BITS, Pilani-Dubai
Dubai International Academic City, Dubai

COMPREHENSIVE EXAMINATION –III YEAR –II SEMESTER 2008-2009

Date: 31/05/09 Course: Numerical Analysis AAOC C341
Duration: 3hours Total Marks: 40

NOTE : ANSWER PART – A AND PART – B SEPARATELY

PART – A

Q1

(a) Evaluate $f(x)=1.107x^3+0.319x^2-0.017x+1.107$ for $x=0.123$ in nested form using 4 digit arithmetic with rounding. [1]

(b) Find a suitable iteration function $g(x)$ in the interval I and check the conditions of convergence of fixed point iteration theorem for the function $x^3-x-1=0$. Further, compute smallest positive root using 5 digit arithmetic with rounding at the end of fifth iteration and find the minimum number of iterations required so that the root is correct to 4 decimal places. [3]

Q2

(a) Find a root of the equation $f(x)=x\sin x+\cos x$ by Newton's method in three iterations starting with $x=\pi$ using 5 digit arithmetic with rounding. [2]

(b) Find a root of the equation $f(x)=3x+\sin x-e^x$ by Muller's method starting with $x_0=0.5, x_1=1$ and $x_2=0$. Do two iterations using 6 digit arithmetic with rounding. [3]

Q3

(a) Find the determinant and inverse of the following matrix by Gauss elimination method using scaling, partial pivoting and 4 digit arithmetic with rounding:

$$A=\begin{pmatrix}4 & -10 & 5\\ 5 & -4 & 10\\ 10 & 5 & -4\end{pmatrix}$$ [4]

(b) Solve the following system of equations by Gauss Seidel method starting with the root $(1,1,1)$.

$$3x-6y+2z=23$$
$$-4x+y-z=-15$$
$$x-3y+7z=16$$

Do 2 iterations using 5 digit arithmetic with rounding. [2]

Q4

(a) Find $f(-0.5)$ by Newton's interpolation method for the following data using 5 digit arithmetic with rounding:

x:	-1	0	1	2	3
$f(x)$:	10	2	0	10	62

[2]

(b) Fit a natural cubic spline curve and evaluate the spline value at $x = 1.5$ for the following data with 5 digit arithmetic. [3]

x:	1	2	3	4
$f(x)$:	1	5	11	8

PART – B

Q5

(a) Express $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$ in terms of Chebyshev polynomial using 5 digit arithmetic. [2]

(b) Find the largest eigen value and the corresponding eigen vector of the matrix $\begin{pmatrix} 3 & -1 & 0 \\ -2 & 4 & -3 \\ 0 & -1 & 1 \end{pmatrix}$. Do four iterations using 5 digit arithmetic with rounding. [2]

Q6

(a) Derive the formula for Fourier series approximation and the Fourier coefficients of a periodic function $f(x)$ defined in $(-\pi, \pi)$. [3]

(b) Find $y'''(2.0)$ from the following data using 5 digit arithmetic with rounding. [1]

x:	2.0	2.1	2.2	2.3
y:	0.12306	0.10571	0.089584	0.074764

Q7

(a) Evaluate $\int_0^1 \int_0^1 (x^2 + y^2) dx dy$ by dividing the interval into four equal parts in both the direction using Trapezoidal rule with 5 digit arithmetic. [3]

(b) Compute $y(0.1)$ by Runge-Kutta method of order 4 for the differential equation $\frac{dy}{dx} = 2x^2 - y$, $y(0) = -1$ with five digit arithmetic. [3]

Q8

(a) Obtain the value of $y(0.3)$ by Taylor's series method of order 3 and hence find $y(0.4)$ by Adams-Moulton predictor corrector method for the equation $\frac{dy}{dx} = -2x - y$, $y(0) = -1, y(0.1) = -0.91451$ and $y(0.2) = -0.85619$ using 5 digit arithmetic with rounding. [4]

(b) Solve the following boundary value problem by finite difference method:

$u'' = u$, $u(1) = 1.17520$, $u(3) = 10.01787$ with $h = 0.5$ using 5 digit arithmetic. [2]

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III Year – Second Semester 2008-2009

Test: II (OB) Course: Numerical Analysis – AAOC C341

Date: 19.04.09 **Duration: 50 min**
Total Marks: 20 **Weightage: 20**

Answer ALL Questions

1. By using the Lagrange's interpolation formula find $f(9)$ from the following data using 5 digit arithmetic with rounding.

x:	5	7	11	13	17
$f(x)$:	150	392	1452	2366	5202

[2]

2. When $x = 0$ to 4, the first five terms of a sequence are 2, 7, 16, 35, 70. Using Newton's interpolation find the general term of the sequence and hence find the tenth term? **[3]**

3. Fit a natural cubic spline curve and evaluate the spline value and the derivative at $x = \frac{\pi}{6}$ for the following data using 5 digit arithmetic with rounding.

x:	0	$\frac{\pi}{2}$	π
$f(x)$:	0	1	0

[4]

4. Expand $\cos x$ in half-range sine series and $\sin x$ in half cosine series over the interval $(0, \pi)$. **[4]**

5. Evaluate $\int_0^1 \frac{x^2}{1+x^3} dx$ with four intervals using Simpson's one third rule and hence evaluate $\log_e 2$ using 5 digit arithmetic with rounding. **[3]**

6. Find A, x_1, B so that the following integration rule is exact for a polynomial of degree as high as possible. Also write all possible formulas.

$$\int_{-1}^{1} xf(x)dx = Af(1) + \frac{2}{3}f(x_1) + Bf(-1)$$ **[4]**

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III Year – Second Semester 2008-2009

Test: I (CB) | **Course: Numerical Analysis – AAOC C341**
Date: 09.03.09 | **Duration: 50 min**
Total Marks: 25 | **Weightage: 25**

Answer ALL Questions

1. (a) Evaluate the cubic polynomial $x^3 - 3.966x^2 + 5.2431x - 2.3105$ using five digit arithmetic with rounding in nested form at $x = 1.372$. **[3]**

   (b) List five different types of errors and describe each of them. **[2]**

2. Do five iterations of Bisection method using five digit arithmetic with rounding to find the point of intersection of the curves $y = \cos x$ and $y = x^3 - 1$ in the interval (0, 2). **[4]**

3. Find a root of the equation $f(x) = x^3 - 3x + 1 = 0$ by Regula Falsi method starting with $a = 0, b = 0.5$. Do two iterations using five digit arithmetic with rounding. **[4]**

4. Find a root of the equation $x^2 + e^x = 5$ by Muller's method starting with the values $1, -0.5, -2$ using five digit arithmetic with rounding. Do one iteration and what would be the starting interval for the next iteration. **[4]**

5. Do three iterations of Newton's method to obtain the double root of $x^3 - 2x^2 - 0.75x + 2.25 = 0$ which is close to 1, using five digit arithmetic with rounding. **[4]**

6. The following are three different rearrangement functions $g(x)$ of the same function $f(x)$. What is $f(x)$ and which of these converge? **[4]**

   (i) $\dfrac{4 + 2x^3}{x^2} - 2x$

   (ii) $\sqrt{\dfrac{4}{x}}$

   (iii) $\dfrac{16 + x^3}{5x^2}$

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III YEAR II SEMESTER 2008-09
QUIZ – 3 (Closed Book)

Course Title: Numerical Analysis **Course No: AAOC C341**
Date: 23.04.2009 **Max marks: 5**

Name of the Student: _____

ID No:

Find $y(1.1)$ & $y(1.2)$ for the differential equation $\frac{dy}{dt} = y^2 + t^2$ given $y(1) = -1$ using Taylor's series of order 3.

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III YEAR II SEMESTER 2008-09
QUIZ – 2 (Closed Book)

Course Title: Numerical Analysis **Course No: AAOC C341**
Date: 16.03.2009 **Max marks: 5**

Name of the Student: _____

ID No:

Solve the following system of equations using Gauss Seidel iterative method starting with (1,1,1). Do three iterations using 5 digit arithmetic.

$$3x_1 - 6x_2 + 2x_3 = 23$$
$$-4x_1 + x_2 - x_3 = -15$$
$$x_1 - 3x_2 + 7x_3 = 16$$

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III YEAR II SEMESTER 2008-09
QUIZ – 1 (Closed Book)

Course Title: Numerical Analysis **Course No: AAOC C341**
Date: 24.02.2009 **Max marks: 5**

Name of the Student: _____

ID No:

Solve the following equations using 5 decimal digits approximation starting with $(1, -1.7)$ by Newton's method.

$$x^2 + y^2 = 4;\ e^x + y = 1$$