BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Comprehensive Examination –III Year –Fifth Semester 2003-2004

Date 6/06/04
Duration : 3 hours

Course:Numerical Analysis
Total Marks : 40
Weightage : 40%

NOTE : (Answer all Questions)

Q1. [2+2+2+2]

(a) Use the method of Bisection to find a real root of $f(x) = x\log_{10} x - 1.2$ in 4 stages . Given the root lies between 2 and 3

(b) Solve the following differential equation

$$y_{n+2} - 2y_{n+1} + y_n = 3^n$$

(c) A river is 80 ft wide. The depth y in feet at a distance x feet from one bank is given by

x	0	10	20	30	40	50	60	70	80
y=f(x)	0	4	7	9	12	15	14	8	3

Find the area of cross section of the river using Simpsons $\frac{1}{3}$rd rule .If area of cross section is given by $A = \int_a^b y dx$

(d) Find the largest eigen value using Power method starting with $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ for the matrix

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Q2. [4+4]

(a) Find the approximate root of $x(1-\ln x) = 0.5$, given root lies between 0.1 and 0.2 Newtons method correct to 3 decimal places.

(b) Use Jacobis method to solve the following system of equations

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

Do 3 iterations starting with (1,1,1) and considering ∞ norm .Use 5digit arithmetic, also find the minimum number of iterations required, so that answer is correct to 6 decimal digits.

Q3. [4+4]

(a) Given f(x)

x	0.1	0.2	0.3	0.4	0.5	0.6
f(x)	0.425	0.475	0.400	0.450	0.525	0.575

Find $f^{(ii)}(0.5)$ and $f^{(iv)}(0.15)$

(b) Apply R-K method of order 4 to approximate the value of y(2.1),y(2.2) as solution of differential equation,using 5 digit arithmetic

$\frac{dy}{dx} = x^2 + y^2 =$ f(x, y) , given y(2)=3.

Q4. [4+4]

(a) Evaluate

$$\int_{-1}^{1} \frac{(1+x)}{\sqrt{1-x^2}} e^x dx$$

using 3 point Gauss Chebyshev Quadrature formula .

(b) Find x_0, x_1, A_0, A_1 *and* error term in the following integration rule so that integration is exact for a polynomial of degree as high as possible

$$\int_{-1}^{1} \frac{f(x)}{1+x^2} dx = A_0 f(x_0) + A_1 f(x_1) + \alpha f^{(iv)}(\xi) \quad \text{,where} \quad -1 < \xi < 1.$$

Q5. [4+4]

(a) Solve the given system of equations

$0.4x + 8y + 5z = -8.2$
$6x + 0.5y - 10z = -19.5$
$5x - 3y + 0.2z = 19.6$

using Gauss algorithm and 5 digit arithmetic with rounding .

(b) Apply one step of 4^{th} order Adam Moultons Predictor Corrector Formula to the system of equations

$$\frac{dy}{dx} = y + z,$$
$$\frac{dz}{dx} = y - z,$$

Given the step length is 0.1and
$y(0) = 1$, $z(0) = -2$
$y(0.1) = 1.2$, $z(0.1) = -1$
$y(0.2) = 1.1$, $z(0.2) = 0$
$y(0.3) = 1.2$, $z(0.3) = 1$

to find $y(0.4)$, $z(0.4)$.

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Date: 25/04/04 **Test :OPEN BOOK(III rd yr)**
Course:Numerical Analysis **Instructor:Priti Bajpai**
Duration :50 min **Total Marks : 20** **Weightage: 20**

NOTE : (Answer all Questions)

Q1. **[4]**

Let f(x) = cosx be defined in the interval [0,1] .Find $f'(x)$ at 0.2 ,using the formula $f'(x) = \frac{f_{i+1} - f_i}{h}$ where h=0.1.

Q2. **[4]**

Given f(x) = $2\sin(\frac{\pi x}{6})$ where x is in radians .Form a Lagranges Interpolation polynomial based on nodes $x_0 = 0$, $x_1 = 1$, $x_2 = 3$ to approximate f(4) .

Q3. **[4]**

Form a Newtons forward difference table and find $P_4(x)$ at x=2.5 given

x	1	2	3	4	5
f(x)	3.60	1.80	1.20	0.90	0.72

Q4. **[4]**

Evaluate $\int_0^{\frac{\pi}{2}} e^{\sin x} dx$ using Simpsons rule by dividing the range into 8 equal parts.

Q5 **[4]**

Find A_0, A_1 and A_2 as functions of β such that the error term vanishes for the formula given below , when f(x) is an arbitrary polynomial of degree 3 or less and $0 \le \beta \le 1$, where

$$\int_{-1}^{1} f(x)dx = A_0 f(-\beta) + A_1 f(0) + A_2 f(\beta) + E(I)$$

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Date: 14/03/04 **Test : QUIZ** **Course:Numerical Analysis (IIIrd yr)**

Duration :50 min **Total Marks : 10** **Weightage: 10**

NOTE : (Answer all Questions)

1.

Given $f(x) = x^3 - 3x^2 + 3x - 1$ .If the nested form of f(x) is denoted by g(x),then **[1]**

(a) g(x) =

If another form of f(x) is given by
$h(x) = (x-1)^3$
then using four digit arithmetic with rounding

(b) f(2.72) =

(c) g(2.72) =

(d) h(2.72) =

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2. **[1]**

If $f(x) = x - e^{-x} = 0$
Then one of the iterative functions g(x) which converges to a root is

g(x) =

---

if

$x_0 = 0.5$

then

$x_1 =$ ----------

$x_2 =$ ----------

[1]

3.

If $1 = x \sin x$

(a)Then one of the real root lies in the interval

(b) Using Bisection method ,the third iteration gives the root as

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[1]

4. When is a system called illconditioned

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[1]

5.

If $x = (x_1, x_2, \ldots x_n)$ then

$\|x\|_1 =$ ----------

$\|x\|_\infty =$ ----------

[1]

6.

Given the following system

$-x + 3y = 1$

$6x - 2y = 2$

On using the Jacobis method

(a) Will the system converge? give reason in support of your answer

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(b) How many iterations are required to get the solution

7. [2]

Write the following system

$$x^{\frac{1}{3}} + y^{\frac{1}{4}} - x - 1.175 = 0$$

$$x^{\frac{1}{4}} + y^{\frac{1}{2}} - y - 0.8412 = 0$$

in the form

(a) x = f(x,y) ,y = g(x,y) , so that the fixed point iteration can be applied for $0.5 \le x \le 1$ and $1 \le y \le 1.5$

(b)Will the system converge ? give reason in support of your answer

8. [2]

If for a system Ax = b

$$A = \begin{bmatrix} 2 & -4 & 3 \\ 4 & 1 & -6 \\ 5 & 8 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} -16 \\ 19 \\ 21 \end{bmatrix}$$

To solve the system Algorithm of Gauss elimination is used

After step1 ,the working augmented matrix is given as

$$\begin{bmatrix} 0.75 & -1.125 & 1.5 & -6.375 \\ 0.66667 & 0.16667 & -1 & 3.1667 \\ 0.9375 & 0.84375 & 1.4375 & -0.3438 \end{bmatrix}$$

for the step 2

$p = (\_, \_, \_)$

$a_{p_3,2} =$ ----------

$a_{p_3,3} =$ ----------

$b_{p_3} =$ ----------

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Date: 14/03/04 **Test : I** **Course:Numerical Analysis**
Duration :50 min **Total Marks : 20** **Weightage: 20**

NOTE : (Answer all Questions)

Q1. **[2+2]**

(a) Evaluate the value of f(x) = $x^3 + 2x^2 + 3x + 5$ at x = 2.39 , in nested form,using 3 digit arithmetic with chopping.

(b) Let $x^{\frac{1}{10}}$ be evaluated correct to 5 significant digits for x lying between 0 and 10 , what digit arithmetic should be used?

Q2. **[4]**

Find one real root of $x = \frac{1}{2} + \sin x$, using Bisection method correct to 4 significant digits .

Q3. **[4]**

Evaluate $\sqrt{12}$, using 5 digit arithmetic using Newtons Method .Do 4 iterations.

Q4. **[4]**

Do two iterations of Mullers Method starting with $x_0 = 0.5, x_2 = 0, x_1 = 1$, to solve f(x) = $x^3 - 3x + 1$.

Q5. **[4]**

Find a suitable interval and corresponding iteration function , so that fixed point iteration converges for the solution of the equation f(x) = $x - x^3$ .Check the conditions of Fixed Point Iteration theorem and find the minimum number of iterations required so that root is correct to 4 significant digits. Also find the root .