BITS PILANI, DUBAI CAMPUS
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

COMPREHENSIVE EXAMINATION –III YEAR –II SEMESTER 2010-2011

Date: 31/05/11 Course: Numerical Analysis AAOC C341

Duration: 3hours Total Marks: 120 Weightage: 40%

**ANSWER PART – A, PART – B AND PART – C SEPARATELY**

**PART – A**

1. (a) Find the largest eigen value for the matrix at the end of third iteration using power method starting with the vector $(0\ 1\ 0)^T$. Use 5 digit arithmetic with rounding.

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ [5]

b). Find the value of $x^3 - 2.5x^2 + 3.1x - 1.5$ at $x = 1.25$ using nesting. Use 5 digit arithmetic with rounding. [5]

2. a). Find the root of $x^4 - x - 10 = 0$ in the interval (1,2) using the method of Regula Falsi. Use 5 digit arithmetic with rounding and do 3 iterations. [5]

b).Compute the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$ by dividing the interval into 4 subintervals. Use Simpsons $\frac{1}{3}$ rule with 5 digit arithmetic and rounding. [5]

3. Find a root of the equation $\log_{10} x - x + 3 = 0$ at the end of second iteration by Muller's method using 7 digit arithmetic with rounding. Use $\frac{1}{4}, \frac{1}{2}$ and 1, as starting values. [10]

4. a ) Given $f(x) = x^3 - x^2 - x + 1 = 0$ has a double root near $x = 0.8$. Find that root using Newton's method at the end of third iteration with 5 digit arithmetic and rounding. [5]

   b). The function $x^2 - 5 = 0$ has a root in the interval [2, 3]. Give three different iterative functions to find the root and verify the fixed point iteration theorem for the three iterative functions. Find the convergent root at the end of fifth iteration using 5 digit arithmetic and rounding. [5]

**PART – B**

1. a) The following table gives the marks obtained by 100 students in Mathematics in a certain examination. Use Newton's Forward difference interpolation formula to find the number of students got more than 55 marks. [7]

| Marks obtained | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| No. of Students | 25 | 35 | 22 | 11 | 7 |

   b) Using Lagrange's interpolating polynomial for the following data find $y(10)$ with 5 digit arithmetic and rounding. [3]

| $x$: | 5 | 6 | 9 |
|---|---|---|---|
| $y$: | 12 | 13 | 14 |

2. Fit a natural cubic spline curve and evaluate the spline value at $x = 3$ for the following data with 5 digit arithmetic and rounding. [10]

| $x$: | 1 | 2 | 4 | 6 |
|---|---|---|---|---|
| $f(x)$: | $-26$ | 12 | 256 | 844 |

3. Using Gauss Jordan Method find the inverse of the matrix:

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$ [10]

4. a) Solve the system of equations by Gauss Seidal Iteration Method. Do three iterations with five digit arithmetic and rounding. [6]

$$\begin{aligned} 2x - 3y + 20z &= 25 \\ 3x + 20y - z &= -18 \\ 20x + y - 2z &= 17 \end{aligned}$$

b) Explain ill-conditioned systems for the following system of equations $AX = B$ by making a small change in the vector B as $B = (-1.60, 7.23, -3.38)^T$. Also find norm-1 and norm-f for the coefficient matrix. [4]

$$3.02x - 1.05y + 2.53z = -1.61$$
$$4.33x + 0.56y - 1.78z = 7.23$$
$$-0.83x - 0.54y + 1.47z = -3.38$$

**PART – C**

1. Find the general Fourier Series expansion of $f(x) = x(2 - x)$ over $[-2, 2]$. [8]

2. (i) Compute $f'(0.268)$ using Newton's Divided Difference Interpolation formula for the following data with 5 digit arithmetic and rounding: [4]

| $x$: | 0.15 | 0.21 | 0.23 | 0.27 |
|---|---|---|---|---|
| $f(x)$: | 0.1761 | 0.3222 | 0.3617 | 0.4314 |

(ii) Solve the following boundary value problem using 5 digit arithmetic with rounding

$$y'' - 3y' + 2y = 0,\ y(0) = 0,\ y(1) = 1 \text{ with } h = 0.25.$$ [4]

3. Find $a, b, c$ in the following Gaussian quadrature rule so that integration is exact for a polynomial of degree as high as possible [8]

$$\int_0^1 \frac{f(x)}{\sqrt{x(1-x)}}dx = af(0) + bf\left(\frac{1}{2}\right) + cf(1)$$

4. Solve the following differential equation by 4th order Runge-Kutta method to find $x(0.1)$ and $x'(0.1)$ using 5 digit arithmetic with rounding

$$\frac{d^2x}{dt^2} + x\frac{dx}{dt} - 3x = 3t,\ x(0) = 1,\ x'(0) = 1$$ [8]

5. Find $y(1.4)$ using 5 digit arithmetic with rounding by Adams Moulton 4th order Predictor Corrector pair with modifier as solution of

$$\frac{dy}{dx} = x^3 + xy,\ y(1) = 2,\ y(1.1) = 1.6,\ y(1.2) = 0.34,\ y(1.3) = 0.594$$

[8]

BITS Pilani, Dubai Campus
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Third year – Second semester 2010 – 2011

AAOC C341 – Numerical Analysis
Test - 2 (Open Book)

Date: 10.04.2011 Max. Marks: 60
Time: 50 Minutes Weightage: 20%

**ANSWER ALL QUESTIONS**

1. (i) Consider the system of equations

$$x_1 + 3x_2 = 8$$
$$1.0001x_1 + 3x_2 = 8.0002$$

Check whether the system is a conditioned system. **[6]**

(ii) Express the function $f(x) = -2x^3 + 7x^2 - 6x + 3$ in terms of Chebyshev polynomial. **[6]**

2. Solve the following system of equations using Gauss-Jacobi iterative method starting with the initial root (0, 0, 0) and find the root at the end of second iteration with 5 decimal digit arithmetic and rounding. Also find the minimum number of iterations required to get the solution correct to 5 decimal digits.

$$4x_1 - 2x_2 + x_3 = 11$$
$$x_1 + 5x_2 - 3x_3 = -6$$
$$2x_1 + 2x_2 + 5x_3 = 7$$ **[12]**

3. The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the independent variable. Obtain the best estimate for the value of the function at the position 6 using Newton's divided difference interpolation. **[12]**

4. Find the cubic spline corresponding to the interval $[-1,2]$ from the following table:

| $x$: | $-1$ | 2 | 4 | 5 |
|---|---|---|---|---|
| $y$: | 5 | 3 | 8 | 5 |

Hence find $y(-1.5)$. **[12]**

5. Find the general Half range Fourier Sine series expansion of the following function:

$$f(x) = \begin{cases} ax, & 0 \le x \le \frac{l}{2} \\ a(l-x), & \frac{l}{2} \le x \le l \end{cases}$$ **[12]**

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**Third year – Second semester 2010 – 2011**

**AAOC C341 – Numerical Analysis**
**Test - 1 (Closed Book)**

**Date: 20.02.2011** **Max. Marks: 75**
**Time: 50 Minutes** **Weightage: 25%**

**ANSWER ALL QUESTIONS**

1. Evaluate the cubic polynomial $4.17x^3 - 1.87x^2 - 3.23x + 8.33$ at $x = 1.82$ using three digit arithmetic with rounding at each arithmetic operation, in nested form. Also find the relative error. **[10]**

2. Use Bisection Method to find the root of the equation $\tan x + x = 0$ in the interval [2, 2.1]. Do four iterations using 5 digits arithmetic with rounding. **[10]**

3. Find a root of the equation $f(x) = \cos x - xe^x$ in the interval [0, 1] at the end of third iteration by Regula-Falsi Method using 5 digits arithmetic with rounding. **[15]**

4. The current $i$ in an electric circuit is given by $i = 10e^{-t} \sin 2\pi t$ where $t$ is in seconds. Using Newton's method, find the value of $t$ for $i = 2$ amp. Do three iterations by taking the starting value as 0 with 5 digits approximation and rounding. **[20]**

5. Use Muller's Method to find a root of the equation $\tan x + 3x^2 - 1$ in the interval [0, 1]. Perform 2 iterations with five digit arithmetic and rounding. **[20]**

BITS Pilani, Dubai Campus
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III YEAR II SEMESTER 2010-11
QUIZ – 2 (Closed Book)

Course Title: Numerical Analysis | Course No: AAOC C341
Date: 27.04.2011 | Weight age: 7%
Time: 20 minutes | Max marks: 21

Name of the Student: _____

ID No: _____

1. Consider the table for $f(x) = \ln(x)$ rounded to four decimal places.

| $x$: | 4.90 | 4.95 | 5.00 | 5.05 |
|---|---|---|---|---|
| $f(x)$: | 1.5892 | 1.5994 | 1.6094 | 1.6194 |

Find the approximate value of $f'(4.92)$ and $f''(4.90)$ rounded to four decimal places. [7]

2. Apply Simpson's $\frac{3}{8}$ rule to find $\int_0^{18} f(x)dx$ for the values given below: [7]

| $x$: | 0 | 2 | 4 | 6 | 8 | 19 | 12 | 14 | 16 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$: | 0 | 22 | 30 | 34 | 40 | 36 | 27 | 18 | 7 | 0 |

3. The three point Gauss-Legendre rule is

$$\int_{-1}^{1} f(x)dx \approx \frac{5f\left(-(0.6)^{1/2}\right)+8f(0)+5f\left((0.6)^{1/2}\right)}{9}$$

Show that the formula is exact for $f(x)=1,\ x,\ x^2$. [7]

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III YEAR II SEMESTER 2010-11
QUIZ – 1 (Closed Book)

Course Title: Numerical Analysis | Course No: AAOC C341
Date: 13.03.2011 | Weight age: 8%
Time: 20 minutes | Max marks: 24

Name of the Student: _____

ID No: _____

1. Solve the following system of equation by Gauss Jordan method:

$$2x + 3y = 5$$

$$3x - y = 2$$

[6]

2. Find the determinant value of the following matrix using Gauss elimination method:

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$$

[8]

3. The cubic $x^3 - 2x^2 - x + 1$ has three roots; one is negative. Following are some different rearrangements:

$$x^3 - 2x^2 + 1, \quad \sqrt{\frac{x^3 - x + 1}{2}}, \quad (2x^2 + x - 1)^{1/3}, \quad \frac{x - 1}{x^2 - 2x}$$

Choose a rearrangement that converges to the negative root. Justify your answer. **[10]**