BITS Pilani, Dubai Campus
Dubai International Academic City, Dubai
Third year – Second semester 2011 – 2012

AAOC C341 – Numerical Analysis
Comprehensive Examination (Closed Book)

Date: 05.06.2012 **Max. Marks: 120**
Time: 3 hours **Weightage: 40%**

ANSWER PART – A AND PART – B SEPARATELY

PART – A

1. Find the truncation error in approximating $y(x)$ by $\tilde{y}_1(x) = x$ at $x = 0.5$, where
$$y(x) = \ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots. \quad \textbf{[5]}$$

2. Find a root of the following equation in the interval (0, 1) at the end of third iteration using Bisection method with 5 digit approximation and rounding:
$$\cos x - xe^x = 0 \quad \textbf{[5]}$$

3. Find a root of the equation $x^3 - 5x + 1 = 0$ at the end of first iteration by Muller's method starting with $x = 0, 0.5, 1$. Use 5 digit approximation and rounding. Also state the starting values for the next iteration. **[10]**

4. Find the 1, 2 and ∞ norms of the following matrix:
$$A = \begin{pmatrix} 5 & -9 & 6 \\ 2 & -7 & 4 \\ 1 & 5 & 8 \end{pmatrix} \quad \textbf{[5]}$$

5. Find the inverse of the following matrix using Gauss elimination method:
$$A = \begin{pmatrix} 0 & 2 & 4 \\ 2 & 4 & 6 \\ 6 & 2 & 2 \end{pmatrix} \quad \textbf{[8]}$$

6. Solve the following system of equations by Gauss Seidel method starting with (0, 0, 0). Do 2 iterations with 5 digit approximation and rounding.
$$\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned} \quad \textbf{[7]}$$

7. Find the missing value using Newton's divided difference interpolation:

x:	1	2	4	5	6
$f(x)$:	14	15	5	-	9

[10]

8. Find $y(1.5)$ and $y'(1)$ for the following data using natural cubic spline with 5 digit approximation and rounding:

x:	-1	0	1	3
y:	-1	1	3	35

[10]

PART – B

1. Find the approximate value of $f'(0.25)$ for the following values of $f(x)$ using 7 digit approximation and rounding:

x:	0.2	0.4	0.6
$f(x)$:	0.9798652	0.9177710	0.8080348

[5]

2. The axial displacement (*du*) of an elemental length (*dx*) of a bar of length 1 unit under a load *P* is given by $\frac{du}{dx}=\frac{P}{EA}$ where *E* is Young's modulus and *A* is the cross-sectional area. Determine the axial displacement of the bar for the data P = 500 lb, $E=30x^{-2}$ psi and $A=2e^{-x}$ by Simpson's $1/3$ rule by taking $h=\frac{1}{6}$ with 5 digit approximation and rounding. **[8]**

3. Find a, x_1, b, x_2 so that the following rule is exact:

$$\int_{-1}^{1}\frac{f(x)}{\sqrt{1-x^2}}dx=af(x_1)+bf(x_2)$$ **[8]**

4. Find the Fourier series of the following function on the given interval:

$$f(x)=\begin{cases}0, & -\pi<x<0\\ x^2, & 0\le x<\pi\end{cases}$$

Use the result to show $\frac{\pi^2}{12}=1-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots$ **[12]**

5. Using 4th order Runge-Kutta method evaluate the value of *y* when $x=0.1$ with 5 digit arithmetic and rounding given that

$$\frac{dy}{dx}=-2x-y;\ y(0)=-1.$$ **[10]**

6. Find the solution of the following initial value problem at $x=0.4$ with $h=0.1$ using 5 digit arithmetic and rounding.

$y'=-1+2x+y,\ y_0=1,\ y_1=1.0103,\ y_2=1.0428,\ y_3=1.0997$

Use the following Admas predictor corrector method to get the solution

$$y_{n+1,p}=y_n+\frac{h}{24}(55f_n-59f_{n-1}+37f_{n-2}-9f_{n-3})$$

$$y_{n+1,c}=y_n+\frac{h}{24}(9f_{n+1}+19f_n-5f_{n-1}+f_{n-2})$$ **[7]**

7. Solve the following boundary value problem by finite difference method using 5 digit arithmetic and rounding:

$$\frac{d^2y}{d\theta^2}+\frac{y}{4}=0,\ y(0)=0,\ y(\pi)=2 \text{ with } h=\frac{\pi}{4}.$$ **[5]**

8. Find the largest eigen value and the corresponding eigen vector of the following matrix starting with $(1, 0.5, -0.5)^T$ at the end of second iteration using 5 digit arithmetic and rounding:

$$A=\begin{pmatrix}12 & 6 & -6\\ 6 & 16 & 2\\ -6 & 2 & 16\end{pmatrix}$$ **[5]**

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Third year – Second semester 2011 – 2012

AAOC C341 – Numerical Analysis
Test - 2 (Open Book)

Date: 22.04.2012 **Max. Marks: 60**
Time: 50 Minutes **Weightage: 20%**

ANSWER ALL QUESTIONS

1. Express the function $f(x) = -3x^3 + 8x^2 - 5x + 4$ in terms of Chebyshev polynomials. **[6]**

2. The thermal conductivity of iron(k) is found to vary with temperature(T) as follows:

T	200	600	1000	1400
k	1	0.4	0.3	0.25

Interpolate at $T = 250$ using Gregory Newton's interpolation formula with 4 digit arithmetic. **[12]**

3. Form the divided difference table for the function $f(x) = \frac{1}{x}$, based on the points x_0, x_1, x_2, x_3. **[12]**

4. Fit a natural cubic spline curve corresponding to the interval [2, 3] for the following data with five digit arithmetic. Hence evaluate the spline value at $x = 2.5$.

x:	1	2	3	5
y:	3	10	29	65

[15]

5. When a sinusoidal voltage $E \sin \omega t$ is passed through a half-wave rectifier which clips the negative portion of the wave, the resulting periodic function is given by

$$f(t) = \begin{cases} 0, & -\pi/\omega < t < 0 \\ E \sin \omega t, & 0 < t < \pi/\omega \end{cases},$$

Develop this function in a Fourier series. **[15]**

BITS PILANI, DUBAI CAMPUS

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

THIRD YEAR – SECOND SEMESTER 2011-2012

AAOC C341 – NUMERICAL ANALYSIS

TEST - 1 (CLOSED BOOK)

DATE: 08.03.2012 MAX. MARKS: 75

TIME: 50 MINUTES WEIGHTAGE:25%

ANSWER ALL QUESTIONS

1. Evaluate the cubic polynomial $3.01x^3+4.87x^2+4.53x+1.45$ at $x=-0.123$ using three digit arithmetic with rounding at each arithmetic operation, in nested form. Also find the relative error. **[10]**

2. Find a root of the equation $f(x)=x^3-5x+1$ in the interval [0, 1] at the end of third iteration by Regula-Falsi Method using 5 digits arithmetic with rounding. **[15]**

3. Find a root of multiplicity three by Newton's method for the equation $f(x)=x^4-8x^3+18x^2-16x+5$ starting with $x=2$ using 5 digits arithmetic with rounding at the end of third iteration. **[10]**

4. Use Muller's Method to find a root of the equation $\cos 3x+1=e^{x^2}$ starting with 0, 0.5, 1 at the end of first iteration with five digit arithmetic and rounding. Also state the starting values for the next iteration. **[20]**

5. Find four different rearrangements to find a fixed point for the function $f(x)=x^4-2x-1$ in the interval [1, 2]. Also verify the condition for convergence of all possibilities. **[20]**

BITS Pilani, Dubai Campus
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III YEAR II SEMESTER 2011-12
QUIZ – II (Closed Book)

Course Title: Numerical Analysis | **Course No: AAOC C341**
Date: 15.05.2012 | **Weightage: 7%**
Time: 20 minutes | **Max marks: 21**

Name of the Student: _____

ID No: _____

1. Find $f'(0.1)$ and $f''(0.3)$ from the given data using 3 digit arithmetic with rounding. [7]

x:	0.10	0.20	0.30	0.40	0.50
$f(x)$:	0.425	0.475	0.400	0.450	0.525

2. Evaluate the integral $I = \int_0^{2\pi} \cos^2 x \, dx$ by splitting the interval into 6 subintervals by Trapezoidal rule using 5 digit arithmetic with rounding. [7]

3. Evaluate the integral $I = \int_0^2 ye^{2y} \, dy$ by 2-point Gauss Legendre quadrature using 5 digit arithmetic with rounding. [7]

B

BITS Pilani, Dubai Campus
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III YEAR II SEMESTER 2011-12
QUIZ – 1 (Closed Book)

Course Title: Numerical Analysis | Course No: AAOC C341
Date: 27.03.2012 | Weight age: 8%
Time: 20 minutes | Max marks: 24

Name of the Student: _____

ID No: _____

1. What do you mean by an ill conditioned system and check whether the following system is conditioned or ill conditioned?

$$x_1 + x_2 = 1; \quad 1.001x_1 + x_2 = 2$$

[6]

2. A fin, with a uniform circular section has a root temperature af $140^{\circ}C$ and an ambient temperature of $40^{\circ}C$. It has a thermal conductivity of $k = 70$ and a heat transfer coefficient of $h = 5$. When the convection loss from the end is also considered, the nodal temperatures T_1, T_2 and T_3 are governed by the equation

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 72.67 & -23.83 \\ 0 & -23.83 & 41.33 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \end{pmatrix} = \begin{pmatrix} 140 \\ 4336 \\ 700 \end{pmatrix}$$

Determine the values of the nodal temperatures using Gauss-Jordan method using 4 digit arithmetic with rounding. [8]

3. Find a root of the following system of equations starting with (0, 0, 0) at the end of first iteration using Gauss-Seidel method with 5 digit arithmetic and rounding. What is the minimum number of iterations required to get a solution correct to 5 significant digits?

$$4.63x_1 - 1.21x_2 + 3.22x_3 = 2.22$$

$$-3.07x_1 + 5.48x_2 + 2.11x_3 = -3.17$$

$$1.26x_1 + 3.11x_2 + 4.57x_3 = 5.11$$

[10]