BITS, PILANI – DUBAI CAMPUS

SECOND SEMESTER – 2013-14

COMPREHENSIVE EXAMINATION(CB)

Course Name: Engineering Optimisation **Course Number:** ME F344

Max. Marks: 80 **Weightage: 40%** **Date: 02-6-2014** **Time:** $2\frac{1}{2}$ **hours**

Write answers of Part A and Part B in separate answer books. All questions are compulsory. Non-programmable calculator is permitted.

PART A

1. A computer project consists of 5 modules- A, B, C, D and E. There are five programmers and each programmer can do each module with different efficiencies. Each programmer will be assigned to exactly one module and each module will be done by a single programmer. Following table gives the computer times(in hours) required for each module when done by different programmers:

PROGRAMMERS	MODULES				
	A	B	C	D	E
1	10	15	12	25	30
2	30	25	15	20	20
3	12	15	32	34	21
4	26	24	28	30	21
5	21	35	42	34	22

Find the optimum assignment which will minimize the total computer time required to complete all the modules. [8]

2. Solve the following LPP by two-phase Method: [10]

$$\text{Minimize } Z = 60x_1 + 50x_2$$
$$\text{subject to } 2x_1 + x_2 \geq 80,$$
$$5x_1 + 2x_2 \geq 60,$$
$$x_1, x_2 \geq 0.$$

3. You need to buy some filing cabinets. You know that Cabinet X costs \$10 per unit, requires six square feet of floor space, and holds eight cubic feet of files. Cabinet Y costs \$20 per unit, requires eight square feet of floor space, and holds twelve cubic feet of files. You have been given \$140 for this purchase, though you don't have to spend that much. The office has room for no more than 72 square feet of cabinets. Formulate the problem as LPP in order to

8. Given the LPP:

Maximize $Z = 3x_1 + 5x_2 + 4x_3$

Subject to $2x_1 + 3x_2 \leq 8,$

$2x_2 + 5x_3 \leq 10,$

$3x_1 + 2x_2 + 4x_3 \leq 15,$

$x_1, x_2, x_3 \geq 0$

Find the range of c_3, the objective coefficient of x_3 in which the optimal solution given by the table below remains unaffected. Also find the range of the value of the objective function. [6]

	c_j	3	5	4	0	0	0	
	Basic	x_1	x_2	x_3	s_1	s_2	s_3	solution
5	x_2	0	1	0	15/41	8/41	-10/41	50/41
4	x_3	0	0	1	-6/41	5/41	4/41	62/41
3	x_1	1	0	0	-2/41	-12/41	15/41	89/41
	Z_j	3	5	4	45/41	24/41	11/41	
	$C_j - Z_j$	0	0	0	-45/41	-24/41	-11/41	-

9. For the network given below, the time estimate (in days) t_o(optimistic), t_m (most likely) and t_p(pessimistic) are given in this order for each of the activities. [10]

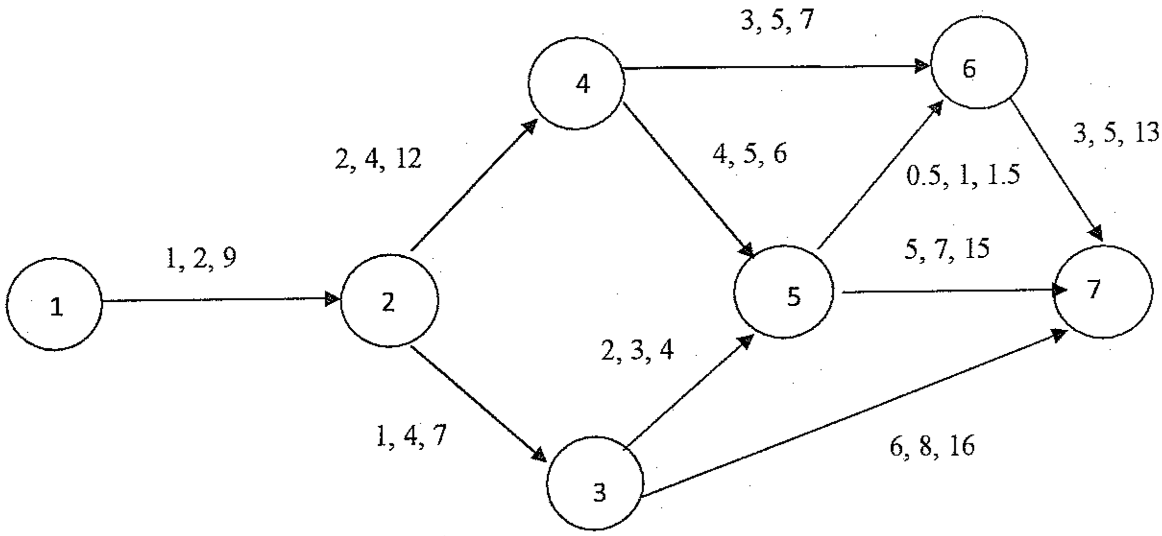
a) Find the expected duration and variance of each activity.

b) Determine the critical path and the standard deviation of the critical path.

c) Find the expected project completion time.

d) What is the probability that the project will be completed in 25 days?

Given $P(Z \leq 1.43) = 0.9236$.



10. A certain design problem is formulated as:

Minimize $f(x) = x_1^2 + 2x_2^2 - 5x_1 - 2x_2 + 10$

subject to $x_1 + 2x_2 - 3 \leq 0,\ 3x_1 + 2x_2 - 6 \leq 0,\ x_1, x_2 \geq 0.$

a) Write KKT's necessary conditions.

b) Find the solution. [10]

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International Academic City, Dubai

SECOND SEMESTER - 2013-2014

TEST – II (OB)

Course Title: Engineering Optimisation **Course No. : ME F344**

Max. Marks: 40 **Weightage: 20%** **Date: 23-04-2014** **Time: 50 min.**

Non-programmable calculator is permitted.

Attempt all the questions.

1. Solve the following transportation problem: [12]

	D1	D2	D3	D4	Supply
S1	55	30	40	50	25
S2	80	32	100	45	30
S3	40	60	95	35	40
Demand	50	10	20	15	

2. Consider the following LPP:

$$\begin{aligned} &\textit{Maximize } Z = 4x_1 + 6x_2 + 2x_3 \\ &\textit{subject to} \quad x_1 + x_2 + x_3 \le 3, \\ &\qquad x_1 + 4x_2 + 7x_3 \le 9, \\ &\qquad x_1, x_2, x_3 \ge 0. \end{aligned}$$

The final simplex table is

	Cj	4	6	2	0	0	
	Basic	x_1	x_2	x_3	S_1	S_2	*Solution*
4	x_1	1	0	-1	4/3	-1/3	1
6	x_2	0	1	2	-1/3	1/3	2
	Zj	4	6	8	10/3	2/3	16
	Cj-Zj	0	0	-6	-10/3	-2/3	

a) What is the optimal solution of the dual of the above LPP(primal)?

b) Find the revised optimal solution of the primal, if right hand side constants of the constraints are changed from (3, 9) to (7, 6). [10]

3. Find the dual of the following LPP: [4]

$$\begin{aligned} &\textit{Maximize } Z = 3x_1 + 4x_2 + x_3 \\ &\textit{subject to} \quad x_1 - 3x_2 + 2x_3 \geq 10, \\ &\qquad\qquad\quad 6x_1 + 2x_2 - 2x_3 \leq 30, \\ &\qquad\qquad\quad x_1 + 3x_2 + x_3 = 5, \\ &\qquad\qquad\quad x_1 \leq 0, x_2 \text{ unrestricted}, x_3 \geq 0. \end{aligned}$$

4. Solve the following assignment problem: [14]

	T1	T2	T3	T4	T5
P1	12	8	11	18	11
P2	14	22	8	12	14
P3	14	14	16	14	15
P4	19	11	14	17	15
P5	13	9	17	20	11

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

SECOND SEMESTER - 2013-2014

TEST – I (CB)

Course Title: Engineering Optimization **Course No. : ME F344**

Max. Marks: 50 **Weightage: 25%** **Date: 05-03-2014** **Time: 50 min.**

Attempt all the questions.

1. Use graphical method to solve the following LPP:
   Minimize $Z = 20x + 10y$
   subject to
   $x + 2y \leq 40,$
   $x \geq 12,$
   $4x + 3y \geq 60,$
   $x, y \geq 0.$ [12]

2. Solve the following LPP by 2-phase method:
   Maximize $Z = 5x_1 + x_2$
   subject to
   $5x_1 + 2x_2 \leq 20,$
   $x_1 \geq 3,$
   $x_2 \leq 5,$
   $x_1, x_2 \geq 0.$ [16]

3. *Maximize* $z = 4x_1 + 3x_2 + 6x_3$
   subject to the constraints:
   $$2x_1 + 3x_2 + 2x_3 \leq 440,$$
   $$4x_1 + 3x_3 \leq 470,$$
   $$2x_1 + 5x_2 \leq 430,$$
   $$x_1, x_2 \geq 0.$$ [16]

4. A company has two grades of inspectors, I and II to undertake quality control inspection. At least 1500 pieces must be inspected in an 8-hour day. Grade I inspector can check 20 pieces in an hour with an accuracy of 96% and Grade II inspector checks 14 pieces an hour with an accuracy of 92%. Wages of Grade I inspector are AED5.00 per hour while those of Grade II inspector are AED4.00 per hour. Any error made by an inspector costs AED3.00 to the company. If there are, in all, 10 Grade I inspectors and 15 Grade II inspectors in the company, formulate the LPP to find the optimal assignment of inspectors that minimizes the daily inspection cost. [6]

AA

BITS PILANI – DUBAI CAMPUS

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SECOND SEMESTER - 2013-2014

QUIZ – II (CB)

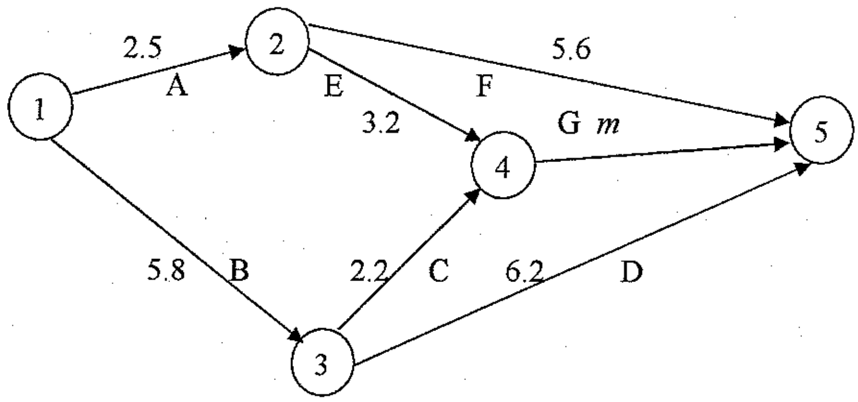
Course Title: ENGINEERING OPTIMISATION **Course No. : ME F344**

Max. Marks: 14 **Weightage: 7%** **Date: 14-5-2014** **Time: 20 min.**

NAME: **ID No:** **Instructor:**

Attempt all the questions. No extra sheets will be given for calculations/rough works.
Fill in the blanks with correct answers.

1.



In the above network of a small project, duration of each activity is given in hours. If the earliest completion time(duration) of the project is 15.4 hours, the critical path is ________________________________ and the value of m is _____ . [2+1]

2. A symmetric matrix of order 10 has ________ number of principal minors of order 5. [1]

3. The values of the leading principal minors of $A = \begin{bmatrix} 2 & -2 & 1 \\ -2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$ are ______, ________, _______. [2]

4. The Hessian matrix for the function $f(x_1, x_2) = 2x_1^2 - 4x_1x_2 + 3x_2^2 - 4x_1$ is: [2]

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

5. In a project, optimistic, pessimistic and most likely time estimates of an activity are 5, 8, 6 days respectively. The expected duration of this activity is _______ and the expected standard deviation of the activity is _________. [2]

6. Write KKT's conditions for the following optimization problem: [4]

$$\text{Maximize } f(x_1, x_2) = 2x_1^2 - 5x_1x_2 + x_2^2$$
$$\text{subject to } \quad 2x_1 - 4x_2 \le 20, \quad x_1, x_2 \ge 0.$$

B

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

SECOND SEMESTER - 2013-2014

QUIZ – II (CB)

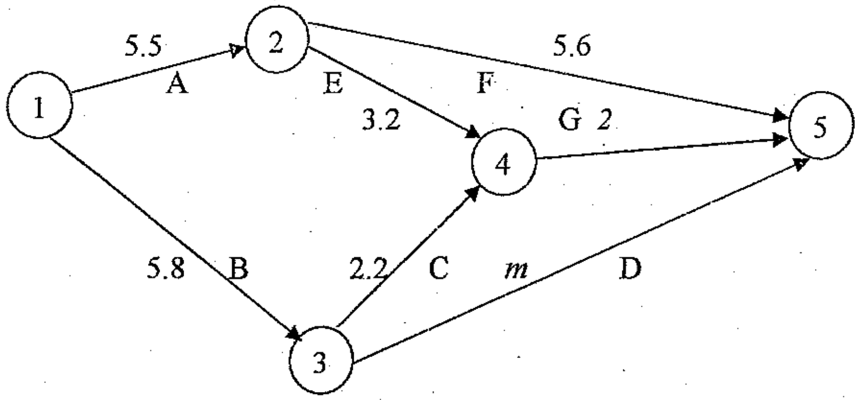
Course Title: ENGINEERING OPTIMISATION **Course No. : ME F344**

Max. Marks: 14 **Weightage: 7%** **Date: 14-5-2014** **Time: 20 min.**

NAME: **ID No:** **Instructor:**

Attempt all the questions. No extra sheets will be given for calculations/rough works.
Fill in the blanks with correct answers.

1.



In the above network of a small project, duration of each activity is given in hours. If the earliest completion time(duration) of the project is 16.4 hours, the critical path is ______________________ and the value of m is _____ . [2+1]

2. A symmetric matrix of order 8 has ________ number of principal minors of order 6. [1]

3. The values of the leading principal minors of $A = \begin{bmatrix} -4 & -2 & 1 \\ -2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$ are _____, _______, _______. [2]

4. The Hessian matrix for the function $f(x_1, x_2) = x_1^2 - 6x_1x_2 + 4x_2^2 - 7x_1$ is: [2]

$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$$

5. In a project, optimistic, pessimistic and most likely time estimates of an activity are 5, 10, 6 days respectively. The expected duration of this activity is _______ and the expected standard deviation of the activity is _________. [2]

6. Write KKT's conditions for the following optimization problem: [4]

$$\text{Maximize } f(x_1, x_2) = 10x_1^2 - 8x_1x_2 + 6x_2^2$$
$$\text{subject to } \quad x_1 - 4x_2 \le 20, \quad x_1, x_2 \ge 0.$$

A

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SECOND SEMESTER - 2013-2014

QUIZ – II (CB)

Course Title: ENGINEERING OPTIMISATION **Course No. : ME F344**

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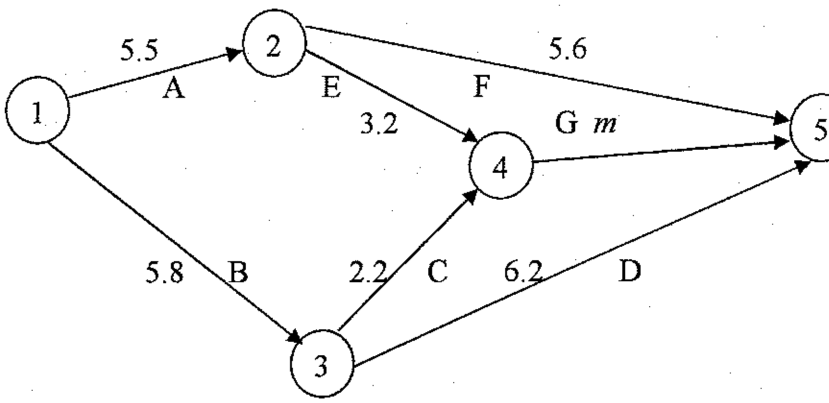
NAME: **ID No:** **Instructor:**

Attempt all the questions. No extra sheets will be given for calculations/rough works.
Fill in the blanks with correct answers.

1.



In the above network of a small project, duration of each activity is given in hours. If the earliest completion time(duration) of the project is 15.4 hours, the critical path is ______________________ and the value of m is _____. [2+1]

2. A symmetric matrix of order 8 has ________ number of principal minors of order 5. [1]

3. The values of the leading principal minors of $A = \begin{bmatrix} -4 & -2 & 1 \\ -2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$ are _____, ________, _______. [2]

4. The Hessian matrix for the function $f(x_1, x_2) = 5x_1^2 - 3x_1x_2 + 2x_2^2 - x_1$ is: [2]

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

5. In a project, optimistic, pessimistic and most likely time estimates of an activity are 5, 10, 6 days respectively. The expected duration of this activity is _______ and the expected standard deviation of the activity is _________. [2]

6. Write KKT's conditions for the following optimization problem:

$$\text{Maximize } f(x_1, x_2) = 4x_1^2 - 3x_1x_2 + 2x_2^2$$
$$\text{subject to } \quad 2x_1 - 4x_2 \le 20, \quad x_1, x_2 \ge 0.$$

[4]

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SECOND SEMESTER - 2013-2014

QUIZ – II (CB)

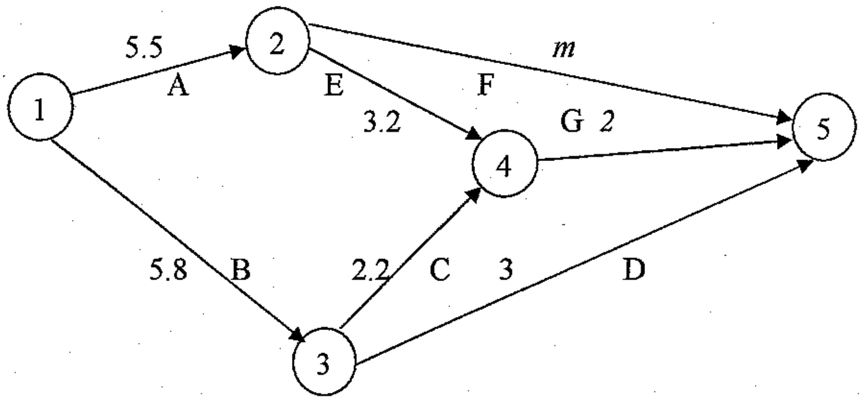
Course Title: ENGINEERING OPTIMISATION **Course No. : ME F344**

Max. Marks: 14 **Weightage: 7%** **Date: 14-5-2014** **Time: 20 min.**

NAME: ________ **ID No:** ________ **Instructor:** ________

Attempt all the questions. No extra sheets will be given for calculations/rough works.
Fill in the blanks with correct answers.

1.



In the above network of a small project, duration of each activity is given in hours. If the earliest completion time(duration) of the project is 14.4 hours, the critical path is ______________________ and the value of m is _____ . [2+1]

2. A symmetric matrix of order 6 has ________ number of principal minors of order 4. [1]

3. The values of the leading principal minors of $A=\begin{bmatrix}4 & 2 & 1\\ 2 & 1 & 3\\ 1 & 3 & 3\end{bmatrix}$ are ______, ________, ______ . [2]

4. The Hessian matrix for the function $f(x_1,x_2)=20x_1^2-5x_1x_2+10x_2^2-7x_1$ is: [2]

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

5. In a project, optimistic, pessimistic and most likely time estimates of an activity are 5, 10, 6 days respectively. The expected duration of this activity is _______ and the expected standard deviation of the activity is _________. [2]

6. Write KKT's conditions for the following optimization problem:

$$\text{Maximize } f(x_1, x_2) = 12x_1^2 - 6x_1x_2 + 3x_2^2$$
$$\text{subject to } \quad x_1 - 4x_2 \le 20, \quad x_1, x_2 \ge 0.$$

[4]

A

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

SECOND SEMESTER - 2013-2014

QUIZ – I (CB)

Course Title: ENGINEERING OPTIMIZATION **Course No. : ME F344**

Max. Marks: 16 **Weightage: 8%** **Date: 26-03-2014** **Time: 20 min.**

NAME: **ID No:** **Sec:**

Attempt all the questions. No extra sheets will be given for calculations/rough works. Do not use pencil. Multiple answers will be treated as incorrect answers.

***Fill in the blanks with correct answers*:**

1. Maximum number of allocated cells in a basic feasible solution of a transportation problem with 5 origins and 3 destinations is ______. [2]
2. To convert the following transportation problem in to balanced one a dummy warehouse should be added with demand ________ units. [2]

Warehouses

Factories

	D1	D2	D3	Capacity
F1	3	5	7	50
F2	5	9	10	34
F3	12	2	6	21
Demand:	32	31	32	

3. In the following u_i, v_j table for a basic feasible solution of a transportation problem where '*' indicates occupied cell, a = _____, b = _____, c = _____, d = _____.

Warehouses

Factories

	D1	D2	D3	u_i
F1	3 *	5 *	7	2
F2	5	9 *	10	6
F3	12 *	2	6 *	d
v_j:	a	b	c	

[4]

4. In a maximization type LPP with two decision variables x, y optimum value of the objective function is obtained at two corner points A(4, 2) and B(2, 6). The objective function is *Maximize* $Z = \_\_\_ x + \_\_\_ y$. *(Fill in the coefficients of x and y)* [2]

5. In Q No. 4, find a point (x, y) different from points A and B which will give optimum value of the objective function. $x =$ ______ and $y =$ ______ . [2]

6. In branch and bound method, one sub-problem SP is chosen for branching. If the optimal solution of SP is $x = 1.5, y = 2, z = 10$, then new sub-problems obtained from SP are
   SP1: SP with additional constraint ____________;
   SP2: SP with additional constraint ____________ . [2]

7. The LPP defined below is ___________ (mixed/pure) integer programming problem:
   Maximize $Z = 20x + 10y$
   subject to

$$x + 2y \leq 40,$$
$$x \geq 12,$$
$$4x + 3y \geq 60,$$

x, y are non-negative integers. [2]

B

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

SECOND SEMESTER - 2013-2014

QUIZ – I (CB)

Course Title: ENGINEERING OPTIMIZATION **Course No. : ME F344**

Max. Marks: 16 Weightage: 8% Date: 26-03-2014 Time: 20 min.

NAME: **ID No:** **Sec:**

Attempt all the questions. No extra sheets will be given for calculations/rough works. Do not use pencil. Multiple answers will be treated as incorrect answers.

***Fill in the blanks with correct answers*:**

1. Maximum number of allocated cells in a basic feasible solution of a transportation problem with 5 origins and 6 destinations is ______ . [2]
2. To convert the following transportation problem in to balanced one a dummy warehouse should be added with demand ________ units. [2]

Warehouses

Factories		D1	D2	D3	Capacity
	F1	3	5	7	50
	F2	5	9	10	34
	F3	12	2	6	21
	Demand:	32	31	20	

3. In the following u_i, v_j table for a basic feasible solution of a transportation problem where '*' indicates occupied cell, a = _____, b = _____, c = _____, d = _____.

Warehouses

Factories		D1	D2	D3	u_i
	F1	3 *	8 *	7	4
	F2	5	9 *	10	5
	F3	10 *	2	8 *	d
	v_j:	a	b	c	

[4]

4. In a maximization type LPP with two decision variables x, y optimum value of the objective function is obtained at two corner points A(4, 2) and B(1, 6). The objective function is *Maximize* $Z = \_\_\_ x + \_\_\_ y$. *(Fill in the coefficients of x and y)* [2]

5. In Q No. 4, find a point (x, y) different from points A and B which will give optimum value of the objective function. $x =$ ______ and $y =$ ______ . [2]

6. In branch and bound method, one sub-problem SP is chosen for branching. If the optimal solution of SP is $x = 0.5$, $y = 2$, $z = 10$, then new sub-problems obtained from SP are
SP1: SP with additional constraint ____________;
SP2: SP with additional constraint ____________ . [2]

7. The LPP defined below is __________ (mixed/pure) integer programming problem:
Maximize $Z = 20x + 10y$
subject to

$$x + 2y \leq 40,$$
$$x \geq 12,$$
$$4x + 3y \geq 60,$$
$$x \geq 0 \text{ and } y \text{ is non-negative integer.}$$

[2]