BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2006-07)

COMPREHENSIVE EXAMINATION

Course Title: Operations Research **Course No. : AAOC UC 312**

Max. Marks: 120 **Weightage: 40%** **Date: 31.05.2007** **Time: 3 Hours**

Instructions:

- Attempt all the questions.
- Write answers of ***SECTION-A*** and ***SECTION-B*** in two separate **Answer Books** and mark the corresponding Answer Books as " SECTION-A" and "SECTION-B".
- Write answers in the sequential order.
- Necessary *statistical table values* are given in the last page.

SECTION-A

1. A machine is set to deliver the packets of a given weight. Ten samples of size 5 each were examined and the following results were obtained:

Sample No.	: 1	2	3	4	5	6	7	8	9	10
Mean	: 43	49	37	44	45	37	51	46	43	47
Range	: 5	6	5	7	7	4	8	6	4	6

(i) Use the above data to find the control limits of $\overline{X}$ - chart.
(ii) Draw the control chart and specify if the process is under control. [10]
(Plot the control chart on the answer book, graph paper is not required)

2. Decide on the basis of the following contingency table whether the student's interest in Statistics is independent of his ability in Mathematics. Use $\alpha = 0.05$.

Ability in Mathematics

Ability in Statistics: Levels	Low	Average	High
Low	82	70	62
Average	93	62	67
High	25	18	21

[10]

3. (i) Simulate a value of x of an exponential random variable with mean 4. Take u = 0.537.
(ii) Generate a value of x of a geometric random variable with parameter p = 0.4. Use the following random numbers in the given order: 0.352, 0.645, 0.245, and 0.456.
[10]

4. A random sample of size 64 is to be used to test the null hypothesis that the mean of a normal population with variance 256 is less than or equal to 40 against the alternative that it is greater than 40. If the null hypothesis is to be rejected if and only if the mean of the random sample exceeds 43, find the level of significance of this test.

[10]

5. A new examination procedure has been proposed for a course. It was realized that the old procedure was more demanding and it is expected that the new procedure will be less demanding. But it is feared that with the adoption of the new procedure, the students may obtain nearly the same marks and the grading may be difficult. In order that the grading be effectively done it is necessary that the scores should have a standard deviation exceeding 6 percent. 25 randomly selected students were examined by the new procedure and the sample standard deviation was found to be 7 percent. Should the new procedure be adopted? Use $\alpha = 0.05$.

[10]

6. An automobile firm requires 1000 bolts of a particular type every day. Shortage cost is estimated to be Rs.2 per hundred bolts per day and holding cost is Re 1 per hundred bolts per day. The firm can either manufacture the bolts or purchase them from an outside source. The set up cost is Rs. 500 for manufacturing and Rs. 50 for purchasing. The unit cost per 100 bolts is Rs 20 if purchased and Rs.10 if manufactured. While the manufacturing rate is 6000 bolts per day, the delivery is instantaneous if the firm purchases from the source. What should be the policy of the firm: to manufacture or to purchase?

[10]

SECTION-B

7. Use Kolmogorov-Smirnov test at 5% level of significance to test whether the following data could have come from uniform distribution in [10, 12]:
10.5, 10.8, 11.0, 11.5, 10.2, 10.6, 11.8, 11.4.

[12]

8. A bank has only one teller counter for cash deposit and withdrawal. The customers arrive at the teller on average 15 customers per hour and the average service time of a customer is 2 minutes. Assume Poisson arrival and exponential service time.

Find
a) utilisation factor;
b) expected number of customers in the system;
c) fraction of time the server is idle;
d) probability that there are two customers in the system. [12]

11. Let X_1 and X_2 are two independent normal variates with standard deviations 4 and 3 respectively. A random sample of size 20 is taken from first population and a random sample of size 25 is taken from the second population. Assume these samples as independent samples. If the sample means are 10 and 8 respectively, test the following hypothesis at 5% level of significance:
$H_0 : \mu_1 = \mu_2$ against the alternative $H_1 : \mu_1 > \mu_2$.
Here μ_1 *and* μ_2 represent the means of these two populations respectively.

[12]

TABLE VALUES: As per the standard notation

For n = 5; $A_1 = 1.596$, $A_2 = 0.577$

$\chi^2_{24;0.95} = 36.415$, $\chi^2_{24;0.05} = 13.848$, $\chi^2_{4;0.95} = 9.488$, $\chi^2_{5;0.95} = 11.071$

$F(z = 1.5) = 0.9332$, $Z_{0.95} = 1.645$

$t_{24,0.95} = 1.711$

For $n = 8$; $D_{0.05} = 0.225$

$F_{4,8,0.95} = 3.84$

BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE

II SEMESTER – III YEAR (2006-07)

TEST – II (OB)

COURSE: Operations Research **COURSE NO.: AAOC UC312**
Max. Marks: 60 **Weightage: 20%** **Date: 13-05-2007** **Time: 50 min.**

Instruction:
1. **Prescribed Text Book (photocopies of EDD Notes) and Class notes are allowed.**
2. **Calculators and Statistical tables are allowed.**
3. **Attempt each question on separate sheet.**

1. An oil company tested four different blends (A, B, C, D) of gasoline for fuel efficiency in order to control the variability of four drivers and four different models of cars. Fuel efficiency was measured in miles/gallon after driving cars over a standard course. A suitable design is adopted and the observations obtained are given in the following table:

	Car Model			
Driver ↓	**Toyota**	**Nissan**	**Mazda**	**Honda**
1	D=15.5	B=33.9	C=13.2	A=29.1
2	B=16.3	C=26.6	A=19.4	D=22.8
3	C=10.8	A=31.1	D=17.1	B=30.3
4	A=14.7	D=34.0	B=19.7	C=21.6

a) Construct the ANOVA table for this design. Take $\alpha = 0.05$. [10]

b) Test the hypothesis H_0 : There is no difference in fuel efficiency of the blends of gasoline against the alternative H_1 : there is a difference in the fuel efficiency of four blends. Take $\alpha = 0.05$. [4]

2. The following table shows the hours of relief provided by two analgesic drugs in 12 pairs of patients suffering from arthritis. Test the hypothesis that both the drugs will provide equal relief against two sided alternative. Test at 5% level of significance.

Pair No.:	1	2	3	4	5	6	7	8	9	10	11	12
Drug I :	2.0	3.6	2.6	2.6	7.3	3.4	14.9	6.6	2.3	2.0	6.8	8.5
Drug II :	3.5	5.7	2.9	2.4	9.9	3.3	16.7	6.0	3.8	4.0	9.1	20.9

Use Wilcoxon's signed rank-sum test. [10]

3. An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 and a sample standard deviation of 5. Test at 5% level of significance the claim that the mean abrasive wear of material 1 exceeds that of material 2. Assume the populations to be approximately normal with equal variances. [12]

4. A commonly prescribed drug for relieving nervous tension is believed to be only 60% effective. Experimental results with a new drug administered to a random sample of 100 adults who were suffering from nervous tension show that 70 received relief. Is this sufficient evidence to conclude that the new drug is superior to the one commonly prescribed? Use $\alpha = 0.05$. [12]

5. In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 180 individuals:

	Non-smokers	Moderate smokers	Heavy smokers
Hypertension	21	36	30
No hypertension	48	26	19

Test the hypothesis that the presence or absence of hypertension is independent of smoking habits. Use $\alpha = 0.10$. [12]

B

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2006-07)

QUIZ – II (CB)

Course Title: Operations Research **Course No. : AAOC UC312**

Max. Marks: 30 **Weightage: 10%** **Time: 30 minutes** **Date: 26-04-2007**

Name: **Id. No.:** **Sec.:**

Instructions:

Write your Name, Id.no., Section in the space provided.
Calculator is permitted.
All questions are compulsory.
Each question carries 3 marks. No marks for incorrect or partly correct answers.

Table values: $\chi^2_{20,0.1} = 12.443$, $Z_{0.95} = 1.645$.

1. If a random sample of size n is taken from $N(\mu, \sigma^2)$ variate, then the sample mean has _____ distribution with mean _____ and standard deviation _____ .

2. A sample of size *25* is taken from a normal population with s.d. *5*. For testing the hypothesis $H_0 : \mu = 5$ versus $H_0 : \mu > 5$ at 5% level of significance a suitable critical region is _____. *Mention the specific critical region, not the formula.*

3. If β denotes the probability of Type-II error, then the power of the test is _____ .

4. Let p be the proportion of defectives in a large dichotomous population. Consider the hypothesis $H_0 : p = 0.4$ *versus* $H_1 : p > 0.4$. If normal approximation to binomial distribution is used, the critical region at 5% level of significance is _____ . Take sample size $n = 100$.. *Mention the specific critical region, not the formula.*

5. Which of the following is/are symmetric probability distributions with mean zero & variance greater than 1.
   (i) z
   (ii) t
   (iii) B(n,p)
   (iv) $\chi 2$

   (a) z & t (b) B(n,p) &$\chi 2$ (c) z (d) t (e) None of the above

6. There are 5 training centres which train candidates for a competitive examination. The following results were recorded:

Training Centres	No. admitted	No. selected	No. rejected
1	80	30	50
2	50	10	40
3	70	20	50
4	80	20	60
5	60	10	50

Given that the inputs to the 5 centres are of the same standard. Test the null hypothesis that the 5 centres are equally good. Use α =0.05.

   (i) Give specific critical region_____

7. The moment generating function of Pascal distribution with parameters r and p is _____.

8. A coin is tossed 100 times with heads & tails occurring 65 & 35 times respectively. Test is to be conducted for null hypothesis that the coin is unbiased. Use α =0.05. The value of $\chi 2$ test statistic is _____

9. To test whether a set of data containing 200 observations has come from a normal distribution (parameters are estimated by maximum likelihood estimators) Chi-square Goodness-of-Fit is used. If the range of the normal variate is divided in 10 cells/bins, the degrees of freedom of χ^2 is _____ . *Mention the specific degrees of freedom, not the formula.*

10. A sample of size 22 is taken from a normal population with mean μ. The sample mean and variance are found to be 500 and 16 respectively. The value of test statistic for testing $H_0 : \mu = 490$ against any suitable alternative is _____ . *Mention the specific value of test statistic, not the formula.*

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2006-07)

TEST – I (CB)

Course Title: Operations Research Course No. : AAOC UC 312

Max. Marks: 60 Weightage: 20% Date: 01.04.2007 Time: 50 min.

NOTE

(i) Attempt all the questions.

(ii) Attempt each question on a separate page.

1. (i) Simulate a value of Binomial variable with parameters n = 5 and p=0.5. Use the following random numbers in the order of occurrence: 0.546, 0.852, 0.125, 0.324, and 0.810.

   (ii) Simulate a value of Pascal variate with parameter r = 3 and p = 0.4. Use the following random numbers in the order of occurrence: 0.145, 0.85, 0.643, 0.325, 0.991, 0.610, 0.551, 0.443.

[5 +5]

2. The demand of a particular item is 12000 units per year. The purchase price is Re.1 per unit and the set-up cost per run is Rs.350. The holding cost is Re. 0.20 per unit per month. If no shortages are allowed and the replenishment is instantaneous, determine

   (i) Economic order quantity

   (ii) Optimum order cycle

   (iii) Minimum total expected cost.

[3+3+4]

3. At a cooperative store, the customers arrive on an average of 1 per minute. There are two clerks A and B who have applied for the job at the counter, Clerk A can attend 2 customers per minute and clerk B can attend 1.75 customers per minute. Assume Poisson input and exponential service. It is estimated that any customer who has to wait in the system loses Rs.6.00 per hour as the cost of lost time. The clerk A has to be paid Rs.4.00 per hour and the clerk B has to be paid Rs.2.25 per hour. Assuming a 8 hour day and that the cooperative store is owned by the customers (so it will naturally take care of their lost time in waiting), which of the clerks should be appointed?

[10]

4. Using central limit theorem, simulate a value of a normal distribution with mean 10 and standard deviation 2. Take the following eight randomly generated numbers: 0.522, 0.244, 0.826, 0.342, 0.666, 0.625, 0.022, and 0.420 for simulation. [10]

5. In a self-service model, let the arrivals be in Poisson process with mean rate 5 per hour and the service times follow exponential distribution with mean time 1/8 hour.
   (i) What is the probability that there are 4 customers in the system?
   (ii) What is the expected queue length?
   (iii) What is the expected number of customers in the system? [10]

6. The demand of a commodity in a single period system has uniform distribution with pdf

$$f(x) = \frac{1}{100}, \quad 0 \le x \le 100$$
$$= 0, \quad otherwise$$

Let the carrying cost is Rs.2 per unit per day, shortage cost is Rs.4 per unit per day and the purchase cost is Re.1 per unit. What is the optimal policy if the initial inventory is 4 units? Assume that there is only one opportunity of procurement. [10]

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B

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2006-07)

QUIZ – I (CB)

Course Title: Operations Research **Course No. : AAOC UC312**

Max. Marks: 30 **Weightage : 10%** **Time : 30 minutes** **Date : 08-03-2007**

Name: **Id. No.:** **Sec.:**

Instructions:

Write your Name, Id.no., Section in the space provided.
Calculator is permitted.
All questions are compulsory.
Each blank carries 3 marks. No marks for incorrect or partly correct answers.

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1. If D = 200, A= ∞, K= 100, C_0= 2, C_1 =1, C_2 = 1. The Economic Backorder quantity should be _____.

2. Holding cost does not include:
   a) Depreciation cost of items stored
   b) taxes
   c) cost of inspection and testing
   d) insurance for the safety of inventory

3. As per the axioms of Birth and Death process, the notation $1-[\lambda_n + \mu_n]\,\Delta T + o(\Delta T)$ implies the Probability of _____ in the interval $(T, T+\Delta T]$ given that $N(T) = n$.

4. If $\lambda = 8$ and $\mu=3$ then to avoid an infinite queue i.e. in order that the system achieves steady-state situation, the minimum number of servers needed are _____.

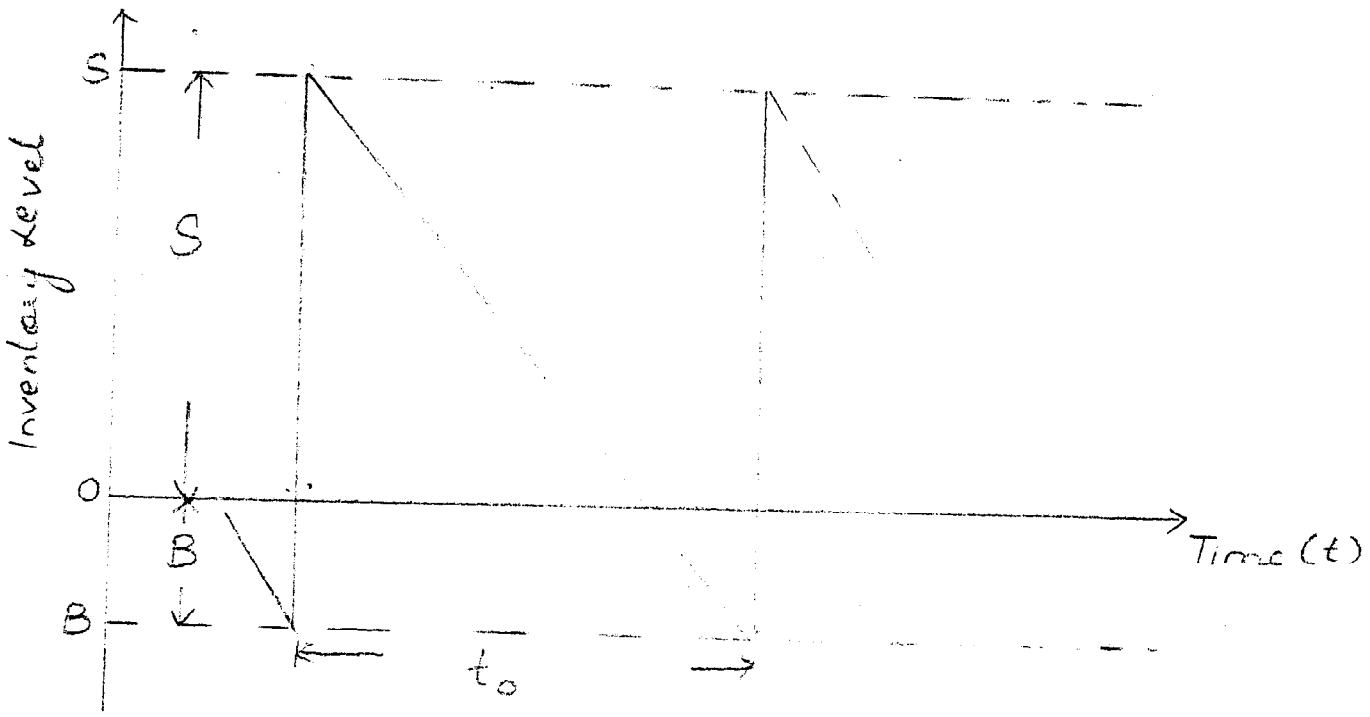
5. In a railway yard, goods trains arrive at the rate of 30 trains per day. Assume that the inter arrival time follows an exponential distribution and the service time is also to be assumed as exponential with mean of 36 minutes. Calculate the probability that the yard is empty _____

6. Patients arrive at a clinic in a Poisson process at the rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate 20 per hour.
   a) Specify model as per Kendall's notation_____.
   b) Proportion of time server is idle_____.

7. Problems arrive at a computing centre in a Poisson process at an average rate 1 per day. The rules of the computing centre are that any man waiting to get his problem solved must aid the man whose problem is being solved. If the time to solve a problem with one man has an exponential distribution with mean time 1/3 day and if the average solving time is inversely proportional to the number of people working on the problem(that is, 1/(3n) days, if n people are working on the problem). The expected number of customers in the queue will be_____.

8. If in a deterministic inventory system with backlogged shortages $EBQ = B^*$ and lead time is L(constant) then reorder point $s^* =$ _____. Take demand rate during lead time as D.

9. Specify the following on the basis of diagram: (1+1+1= 3 marks)
   (a) Replenishment policy _____
   (b) Delivery rate (A) _____
   (c) Demand is _____(static, dynamic)



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