goal is to choose a job that offers a *high salary* under the constraints that the job is *interesting* and within *close driving distance*. Apply the fuzzy logic to arrive at an appropriate conclusion. 10

Job	Salary	Nature of job	Distance from home
I	\$40,000	Less interesting	27 miles
II	\$45,000	Interesting	7.5 miles
III	\$50,000	Least interesting	12 miles
IV	\$60,000	Least interesting	2.5 miles

7. What do you mean by the *transitive closure* of a crisp relation. Using the algorithm for obtaining the transitive closure, determine the transitive max-min closure $R_T(X, X)$ for a fuzzy relation $R(X, X)$ defined by the following membership matrix: 10

$$R = \begin{bmatrix} .7 & .5 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & .4 & 0 & 0 \\ 0 & 0 & .8 & 0 \end{bmatrix}$$

8. Let A = $\{a_1, a_2, a_3\}$ and B = $\{b_1, b_2, b_3, b_4\}$ and C = $\{c_1, c_2, c_3\}$. Let P be a relation from A to B and Q be a relation from B to C, given by the sagittal diagram shown in figure 1. Obtain the join S = P * Q and then convert this join into corresponding standard composition R = P □ Q. (10)

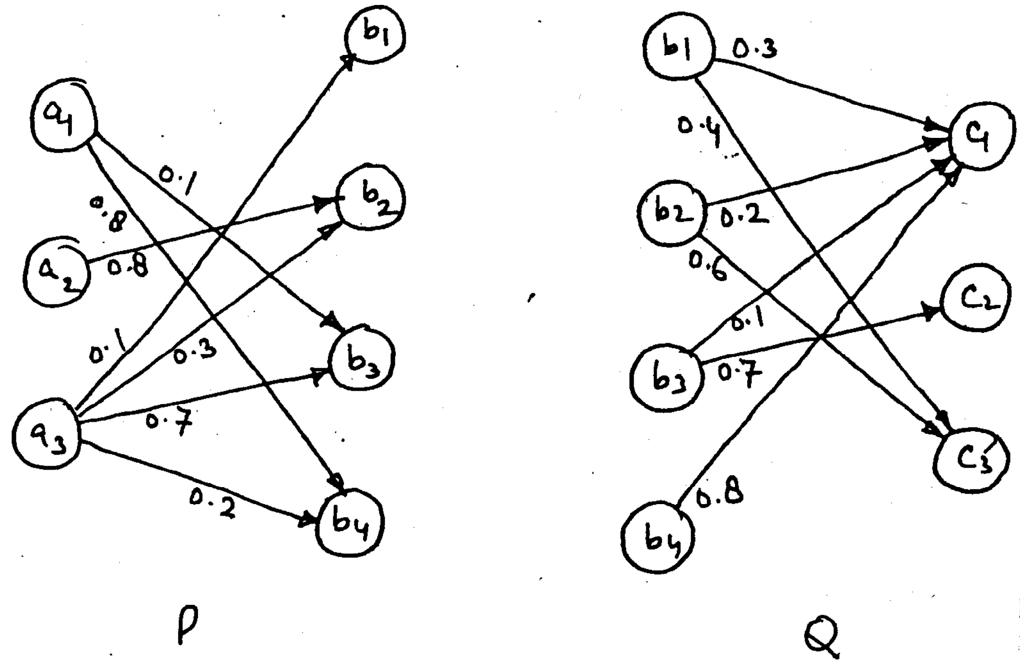

Figure 1

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Year IV – Semester I 2003 – 2004

MAKE-UP

TEST I (Closed Book)

Course No.: EA UC482 | Course Title: Fuzzy Logic & Applications

Date: October 26, 2003 | Time: 50 Minutes | M.M. = 40 (20 %)

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1. **Define the following terms, in reference with 'Fuzzy Logic' Theory with one example of each.** (3)
   - (i) **Fuzzification**
   - (ii) **Linguistic variable**
   - (iii) **Membership Function**

2. **Simplify the following expressions using Venn diagrams.** (3)
   - (i) $A \cup (A \cap B)$
   - (ii) $A \cap (A \cup B)$
   - (iii) $(A \cap B) \cup (A \cap \overline{B})$

3. **Given that $A = \{\alpha_1, \alpha_2, \alpha_3\}$, and $B = \{\beta_1, \beta_2\}$; determine $(A \times B) \cup (B \times A)$** (2)

4. **If R is a binary relation which gives a correspondence from A to A then Define the following properties (i) Reflexive (ii) Symmetric (iii) Anti-symmetric, and (iv) Transitive.** (2)

5. **Given that $A = \{\alpha_1, \alpha_2, \alpha_3\}$, and $B = \{\beta_1, \beta_2\}$, and $C = \{\gamma_1, \gamma_2, \gamma_3\}$, and let the relation R and S be defined as**

   $R = \{(\alpha_1, \beta_1), (\alpha_2, \beta_1), (\alpha_2, \beta_2)\}$ **and**

   $S = \{(\beta_1, \gamma_1), (\beta_1, \gamma_2), (\beta_2, \gamma_2), (\beta_2, \gamma_3)\}$

   - (i) **Determine the composite relation $R \circ S$ using the arrow diagram.**
   - (ii) **Also Determine M_R^{-1}.** (3)

6. Define cardinality and relative cardinality of a fuzzy set. Obtain cardinality and relative cardinality of fuzzy set A, if 4

$X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and

$A = \{ (1, 0), (2, 0.1), (3, 0.2), (4, 0.5), (5, 0.3), (6, 0.1), (7, 0), (8, 0), (9, 0), (10, 0) \}$

7. Describe the operations concentration and dilation used in the fuzzy set theory and show the effect of these operations on the shape of the membership function with the help of one example. 6

8. Let $A = 0.5/4 + 1/5 + 0.6/7$ and $B = 0.3/4 + 0.6/5$ then obtain $A \oplus B$ and $A \ominus B$. 4

9. Let $X = 1 + 2 + 3 + 4$, and $A = 0.8/1 + 0.5/2$. Also assume that $K(1) = 1/1 + 0.3/2$ and $K(2) = 1/2 + 0.3/1 + 0.2/3$. Then obtain SF (A; K). 2

10. Define the terms 'Supermum' and 'infimum' with respect to R, which denotes a set of real numbers. 4

11. Describe an 'interval-valued fuzzy set' and give one example of the same. 3

12. Define 'Absolute' and 'Relative' complement of a fuzzy set. If sets A and B are defined as follows then obtain the absolute complement of A and also complement of A with respect to B.

$$A = \{ (0, 0.1), (1, 0.2), (2, 0.3), (3, 0.4) \}$$

and $$B = \{ (1, 0.1), (2, 0.2), (3, 0.3) \}$$ 4

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TEST I (Closed Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: October 26, 2003 Time: 50 Minutes M.M. = 40 (20 %)

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1. Cite any three reasons, which make fuzzy variables more attuned to reality than crisp variables. (3)

2. If $A = \{x | 100K \le x \le 200K, \quad x \in U\}$ (3)

   and $B = \{x | 50K \le x \le 120K, \quad x \in U\}$

   where U is the universe of discourse [0, 1000K]. Determine

   (a) $A \cap B$ (b) $A \cup B$ (c) $\overline{A}$

3. Let A = {1, 2, 3}, B = {a, b, c} and C = {α, β, γ} and let the relations R and S be defined by as: (4)

   R = { (1, a), (1, c), (2, b), (3, a), (3, b) } and S = { (a, β), (b, α), (c, γ) }

   Determine M_R, M_S and $M_{R \circ S}$.

4. Let fuzzy sets A and B are given as: (4)

   A = { (4, 0.1), (6, 0.3), (8, 0.6), (10, 1) }

   and B = { (0, 0.3), (2, 0.6), (4, 1), (6, 1), (8, 0.6), (10, 0.3)}

   Determine (a) $A \cup B$ and (b) $A \cap B$

5. Define 'Support' of a fuzzy set. Obtain the support of the fuzzy set A described below: (3)

   X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and

   A = { (1, 0), (2, 0.1), (3, 0.2), (4, 0.5), (5, 0.3), (6, 0.1), (7, 0), (8, 0), (9, 0), (10, 0) }

6. **Describe the concept of α-cut and strong α-cut used in the fuzzy set theory with the help of one example.** (5)

7. **Let the universe of discourse be X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}** (4)

and $A = \{ (1, 0.3), (2, 0.5), (3, 1), (4, 0.7), (5, 0.2) \}$

Then obtain $|A|$ **and** $\|A\|$.

8. **Define 'Absolute' and 'Relative' complement of a fuzzy set. If sets A and B are defined as follows then obtain the absolute complement of A and also complement of A with respect to B.** (6)

$A = \{ (0, 0.3), (1, 0.4), (2, 0.6), (3, 0.7) \}$

and $B = \{ (0, 0.4), (1, 0.6), (2, 0.8), (3, 0.8) \}$

9. **Let A = $\{a_1, a_2\}$, B = $\{b_1, b_2, b_3\}$, and C = $\{c_1, c_2\}$. Let R be a relation from A to B defined by the matrix** (4)

$$\begin{array}{c} \\ a_1 \\ a_2 \end{array} \begin{array}{c} \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \begin{bmatrix} 0.4 & 0.5 & 0 \\ 0.2 & 0.8 & 0.2 \end{bmatrix} \end{array}$$

Let S be a relation from B to C defined by the matrix

$$\begin{array}{c} \\ b_1 \\ b_2 \\ b_3 \end{array} \begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \begin{bmatrix} 0.2 & 0.7 \\ 0.3 & 0.8 \\ 1 & 0 \end{bmatrix} \end{array}$$

Then obtain the max-av composition of R and S.

10. **If α-cuts of a fuzzy set A are as follows then obtain the fuzzy set A.** (4)

$$^{.2}A = 1/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$^{.4}A = 0/x_1 + 1/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$^{.6}A = 0/x_1 + 0/x_2 + 1/x_3 + 1/x_4 + 1/x_5$$

$$^{.8}A = 0/x_1 + 0/x_2 + 0/x_3 + 1/x_4 + 1/x_5$$

$$^{1}A = 0/x_1 + 0/x_2 + 0/x_3 + 0/x_4 + 1/x_5$$

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Year IV – Semester I 2003 – 2004

TEST II (Open Book)

Course No.: EA UC482 **Course Title: Fuzzy Logic & Applications**

Date: November 30, 2003 **Time: 50 Minutes** **M.M. = 40 (20 %)**

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1. **Which are the two laws of crisp set theory that are violated in fuzzy set theory? Prove any one of them.** 3

2. **Prove that law of absorption holds good for fuzzy sets.** 2

3. **If** 3

$$^{.2}A = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7}$$

$$^{.4}A = \frac{1}{1} + \frac{0}{3} + \frac{1}{5} + \frac{1}{7}$$

$$^{.8}A = \frac{1}{1} + \frac{0}{3} + \frac{1}{5} + \frac{0}{7}$$

$$^{1}A = \frac{0}{1} + \frac{0}{3} + \frac{1}{5} + \frac{0}{7}$$

and B = {(1, 0.3), (3, 0.1), (5, 0.4), (7, 0.1)}. Then find the value of B – A.

4. **The fuzzy relation R is defined on sets U = {&, *}, V = {x, y}, and W = {a, b, c} as follows**

   R = 0.9/(a, x, &) + 0.4/(b, x, &) + 1/(a, y, &) + 0.7/(a, y, *) + 0.8/(b, y, *)

   How many different projections of the relation can be taken? Determine all one-dimensional projections. 4

5. **Suppose expression "medium" for a fuzzy variable is defined as given in figure 1, then draw the graphs for expressions "more medium" and "less medium", if they are defined by the relations *more medium = CON(medium)*; and *less medium = DIL(medium)*** 2

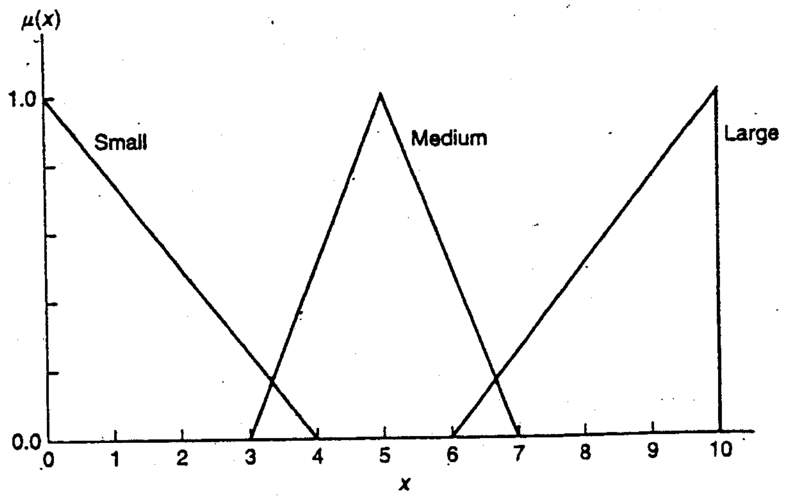

Figure 1

6. **What are the key benefits of fuzzy design? List few prominent applications of fuzzy logic.** 3

7. **If $A_1 = 0.2/3 + 0.5/4 + 0.8/5$ and $A_2 = 0.4/3 + 0.7/4 + 1/5$, then obtain the following expression** 3

$$C \wedge \{\mu_{A_1}(x) + (\mu_{A_2}(x) \Theta \mu_{A_1}(x))\}$$

where 'C' stands for convex combination of A_1 and A_2 with $\omega_1 = 0.3$ and $\omega_2 = 0.7$ and symbol '+' represents arithmetic sum.

8. **Show that the reasoning pattern given by $[\, p \wedge (p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow r$ is a tautology and thus a valid one.** 2

9. **What are the main reasoning patterns used in the propositional logic? Explain them with the help of one example each.** 3

10. **Let U = {a, b, c} and V = {x, y}. If R be a FUR on U ×V, defined by** 3

$$\begin{array}{cc} & \begin{array}{cc} x & y \end{array} \\ \begin{array}{c} a \\ b \\ c \end{array} & \begin{bmatrix} 0.3 & 1 \\ 0.6 & 0.2 \\ 0.4 & 0.5 \end{bmatrix} \end{array}$$

Then obtain Dom(R), Ran (R) and h(R).

11. Let A = {a_1, a_2, a_3} and B = {b_1, b_2, b_3, b_4} and C = {c_1, c_2, c_3}. Let P be a relation from A to B and Q be a relation from B to C, given by the sagittal diagram given in figure 2. Obtain the join S = P * Q and then convert this join into corresponding standard composition R = P □ Q. (4)

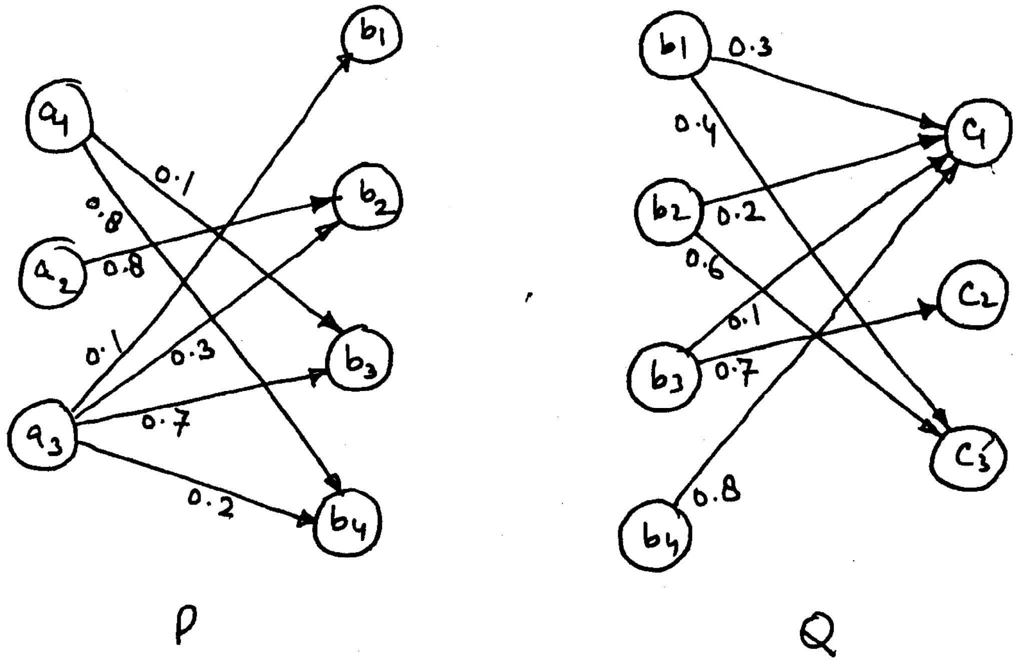

Figure 2

12. Solve the following puzzle of matching Kapil, Leander and Pilai with the sports Cricket, Hockey and tennis, applying the basic laws of propositional logic on the following details available to us: (3)

(a) It is not the case that both Leander is not tall and Kapil does not play Tennis.

(b) Leander is tall implies that Kapil is short and kapil is short implies that he plays Tennis.

(c) Either Kapil is short or he does not play Tennis.

(d) Kapil is short implies that leander is tall and leander is tall implies that Kapil plays Tennis and Kapil plays Tennis implies that pilai plays Hockey.

13. A switching function is given by the following truth table (5)

a	b	c	d
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

Write down the form of f. Draw the corresponding circuit. Find the simplified circuit.

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QUIZ I (Closed Book)

Course No.: EA UC482 Course Title: Fuzzy Logic & Applications

Date: November 11, 2003 Time: 40 Minutes M.M. = 20 (10 %)

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1. The family of *all subsets* of a given set A is called the .... of A.

2. The number of members of a finite set A is called the .... of A.

3. If the complement of a complement yields the original set, then this property is referred to as ....

4. The family of all subsets of P(A) is called .... of A.

5. A set A in $\Re^n$ is ....: iff, for every pair of points r and s in A, all points located on the straight-line segment connecting r and s are also in A.

6. For any set of real numbers R that is bounded above, a real number r is called .... of R iff (a) r is an upper bound of R; and (b) no number less than r is an upper bound of R.

7. Interval-valued fuzzy sets can further be generalized by allowing their intervals to be fuzzy, the sets so obtained are referred as ....

8. The height, h(A), of a fuzzy set A is defined as ....

9. Fuzzy sets, by definition, violate two properties of the complement of crisp sets, these properties are ....

10. Let A = 0.4/3 + 1/5 + 0.6/7 and B = 1/5 + 0.6/6 Then A × B = ...

11. Let A = 0.5/3 +1/5 + 0.6/7 and B = 1/3 + 0.6/5 Then the value of A + B = ...

12. The operation needed to be performed on the special fuzzy sets defined by ${}_{\alpha}A(x) = \alpha \cdot {}^{\alpha}A(x)$, in order to obtain the original fuzzy set A is

(i) Standard Fuzzy Union

(ii) Standard Fuzzy intersection

(iii) Bounded Sum operation

(iv) Max-min composition

13. Classical (standard) fuzzy complement is one which

(i) satisfies the axiomatic skeleton for fuzzy complements.

(ii) is continuous fuzzy complement.

(iii) is involutive fuzzy complement.

(iv) satisfies all the above conditions

14. Sugeno class of fuzzy complements is defined by

(i) $c_\lambda(a) = \dfrac{1-a}{1+\lambda a}$, Where $\lambda \in (-1, \infty)$.

(ii) $c(a) = \dfrac{1}{2}(1+\cos \pi a)$,

(iii) $c(a) = \begin{cases} 1 & for \quad a \le t \\ 0 & for \quad a > t \end{cases}$

(iv) All of the above

15. The equilibrium of a complement c is that degree of membership in a fuzzy set A which equals the degree of membership

(i) in the complement of A

(ii) in the complement of P(A)

(iii) in the Complement of universe of discourse.

(iv) None of the above

16. A *fuzzy intersection/t-norm* i is a binary operation on the unit interval that satisfies the properties of, for all a, b, d $\in$ [0, 1]:

(i) monotonicity

(ii) commutativity

(iii) associativity

(iv) All of the above

17. The standard fuzzy intersection is the only

(i) Subidempotent t-norm.

(ii) Idempotent t-norm.

(iii) Super idempotent t-norm.

(iv) All of the above

18. **Which one of the following is an incorrect relationship**

(i) $A + A = A$

(ii) $A\ I = A$

(iii) $A + \Phi = A$

(iv) None of them

19. **Let A = 0.7/5 + 0.5/7 + 1/8, then the value of $CON^2(A)$ will be**

(i) 49/5 + 0.25/7 + 1/8

(ii) .84/5 + 0.7/7 + 1/8

(iii) .24/5 + .30/7 + 1/8

(iv) .71/5 + .49/7 + 1/8

20. **which one of the following is a correct relationship**

(i) $\mu_A(x) \vee \mu_B(x) = \mu_A(x) + (\mu_B(x) \ominus \mu_A(x))$

(ii) $|\mu_A(x) - \mu_B(x)| = (\mu_A(x) \ominus \mu_B(x)) + (\mu_B(x) \ominus \mu_A(x))$

(iii) $\mu_A(x) \leftrightarrows \mu_B(x) = 1 \ominus (\mu_A(x) \ominus \mu_B(x) + (\mu_B(x) \ominus \mu_A(x))$

(iv) All of them