

**BITS PILANI, DUBAI CAMPUS  
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI**

**COMPREHENSIVE EXAMINATION – III YEAR – II SEMESTER 2012-2013**

Date: 02/06/2013  
Duration: 3 hours

Total Marks: 120

Course: Numerical Analysis AAOC C341  
Weightage: 40%

**ANSWER PART – A, PART – B AND PART – C SEPARATELY**

**Use FIVE digit arithmetic with rounding wherever applicable**

**PART – A**

1. Use Muller's method to find a root of the equation  $e^{-x} = 3 \ln x$  at the end of first iteration if the root is near the points 1, 1.2, 1.5. [10]
2. Find the Fourier series of  $f(x) = x$  in  $[0, 4]$ . Assume the periodicity of the function in the given interval. [8]
3. Use fixed point iterative method to find a root of  $e^x - 3x^2 = 0$ , with the iterative scheme  $x = \ln(3x^2)$ . Perform three iterations, by taking the initial root as 3. [6]
4. Find the double root of  $x^3 - x^2 - x + 1 = 0$  by Newton's method at the end of third iteration with  $x_0 = 1.5$ . [6]
5. Let  $f(x) = \sqrt{x}$  defined in  $[15, 21]$ . Use Newton's forward interpolation method to find  $f''(x)$  at  $x = 20$  with  $h = 2$ . [8]
6. Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by using Simpson 1/3<sup>rd</sup> rule with  $h = \frac{1}{6}$ , also find the approximate value of  $\pi$ . [8]
7. Evaluate  $\int_0^1 \frac{\sin \pi x}{\{x(1-x)\}^{3/2}} dx$  by using 2-point Guass Chebyshev quadrature. [8]
8. Find the largest eigen value for the given matrix at the end of third iteration using power method starting with the vector  $(0 \ 0 \ 1)^T$

$$\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix} \quad [6]$$

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**MARKING SCHEME**

$$\textcircled{1} \quad x_2 = 1, x_0 = 1.2, x_1 = 1.5, \quad f(x) = e^{-x} - 3 \ln x$$

$$f_2 = f(x_2) = 0.36788$$

$$f_0 = f(x_0) = -0.24577$$

$$f_1 = f(x_1) = -0.99327$$

— [1/2]

$$h_1 = x_1 - x_0 = 0.3, \quad h_2 = x_0 - x_2 = 0.2. \quad — [1]$$

$$\gamma = \frac{h_2}{h_1} = 0.66667. \quad — [1/2]$$

$$c = f_0 = -0.24577.$$

$$a = \frac{\gamma f_1 - f_0(1+\gamma) + f_2}{\gamma h_1^2(1+\gamma)} = 1.1531. \quad — [2]$$

$$b = \frac{f_1 - f_0 - a h_1}{h_1} = -2.8376. \quad — [2]$$

∴ Root after 6<sup>th</sup> iteration is

$$x_1 = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}} = 1.1162 \quad — [3]$$

(2) Let the F.S. be

$$x = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[ A_n \cos\left(\frac{n\pi x}{2}\right) + B_n \sin\left(\frac{n\pi x}{2}\right) \right]$$

$$\therefore A_0 = \frac{1}{2} \int_0^4 x dx = 4 \quad \longrightarrow [1]$$

$$A_n = \frac{1}{2} \int_0^4 x \cos \frac{n\pi x}{2} dx \quad \longrightarrow [1]$$

$$= 0, \quad n=1, 2, \dots \quad \longrightarrow [1]$$

$$B_n = \frac{1}{2} \int_0^4 x \sin \frac{n\pi x}{2} dx \quad \text{②} \quad \longrightarrow [3]$$

$$= -\frac{4}{n\pi}, \quad n=1, 2, 3, \dots \quad \longrightarrow [1]$$

$$\therefore x = 2 - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{2}.$$

$$\textcircled{3} \quad x_{n+1} = \ln(3 * x_n^2), \quad n=0,1,2,\dots$$

$$x_0 = 3.$$

$$\therefore x_1 = \ln(3 * 3^2) = 3.2958 \quad \rightarrow [2]$$

$$x_2 = \ln(3 * 3.2958^2) = 3.4839 \quad \rightarrow [2]$$

$$x_3 = \ln(3 * 3.4839^2) = 3.5949. \quad \rightarrow [2]$$

$$\textcircled{4} \quad \text{Here } f(x) = x^3 - x^2 - x + 1 \text{ and } x_0 = 1.5.$$

Newton's iterative formula for double root is

$$\begin{aligned} x_{n+1} &= x_n - \frac{2f(x_n)}{f'(x_n)} \\ &= x_n - \frac{2(x_n^3 - x_n^2 - x_n + 1)}{3x_n^2 - 2x_n - 1}, \quad \rightarrow [3] \end{aligned}$$

$$n=0,1,2,\dots$$

$$x_1 = 1.0455 \quad \rightarrow [1]$$

$$x_2 = 1.0005 \quad \rightarrow [1]$$

$$x_3 = 1. \quad \rightarrow [1]$$

(5)

$$f(x) = \sqrt{x}; \quad x \in [15, 21]; \quad h = 2$$

$x$	$f(x)$	$\Delta f$	$\Delta^2 f$	$\Delta^3 f$	
15	3.8730	0.2501			
17	4.1231	0.2358	-0.0143	0.0022	
19	4.3589	0.2237	-0.0121		
21	4.5826				— [4M]

$$f(x) = f_0 + s \Delta f_0 + \frac{s(s-1)}{2} \Delta^2 f_0 + \frac{s(s-1)(s-2)}{3!} \Delta^3 f_0$$

$$f''(x) = \frac{1}{h^2} (\Delta^2 f_0 + (s-1) \Delta^3 f_0) \quad — [2M]$$

$$s = \frac{x - x_0}{h} \Rightarrow s = 2.5$$

$$f''(x_0) = -0.00275 \quad — [2M]$$

6)

$$h = \frac{1}{6}$$

$x:$	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	$\frac{6}{6} = 1$
$y = f(x):$	1 $y_0$	$0.97297$ $y_1$	$0.9$ $y_2$	$0.8$ $y_3$	$0.69231$ $y_4$	$0.59016$ $y_5$	$0.5$ $y_6$

Simpson's  $\frac{1}{3}rd$  rule $\longrightarrow [3M]$ 

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1}{18} [(y_0+y_6) + 2(y_2+y_4) + 4(y_1+y_3+y_5)]$$

$$\int_0^1 \frac{dx}{1+x^2} = 0.785396 \quad \longrightarrow [3M]$$

Again  $\int_0^1 \frac{dx}{1+x^2} = (\tan^{-1} x)_0^1 = \frac{\pi}{4}$

$$\Rightarrow \frac{\pi}{4} = 0.785396$$

$$\pi = 3.141587 \quad \longrightarrow [2M]$$

$$7- \quad x = \frac{(b-a)t + a+b}{2}$$

$$x = \frac{t+1}{2} \quad \text{—— [1m]}$$

$$dx = \frac{1}{2} dt$$

$$\int_0^1 \frac{\sin \pi x \, dx}{\{x(1-x)\}^{3/2}} = \int_{-1}^1 \frac{4 \sin \frac{\pi}{2}(t+1)}{(1-t^2) \sqrt{1-t^2}} dt$$

$$f(t) = \frac{4 \sin \frac{\pi}{2}(t+1)}{1-t^2} \quad \text{—— [4m]}$$

$$\int_{-1}^1 f(t) dt = \frac{\pi}{2} \left( f(-\frac{1}{\pi}) + f(\frac{1}{\pi}) \right)$$

$$= 8\pi \times 0.444016$$

$$= 11.1593 \quad \text{—— [3m]}$$

$$8) \quad AX_0 = \begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ 10 \end{pmatrix} = 10 \begin{pmatrix} -0.1 \\ 0.4 \\ 1 \end{pmatrix} \quad [2]$$

$$AX_1 = \begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix} \begin{pmatrix} -0.1 \\ 0.4 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 4.5 \\ 11.7 \end{pmatrix} = 11.7 \begin{pmatrix} 0.008547 \\ 0.38462 \\ 1 \end{pmatrix} \quad [2]$$

$$AX_2 = \begin{pmatrix} 0.16241 \\ 4.7949 \\ 11.530 \end{pmatrix} = 11.530 \begin{pmatrix} 0.014086 \\ 0.41586 \\ 1 \end{pmatrix} \quad [2]$$

Part - B

$$1) \text{ Relative error} = \frac{\text{exact value} - \text{approximate value}}{\text{exact value}}$$

$$= 0.13616 \quad [4]$$

$$2) \quad \eta_1 = \frac{1}{3}(1 + \eta_2 - \eta_3); \quad \eta_2 = \frac{1}{6}(-3\eta_1 - 2\eta_3) \quad \text{and}$$

$$\eta_3 = \frac{1}{7}(4 - 3\eta_1 - 3\eta_2)$$

$$\eta_1^{(1)} = \frac{1}{3} = 0.33333; \quad \eta_2^{(1)} = -\frac{1}{6} = -0.16667; \quad \eta_3^{(1)} = \frac{1}{2} = 0.5$$

$$\eta_1^{(2)} = \frac{1}{9} = 0.11111; \quad \eta_2^{(2)} = -\frac{2}{9} = -0.22222; \quad \eta_3^{(2)} = \frac{13}{21} = 0.61905$$

$x$	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	1			
0.1	1.2214	0.1714	-0.076	
0.5	1.4918	0.676	5.254	13.883
0.6	1.8221	0.303		

[5]

$$f(n) = f(n_0) + (n - n_0) \Delta y_0 + (n - n_0)(n - n_1) \Delta^2 y_0 \\ + (n - n_0)(n - n_1)(n - n_2) \Delta^3 y_0$$

$$= 1.1340 \quad [3]$$

$$4) \quad (A, B) = \begin{pmatrix} 6 & 3 & 1 & 12 \\ 1 & 5 & 2 & 3 \\ 2 & 4 & 7 & 21 \end{pmatrix}$$

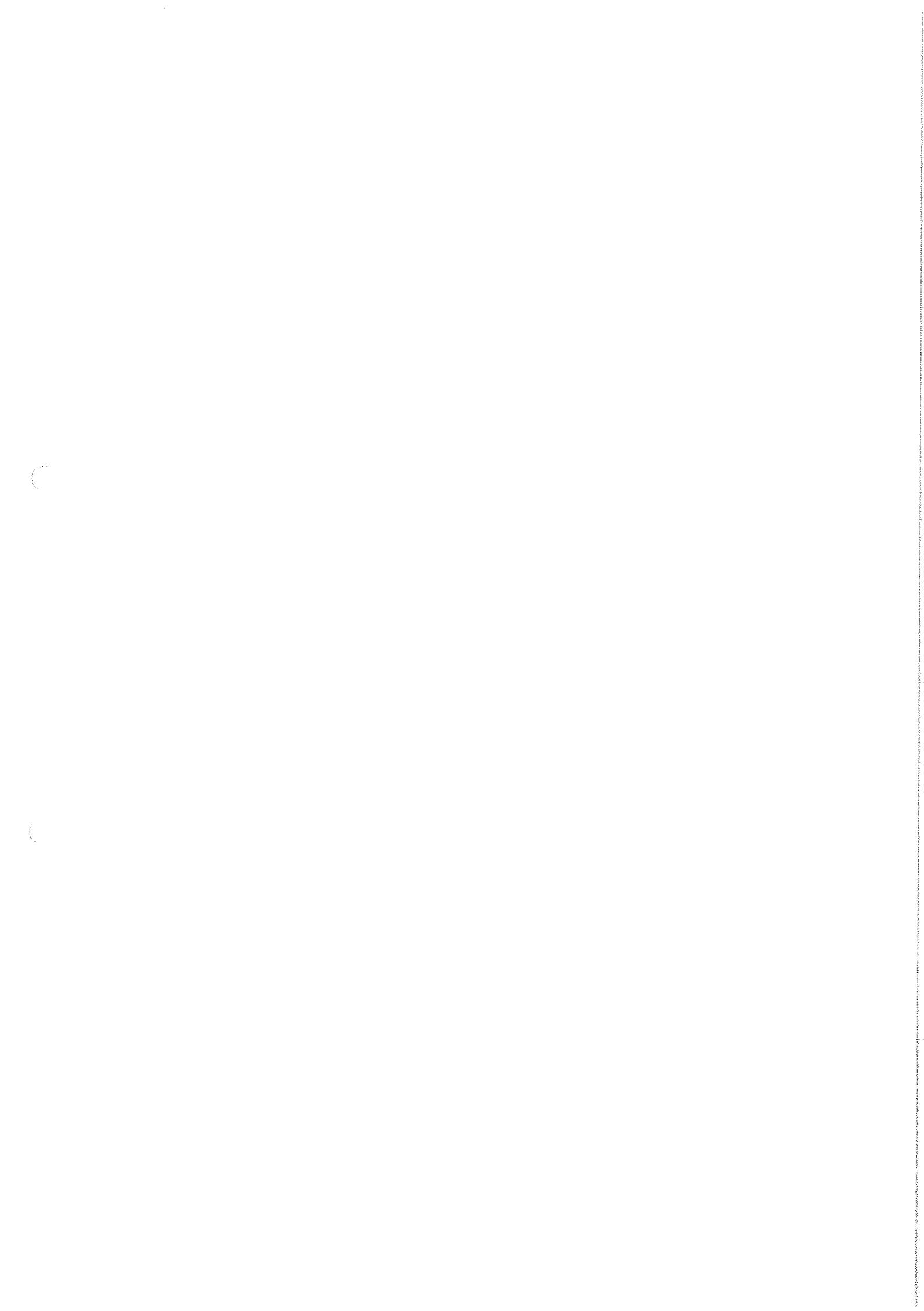
$$= \begin{pmatrix} 1 & 0.5 & 0.16667 & 2 \\ 0.2 & 1 & 0.4 & 0.6 \\ 0.28571 & 0.57143 & 1 & 3 \end{pmatrix} \quad R_2 \rightarrow R_2 - 0.2R_1 \\ R_3 \rightarrow R_3 - 0.28571R_1$$

$$= \begin{pmatrix} 1 & 0.5 & 0.16667 & 2 \\ 0 & 0.9 & 0.36667 & 0.2 \\ 0 & 0.42857 & 0.95238 & 2.4286 \end{pmatrix} \quad R_3 \rightarrow R_3 - \frac{0.42857}{0.9}R_2 \\ R_3 \rightarrow R_3 - 0.4762R_2$$

$$= \begin{pmatrix} 1 & 0.5 & 0.16667 & 2 \\ 0 & 0.9 & 0.36667 & 0.2 \\ 0 & 0 & 0.77778 & 2.3334 \end{pmatrix}$$

$$\eta_3 = 3 ; \quad \eta_2 = \frac{0.2 - 0.36667 \times 3}{0.9} = -1$$

$$\eta_1 = 2 - 0.5(-1) - 0.16667(3) = 2 \quad [10]$$



$$3) \quad y_{4,1} = y_3 + \frac{h}{24} [55f_3 - 59f_2 + 37f_1 - 9f_0]$$

$$= 1.0997 + \frac{0.1}{24} [55 \times 0.6997 - 59 \times 0.4428$$

$$+ 37 \times 0.2103 - 9 \times 0]$$

$$= 1.1836$$

[5]

$$y_{4,2} = y_3 + \frac{h}{24} [9f_4 + 19f_3 - 5f_2 + f_1]$$

$$= 1.0997 + \frac{0.1}{24} [9 \times 0.9836 + 19 \times 0.6997$$

$$- 5 \times 0.4428 + 0.2103]$$

$$= 1.1836$$

[3]

$$4) \quad \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} - \left( \frac{y_{i+1} - y_i}{h} \right) + y_i = 3e^{2x_i} - 2\sin x_i$$

$$-1.6875y_1 + 0.75y_2 = -4.1428$$

$$y_1 - 1.6875y_2 + 0.75y_3 = 3.6414$$

$$y_2 - 1.6875y_3 = -35.486$$

[5]

$$y_1 = 10.572; \quad y_2 = 18.263; \quad y_3 = 31.851$$

[3]



**BITS Pilani, Dubai Campus**  
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**Third year – Second semester 2012 – 2013**  
**AAOC C341 – Numerical Analysis**

**Test - 2 (Open Book)**

**Date: 25.04.2013**

**Time: 50 Minutes**

**Max. Marks: 60**

**Weightage: 20%**

**ANSWER ALL QUESTIONS**

1. (a) Let  $f(x) = e^{2x}$  defined in [2,3]. Use Newton's forward interpolation method to find  $f^{iv}(x)$  at  $x = 2.5$ . Take  $h = 0.2$  and use 4 digit arithmetic with rounding. [10]

(b) Express first three terms of  $\frac{e^{2x} - e^{-2x}}{2}$  in terms of Chebyshev polynomial. [8]

2. a) Evaluate the following integral by dividing the range of  $x$  in 9 equal subintervals using Simpson's  $\frac{3}{8}$  rule. Use 6-digit arithmetic with rounding. [8]

$$\int_0^{0.9} \frac{x + e^{\cos x}}{1 + x^2} dx$$

- b) Evaluate the following integral by 3-point Gauss-Legendre quadrature. Use 6-digit arithmetic with rounding. [7]

$$\int_1^4 \frac{x^2 + 2x - 1}{\sqrt{1+x^2}} dx$$

3. Fit a natural cubic spline curve and evaluate the spline value at  $x = 5$  for the following data using 6 digit arithmetic. [15]

x:	3	4.5	7	9
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f(x):	2.5	1	2.5	0.5
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4. Find the half range Fourier Sine series approximation of the given function in the interval  $(0, 2)$ .

$$f(x) = e^{-x} \sin(2x - 1) \quad [12]$$

AAOC C341 – Numerical Analysis  
 Test - 2 (Open Book)

Date: 25.04.2013  
 Time: 50 Minutes

Max. Marks: 60  
 Weightage: 20%

ANSWER ALL QUESTIONS

1) (a)

$x$	$f(x)$	$\Delta f_0$	$\Delta^2 f_0$	$\Delta^3 f_0$	$\Delta^4 f_0$	$\Delta^5 f_0$
2	54.60	26.85				
2.2	81.45	40.05	13.2			
2.4	121.5	59.8	19.75	6.55		
2.6	181.3	89.1	29.3	9.55	3	
2.8	270.4	109.1	43.9	14.6	5.05	
3.0	403.4	133				2.05

[6]

$$f(6) = f_0 + s \Delta f_0 + \frac{s^2 - s}{2} \Delta^2 f_0 + \frac{s^3 - 3s^2 + 2s}{6} \Delta^3 f_0 + \frac{s^4 - 6s^3 + 11s^2 - 6s}{24} \Delta^4 f_0 \\ + \frac{s^5 - 10s^4 + 35s^3 - 50s^2 + 24s}{120} \Delta^5 f_0$$

$$f^{(IV)}(x) = \frac{1}{h^4} [\Delta^4 f_0 + (s-2) \Delta^5 f_0]$$

$$h = 0.2 \quad s = \frac{x - x_0}{h} = \frac{2.5 - 2}{0.2} = 2.5$$

$$f^{(IV)}(2.5) = 2515.6 \approx 2516$$

[7]

1) (b)

$$e^{2x} = 1 + \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} + \frac{(2x)^5}{5!}$$

$$e^{-2x} = 1 - \frac{2x}{1!} + \frac{(2x)^2}{2!} + \frac{(2x)^3}{3!} + \frac{(2x)^4}{4!} - \frac{(2x)^5}{5!}$$

$$\frac{e^{2x} - e^{-2x}}{2} = 2x + \frac{8}{3} x^3 + \frac{32}{15} x^5 \quad (4)$$

$$= 2T_1 + \frac{8}{3} \times \frac{1}{4} (T_3 + 3T_1) + \frac{32}{15} \times \frac{1}{16} (T_5 + 5T_3 + 10T_1)$$

$$\boxed{\frac{e^{2x} - e^{-2x}}{2} = \frac{19}{6} T_1 + \frac{5}{12} T_3 + \frac{1}{60} T_5} \quad (4)$$

$$= 3.17 T_1 + 0.417 T_3 + 0.0167 T_5$$

Q.No.: 2(a)

$i$	0	1	2	3	4	5	6
$x_i$	0	0.1	0.2	0.3	0.4	0.5	0.6
$f_i$	2.71828	2.77697	2.75446	2.66013	2.51031	2.32406	2.11959

$i$	7	8	9
$x_i$	0.7	0.8	0.9
$f_i$	1.91185	1.71167	1.52592

(5 marks)  
 $(\frac{1}{2} \text{ mk. for each correct pair } (x_i, f_i))$

Therefore,

$$\int_0^{0.9} \frac{x + e^{\cos x}}{1+x^2} dx = \frac{3 \cdot h}{8} [f_0 + f_9 + 2 * (f_3 + f_6) + 3 * (f_1 + f_2 + f_4 + f_5 + f_7 + f_8)] \quad [1]$$

$$= 2.09143. \quad [2]$$

Q.No.: 2(b)

By substituting  $x = \frac{3u+5}{2}$ , we get  $u = \frac{2x-5}{3}$  and  $dx = \frac{3}{2} du$ . [1]

Integrand is

$$g(u) = \frac{3\{(3u+5)^2 + 4(3u+5) - 4\}}{4 * \sqrt{4 + (3u+5)^2}} \quad [1]$$

Hence the given integral becomes,

$$\int_{-1}^1 g(u) du.$$

By 3-point Gauss-Legendre quadrature,

$$\int_{-1}^1 g(u) du = \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right) \quad [1] \rightarrow [3]$$

$[i \text{ for each correct term}]$

$$= \int_{-1}^1 g(u) du = \frac{5}{9} * 3.11293 + \frac{8}{9} * 5.71013 + \frac{5}{9} * 7.79769$$

$$= 11.1371. \quad [1]$$

$$3) \quad h_0 = 1.5, \quad h_1 = 2.5, \quad h_2 = 2$$

$x$	$f(x)$	$f[x_i, x_{i+1}]$
3	2.5	-1
4.5	1	0.6
7	2.5	-1
9	0.5	

$$\begin{pmatrix} 8 & 2.5 \\ 2.5 & 9 \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = 6 \begin{pmatrix} 1.6 \\ -1.6 \end{pmatrix} = \begin{pmatrix} 9.6 \\ -9.6 \end{pmatrix}$$

Solving we get  $s_1 = 1.67909$ ;  $s_2 = -1.53308$  [7]

$$a_1 = \frac{s_2 - s_1}{6h_1} = -0.214145$$

$$b_1 = \frac{s_1}{2} = 0.839545$$

$$c_1 = \frac{y_2 - y_1}{h_1} - \frac{2h_1 s_1 + h_1 s_2}{6} = -0.160458$$

$$d_1 = y_1 = 1$$

$$g_1(x) = -0.214145(x - 4.5)^3 + 0.839545(x - 4.5)^2 - 0.160458(x - 4.5) + 1$$

$$g_1(5) = 11.0289 \quad [8]$$

$$4) f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right)$$

$$\begin{aligned}
 b_n &= \int_0^2 e^{-x} \sin((2n-1)\sin\left(\frac{n\pi x}{2}\right)) dx \\
 &= \int_0^2 e^{-x} \sin\left(\frac{n\pi x}{2}\right) (\sin 2n \cos 1 - \cos 2n \sin 1) dx \\
 &= \cos 1 \int_0^2 \left[ e^{-x} \sin\left(\frac{n\pi x}{2}\right) \sin 2n - \sin\left(\frac{n\pi x}{2}\right) \right] dx \\
 &\quad - \sin 1 \int_0^2 e^{-x} \sin\left(\frac{n\pi x}{2}\right) [\cos 2n] dx \\
 &= \frac{\cos 1}{2} \int_0^2 e^{-x} \left[ \cos\left(\frac{n\pi x}{2} - 2n\right) - \cos\left(\frac{n\pi x}{2} + 2n\right) \right] dx \\
 &\quad - \frac{\sin 1}{2} \int_0^2 e^{-x} \left[ \sin\left(\frac{n\pi x}{2} + 2n\right) + \sin\left(\frac{n\pi x}{2} - 2n\right) \right] dx \tag{5} \\
 &= \frac{\cos 1}{2} \left[ \frac{e^{-x}}{1 + \left(\frac{n\pi}{2} - 2\right)^2} \left( (-1) \cos\left(\frac{n\pi x + 4\pi}{2}\right) + \left(\frac{n\pi - 4}{2}\right) \sin\left(\frac{n\pi x - 4\pi}{2}\right) \right) \right. \\
 &\quad \left. - \left[ \frac{e^{-x}}{1 + \left(\frac{n\pi}{2} + 2\right)^2} \left( -\cos\left(\frac{n\pi x + 4\pi}{2}\right) + \left(\frac{n\pi + 4}{2}\right) \sin\left(\frac{n\pi x + 4\pi}{2}\right) \right) \right] \right. \\
 &\quad \left. - \frac{\sin 1}{2} \left[ \frac{e^{-x}}{1 + \left(\frac{n\pi}{2} + 2\right)^2} \left( -\sin\left(\frac{n\pi x + 4\pi}{2}\right) - \left(\frac{n\pi + 4}{2}\right) \cos\left(\frac{n\pi x + 4\pi}{2}\right) \right) \right. \right. \\
 &\quad \left. \left. + \left( \frac{e^{-x}}{1 + \left(\frac{n\pi}{2} - 2\right)^2} \left( -\sin\left(\frac{n\pi x - 4\pi}{2}\right) - \left(\frac{n\pi - 4}{2}\right) \cos\left(\frac{n\pi x - 4\pi}{2}\right) \right) \right) \right] \right. \\
 &= \frac{\cos 1}{2} \left[ \frac{e^{-2}}{1 + \left(\frac{n\pi}{2} - 2\right)^2} \left( -\cos(n\pi - 4) + \left(\frac{n\pi - 4}{2}\right) \sin(n\pi - 4) \right) \right. \\
 &\quad \left. + \frac{1}{1 + \left(\frac{n\pi}{2} - 2\right)^2} \right]
 \end{aligned}$$

$$\begin{aligned}
& - \frac{\cos 1}{2} \left[ \frac{e^{-2}}{1 + \left(\frac{n\pi}{2} + 2\right)^2} \left( -\cos(n\pi + 4) + \left(\frac{n\pi}{2} + 4\right) \sin(n\pi + 4) \right) \right. \\
& \quad \left. + \frac{1}{1 + \left(\frac{n\pi}{2} + 2\right)^2} \right] \\
& - \frac{\sin 1}{2} \left[ \frac{e^{-2}}{1 + \left(\frac{n\pi}{2} + 2\right)^2} \left( -\sin(n\pi + 4) - \left(\frac{n\pi}{2} + 4\right) \cos(n\pi + 4) \right) \right. \\
& \quad \left. + \frac{1}{1 + \left(\frac{n\pi}{2} + 2\right)^2} \left( \frac{n\pi}{2} + 4 \right) \right] \\
& - \frac{\sin 1}{1} \left[ \frac{e^{-2}}{1 + \left(\frac{n\pi}{2} - 2\right)^2} \left( -\sin(n\pi - 4) - \left(\frac{n\pi}{2} - 4\right) \cos(n\pi - 4) \right) \right. \\
& \quad \left. + \frac{1}{1 + \left(\frac{n\pi}{2} - 2\right)^2} \left( \frac{n\pi}{2} - 4 \right) \right] \quad (\#)
\end{aligned}$$

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**Third year – Second semester 2012 – 2013**

**Numerical Analysis (AAOC C341)**

**Test - 1 (Closed Book)**

**Date: 07.03.2013**

**Time: 50 Minutes**

**Max. Marks: 75**

**Weightage: 25%**

**Answer ALL Questions**

1. Evaluate the cubic polynomial  $f(x) = 3.178x^3 + 0.931x^2 - 0.079x + 0.541$  at  $x = 0.234$  using five digit arithmetic with rounding in nested form. Write the advantage of using the nested form and also find the relative error. **[10]**
2. Consider the following equation:  $4e^x - 15 = 0$ . Perform two iterations of secant method using 5-digit arithmetic with rounding starting with  $x_0 = 1$ ,  $x_1 = 1.2$ . **[15]**
3. Find a root of the nonlinear equation  $f(x) = x^3 + 2x - 1$  with the values 0.5, 1.0, 1.5 at the end of first iteration by Muller's method using 5 digit arithmetic with rounding. Also write the starting values for the next iteration. **[15]**
4. Find a root of the equation  $x^4 - x - 9 = 0$  using Newton's method starting with  $x_0 = 2$ . Do two iterations using 5-digit arithmetic with rounding. **[10]**
5. For  $f(x) = e^{-x} - 3 \ln x$  defined in the interval [1,2], write two different possible iterative functions and check the conditions of convergence of fixed point method. **[10]**
6. Solve the system of nonlinear equations  $y - x^3 = 0$  and  $36 - 4x^2 - 9y^2 = 0$  using Newton's method starting with  $x = 1$ ,  $y = 2$ . Find the root at the end of first iteration using 5-digit arithmetic with rounding. **[15]**

BITS Pilani, Dubai Campus  
Dubai International Academic City, Dubai  
Third year – Second semester 2012 – 2013

AAOC C341 – Numerical Analysis  
Test - 1 (Closed Book)

Date: 07.03.2013

Time: 50 Minutes

Max. Marks: 75

Weightage: 25%

MARKING SCHEME

$$\begin{aligned}
 1) \quad f(x) &= 3.178x^3 + 0.931x^2 - 0.079x + 0.541 \\
 &= ((3.178x + 0.931)x - 0.079)x + 0.541 \quad [3] \\
 &= 0.61421 \quad [3]
 \end{aligned}$$

$$\text{Exact value} = 0.614211244$$

$$\begin{aligned}
 \text{Relative error} &= \frac{0.614211244 - 0.61421}{0.614211244} \\
 &= 2.0254 \times 10^{-6} \quad [2]
 \end{aligned}$$

Advantage is we use less number of operations [2]

$$2) \quad x_{n+1} = x_n - \frac{f(x_n)}{f(x_{n-1}) - f(x_n)} \cdot \frac{x_{n-1} - x_n}{f(x_{n-1}) - f(x_n)}$$

Iteration (n)	$x_{n-1}$	$f(x_{n-1})$	$f(x_n)$	$x_{n+1}$
1	1	1.2	-1.7195	1.3429
2	1.2	1.3429	0.32054	1.3204

$$3) \quad f(x) = x^3 + 2x - 1$$

$$x_2 = 0.5 \quad x_0 = 1.0 \quad x_1 = 1.5$$

$$h_1 = 0.5 \quad h_2 = 0.5$$

$$V = 1$$

$$a = \frac{f_1 - 2f_0 + f_2}{2h_1^2} \Rightarrow a = 3$$

$$b = \frac{f_1 - f_0 - 3h_1^2}{h_1} \Rightarrow b = 5.25$$

$$c = 2$$

$$x_8 = x_0 - \frac{2c}{b \pm \sqrt{b^2 - 4ac}}$$

$$x_1 = 0.43958$$

The starting values for the next iteration

will be 0.43958, 0.5 and 1.

$$f(x) = x^4 - x - 9 = 0 \quad f'(x) = 4x^3 - 1$$

$$\therefore f(0) = -9 \quad f(1) = -9 \quad f(2) = 5$$

$\Rightarrow f(2)$  is nearer to 0

$\Rightarrow$  Initial approximate root  $x_0 = 2$ .

Iterations	$x_{i-1}$	$f(x_{i-1})$	$f'(x_{i-1})$	$x_i$
1	2	5	31	1.8387
2	1.8387	0.59123	23.865	1.8139
3	1.8139	0.011735	22.873	1.8134
4	1.8134	0.0030320	22.853	1.8134

[10]

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

5)  $f(x) = e^{-x} - 3 \ln x = 0$

$$\begin{aligned} i) \quad x &= -\ln[3 \ln x] \\ ii) \quad x &= e^{\frac{-x}{3}} \end{aligned} \quad ] \quad (5m)$$

(i) does not converge in the interval  $[1, 2]$ , since

if  $g(x) = -\ln(3 \ln x)$ ,  $g(1)$  does not exist.

(ii) converges in the interval  $[1, 2]$ . — [5m]

6) Newton's method:

$$f(x, y) = y - x^3 = 0$$

$$g(x, y) = 36 - 4x^2 - 9y^2 = 0,$$

$$x_i = x_{i-1} - \left[ \frac{fg_y - g f_y}{f_x g_y - f_y g_x} \right]_{(x_{i-1}, y_{i-1})}$$

$$y_i = y_{i-1} - \left[ \frac{g f_x - f g_x}{f_x g_y - f_y g_x} \right]_{(x_{i-1}, y_{i-1})} \quad (5m)$$

Iteration 1:  $x_0 = 1, y_0 = 2$ .

$$f_x = -3x^2 \quad f_y = 1, \quad g_x = -8x \quad g_y = -18y$$

$$x_1 = 1 - (-0.27586) = 1.2759$$

$$y_1 = 2 - (0.17241) = 1.8276 \quad ] \quad (10m)$$

BITS Pilani, Dubai Campus  
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III YEAR II SEMESTER 2012-13  
QUIZ – 2 (Closed Book)

B

**Course Title:** Numerical Analysis  
**Date:** 16.05.2013  
**Time:** 20 minutes

**Course No:** AAOC C341  
**Weightage:** 7%  
**Max marks:** 21

**Name of the Student:** \_\_\_\_\_

**ID No:** \_\_\_\_\_ **Name of the Faculty:** \_\_\_\_\_

1. Consider the initial value problem  $\frac{dy}{dx} = x^2 y, y(2) = 1.5$ . For step size  $h$ , the 3<sup>rd</sup> order Taylor series for determining  $y(2+h)$  is obtained as  $y(2+h) = A + Bh + Ch^2 + Dh^3$ . Find the values of  $A, B, C, D$ . [6]

2. Solve the following differential equation by modified Euler's method to find  $y(1.5)$  with  $h = 0.5$ . Use 5 digit arithmetic with rounding.

$$\frac{dy}{dx} = -2x + y, y(1) = -1 \quad [5]$$

3. Solve  $\frac{dy}{dx} = x^2 + y^2$  with initial condition  $y(2) = 3$  by using 2<sup>nd</sup> order Runge Kutta (RK) method. By taking  $h = 0.1$ , find  $y(2.1)$ . Use five digit arithmetic with rounding. [5]

4. Given  $\frac{dy}{dx} = \frac{1}{x+y}$ ;  $y(0) = 2$ . If  $y(0.2) = 2.09$ ,  $y(0.4) = 2.17$ ,  $y(0.6) = 2.24$  find  $y(0.8)$   
using Milne's predictor formula  $y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$ . Use 5 digit arithmetic  
with rounding. [5]

BITS Pilani, Dubai Campus  
 Dubai International Academic City, Dubai  
 III YEAR II SEMESTER 2012-13  
 QUIZ – 2 (Closed Book)

B

**Course Title:** Numerical Analysis  
**Date:** 16.05.2013  
**Time:** 20 minutes

**Course No:** AAOC C341  
**Weightage:** 7%  
**Max marks:** 21

Name of the Student: \_\_\_\_\_

ID No: \_\_\_\_\_ Name of the Faculty: \_\_\_\_\_

1. Consider the initial value problem  $\frac{dy}{dx} = x^2 y, y(2) = 1.5$ . For step size  $h$ , the 3<sup>rd</sup> order Taylor series for determining  $y(2+h)$  is obtained as  $y(2+h) = A + Bh + Ch^2 + Dh^3$ . Find the values of  $A, B, C, D$ . [6]

$$y'_0 = 6, \quad y''_0 = 30, \quad y'''_0 = 171$$

$$\begin{aligned} y(2+h) &= y_0 + h y'_0 + \frac{h^2}{2} y''_0 + \frac{h^3}{6} y'''_0 \\ &= 1.5 + 6h + 15h^2 + \frac{171}{6} h^3. \end{aligned}$$

$$\therefore A = 1.5, \quad B = 6, \quad C = 15, \quad D = 28.5$$

[67]

2. Solve the following differential equation by modified Euler's method to find  $y(1.5)$  with  $h = 0.5$ . Use 5 digit arithmetic with rounding.

$$\frac{dy}{dx} = -2x + y, y(1) = -1 \quad [5]$$

$$y_1^* = y_0 + h f(x_0, y_0)$$

$$= -2.5$$

$$y_1 = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^*)]$$

$$= -3.125$$

[5]

3. Solve  $\frac{dy}{dx} = x^2 + y^2$  with initial condition  $y(2) = 3$  by using 2<sup>nd</sup> order Runge Kutta (RK) method. By taking  $h = 0.1$ , find  $y(2.1)$ . Use five digit arithmetic with rounding. [5]

$$k_1 = h f(x_0, y_0) = 1.3$$

$$k_2 = h f(x_0 + h, y_0 + k_1) = 2.29$$

$$y = y_0 + \frac{1}{2} (k_1 + k_2) = 4.795$$

[5]

4. Given  $\frac{dy}{dx} = \frac{1}{x+y}$ ;  $y(0) = 2$ . If  $y(0.2) = 2.09$ ,  $y(0.4) = 2.17$ ,  $y(0.6) = 2.24$  find  $y(0.8)$   
 using Milne's predictor formula  $y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$ . Use 5 digit arithmetic  
 with rounding. [5]

$$y'_1 = \frac{1}{y_1 + y_1} = 0.43668$$

$$y'_2 = \frac{1}{y_2 + y_2} = 0.38911$$

$$y'_3 = \frac{1}{y_3 + y_3} = 0.35211$$

$$y'_{4,p} = 2.3169$$

[5]

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III YEAR II SEMESTER 2012-13  
QUIZ - I (Closed Book)

A

Course Title: Numerical Analysis  
Date: 28.03.2013  
Time: 20 minutes

Course No: AAOC C341  
Weightage: 8%  
Max marks: 24

Name of the Student: \_\_\_\_\_

ID No: \_\_\_\_\_ Name of the Faculty: \_\_\_\_\_

1. For the matrix  $A = \begin{pmatrix} 6 & -2 & 4 \\ -3 & 6 & -1 \\ -5 & -2 & 6 \end{pmatrix}$  find  $\|A\|_F$ ,  $\|A\|_\infty$ ,  $\|A\|_1$ . [6]

2. Find a solution of the following system of equation at the end of first iteration by Gauss Seidel method starting with  $x_1^{(0)} = 2.756$ ,  $x_2^{(0)} = 1.857$ ,  $x_3^{(0)} = -3.659$ . Use 5 digit arithmetic with rounding.

[6]

$$3x_1 + 0.4x_2 - 5x_3 = 29.8$$

$$4x_1 + 3x_2 + 0.7x_3 = 15.2$$

$$0.6x_1 + 4x_2 - 3x_3 = 21.8$$

3. Fit a cubic through the first four points of the following data and use it to find the interpolated value for  $x = 2.96$  using Lagrange's interpolation formula. Use 5 digit arithmetic with rounding.

$x:$	3.2	2.7	1	4.8	5.6	
$f(x):$	22.0	17.8	14.2	38.3	51.7	[6]

4. Form the divided difference table for the following data using 5 digit arithmetic with rounding.

[6]

$x:$	2	2.5	2.8	3.2
$f(x):$	0.301	0.345	0.380	0.915

BITS Pilani, Dubai Campus  
 Dubai International Academic City, Dubai  
 III YEAR II SEMESTER 2012-13  
 QUIZ - I (Closed Book)

A

**Course Title:** Numerical Analysis  
**Date:** 28.03.2013  
**Time:** 20 minutes

**Course No:** AAOC C341  
**Weightage:** 8%  
**Max marks:** 24

**Name of the Student:** \_\_\_\_\_

**ID No:** \_\_\_\_\_ **Name of the Faculty:** \_\_\_\_\_

1. For the matrix  $A = \begin{pmatrix} 6 & -2 & 4 \\ -3 & 6 & -1 \\ -5 & -2 & 6 \end{pmatrix}$  find  $\|A\|_f, \|A\|_\infty, \|A\|_1$ . [6]

$$\|A\|_1 = \max \{14, 10, 11\} = 14 \quad [2]$$

$$\|A\|_\infty = \max \{12, 10, 13\} = 13 \quad [2]$$

$$\begin{aligned} \|A\|_F &= \sqrt{(36+4+16+9+36+1+25+4+36)} \\ &= \sqrt{167} \end{aligned} \quad [2]$$

2. Find a solution of the following system of equation at the end of first iteration by Gauss Seidel method starting with  $x_1^{(0)} = 2.756$ ,  $x_2^{(0)} = 1.857$ ,  $x_3^{(0)} = -3.659$ . Use 5 digit arithmetic with rounding. [6]

$$3x_1 + 0.4x_2 - 5x_3 = 29.8$$

$$4x_1 + 3x_2 + 0.7x_3 = 15.2$$

$$0.6x_1 + 4x_2 - 3x_3 = 21.8$$

The coefficient matrix is not diagonally dominant. Hence we rearrange as follows:

$$x_1 = \frac{1}{4} (15.2 - 3x_2 - 0.7x_3)$$

$$x_2 = \frac{1}{4} (21.8 - 0.6x_1 + 3x_3)$$

$$x_3 = \frac{1}{5} (3x_1 + 0.4x_2 - 29.8)$$

$$x_1^{(1)} = \frac{1}{4} [15.2 - 3 \cdot 1.857 - 0.7(-3.659)] = 3.0476 \quad (2)$$

$$x_2^{(1)} = \frac{1}{4} [21.8 - 0.6 \cdot 3.0476 + 3(-3.659)] = 2.4736 \quad (2)$$

$$x_3^{(1)} = \frac{1}{5} [3 \cdot 3.0476 + 0.4 \cdot 2.4736 - 29.8] = -3.9336 \quad (2)$$

3. Fit a cubic through the first four points of the following data and use it to find the interpolated value for  $x = 2.96$  using Lagrange's interpolation formula. Use 5 digit arithmetic with rounding.

$x:$	3.2	2.7	1	4.8	5.6	
$f(x):$	22.0	17.8	14.2	38.3	51.7	[6]

$$\begin{aligned}
 f(2.96) &= \frac{(2.96 - 2.7)(2.96 - 1)(2.96 - 4.8)}{(3.2 - 2.7)(3.2 - 1)(3.2 - 4.8)} 22 + \\
 &\quad \frac{(2.96 - 3.2)(2.96 - 1)(2.96 - 4.8)}{(2.7 - 3.2)(2.7 - 1)(2.7 - 4.8)} 17.8 + \\
 &\quad \frac{(2.96 - 3.2)(2.96 - 2.7)(2.96 - 4.8)}{(1 - 3.2)(1 - 2.7)(1 - 4.8)} 14.2 + \\
 &\quad \frac{(2.96 - 3.2)(2.96 - 2.7)(2.96 - 1)}{(4.8 - 3.2)(4.8 - 2.7)(4.8 - 1)} 38.3 \\
 &= 19.870 \quad [6]
 \end{aligned}$$

4. Form the divided difference table for the following data using 5 digit arithmetic with rounding.

[6]

$x:$	2	2.5	2.8	3.2
$f(x):$	0.301	0.345	0.380	0.915

$x$	$f(x)$	$\Delta$	$\Delta^2$	$\Delta^3$
2	0.301			
2.5	0.345	0.088	0.035838	
2.8	0.380	0.11667	1.7440	
3.2	0.915	1.3375		1.4235