BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

COMPREHENSIVE EXAMINATION –III YEAR –II SEMESTER 2006-2007

Date: 30/05/07
Duration: 3hours
Course: Numerical Analysis AAOC UC341
Instructors: Dr Priti Bajpai, Dr A.Somasundaram
Total Marks: 40

NOTE : ANSWER PART – A AND PART – B SEPARATELY

PART – A

Q1

(a) Prove that $r_{x-y} = r_x \dfrac{x}{x-y} - r_y \dfrac{y}{x-y}$ **[1]**

(b) Use Bisection method to find the root of $x^{10} - 1$ between 0 and 1.3. Do four iterations with 5 digit arithmetic. **[1]**

Q2

(a) Do two iterations of Muller's method to find the real root of $f(x) = x^3 + 2x - 1 = 0$ using 5 digit arithmetic starting with values $-0.5, 0, 0.5$. **[2]**

(b) If $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$ then find $f'''(0.5)$ with $h = 0.2$ by taking the function values in the interval [0,0.6] using 5 digit arithmetic. **[3]**

Q3.

(a) Verify fixed point theorem for the function $f(x) = e^x - x^2$ and locate the root after 5 iterations starting with $x_0 = 0$ using five digit arithmetic and also find the minimum number of iterations required to get 8 significant digit accuracy. **[3]**

(b) Find the minimum number of equispaced tabular points required for piecewise cubic interpolation of the function $f(x) = e^{-2x} + \dfrac{22}{3}x^4$ on the interval [0,3] to get 4 decimal place accuracy. **[3]**

Q4.

(a) Do three iterations of Gauss Jacobi method to solve the following system of equation using 5 digit arithmetic, given initial value is (0, 0, 0). Also find the minimum number of iterations required to get solutions correct to 6 decimal places. **[3]**

$$3x_1 + 4x_2 + 15x_3 = 54.8$$
$$x_1 + 12x_2 + 3x_3 = 39.66$$
$$10x_1 + x_2 - 2x_3 = 7.74$$

(b) Solve the following system of equations using Gauss elimination algorithm with scaling, partial pivoting and back substitution. Store multipliers and use 5 digit arithmetic. **[4]**

$$0.4x_1 + 5x_2 + 2x_3 = 7.4$$
$$6x_1 - 0.3x_2 + 4x_3 = 9.7$$
$$-5x_1 + 2x_2 + 0.8x_3 = -2.2$$

PART – B

Q5.

(a) Evaluate $\int_{-1}^{1} \frac{e^{-x^2}}{\sqrt{1-x^2}} dx$ by 3 point Gauss-Chebyshev quadrature with 5 digits. **[1]**

(b) Do one iterations of Newton's method to solve the following using 5 digit arithmetic with $x_0 = 1.5, y_0 = 3.5$ **[2]**

$$x^2 + xy = 10 \text{ and } y + 3xy^2 = 57$$

Q6.

(a) Find the largest eigen value and eigen vector of the following matrix after 5 iterations with 5 digit arithmetic, using power method with initial vector $(1\,1\,1)^T$. **[2]**

$$\begin{pmatrix} 40 & -20 & 0 \\ -20 & 40 & -20 \\ 0 & -20 & 40 \end{pmatrix}$$

(b) Find x_0, x_1, A_0, A_1 in the following integration rule so that integration is exact for a polynomial of degree as high as possible **[3]**

$$\int_{-1}^{1} \frac{f(x)}{1+x^2} dx = A_0 f(x_0) + A_1 f(x_1) + E$$

Q7.

(a) Evaluate the integral $\int_{1}^{4} \frac{xe^{2x}}{1+x^2} dx$ using composite Simpson's 3/8 rule using 5 digit arithmetic, with $h = 0.3$. **[2]**

(b) Compute values of y as a solution of differential equation $\frac{dy}{dx} = x^2 y^2$ at $x = 0.1, 0.2$ using Taylor's series of order 3, given $y(0) = 1$ **[3]**

Q8.

(a) Solve $y'' + 4y = 4x, y(0) = -1, y(1) = 1, h = 0.25$ using finite difference method with 5 digit arithmetic. **[3]**

(b) Find $y(1.4), z(1.4)$ as solution of pair of differential equations using 4th order Adams-Moulton predictor corrector formula. Use 5 digit arithmetic. **[4]**

$$\frac{dy}{dx} = yz - x, y(1) = -2, y(1.1) = -1.5, y(1.2) = -1.8, y(1.3) = -1$$

$$\frac{dz}{dx} = yz + x, z(1) = 2, z(1.1) = 1.8, z(1.2) = 1.5, z(1.3) = 1 \text{ with } h = 0.1.$$

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Test: II (CB) Course: Numerical Analysis – AAOC UC341
Date: 06.05.07
Total Marks: 20
Duration: 50 min
Weightage: 20

Answer ALL Questions (USE 5 DIGIT ARITHMETIC)

1. Find $y'''(50)$ when $y = \sqrt[3]{x}$ from the table of values given below: [3]

X:	50	51	52	53	54	55	56
Y:	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

2. Using Taylor's series method of order 2 find the value of $y(1.1)$ and $y(1.2)$ given that $y' = xy^{\frac{1}{3}}, y(1) = 1$ [4]

3. Find $\int_0^{\frac{\pi}{2}} \sin x \, dx$ by 3 point Gauss-Legendre Quadrature and compare with the exact solution. [4]

4. Solve $y_{n+1} = \sqrt{y_n}$ [2]

5. The velocity v of a particle at distance s from a point on its path is given below:

s in meters	:	0	10	20	30	40	50	60
v in m/sec	:	47	58	64	65	61	52	38

Estimate the time taken to travel 60m by Simpson's 1/3 rule. [3]

6. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by composite Trapezoidal rule using 6 coordinates and hence determine the value of π. [4]

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III YEAR II SEMESTER

A

QUIZ – 2 (Closed Book)

Course Title: Numerical Analysis
Date: 19.4.2007
Time: 30 min

Course No: AAOC UC341
Max marks: 10
Weightage: 10%

Name of the Student: ____________________

ID No: ____________________

Branch: ____________________

Set: A

Recheck Request:

Answer all Questions

1. What is an ill conditioned system?

[0.5]

2. What is a condition number and what does it tell us?

[0.5]

3. For any system $Ax = b$ if $\bar{x}$ is approximate solution and x is true value then the residue vector is defined as ______________

[0.25]

4. The formula used for iterative improvement is given by ___________

[0.25]

5. On using Gauss algorithm for the system with the coefficient matrix $A = \begin{pmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{pmatrix}$

the working matrix after step 1 is $A = \begin{pmatrix} 1 & 0.3 & 0.4 \\ 0.2 & -1.06 & 0.22 \\ 0.3 & 0.11 & -1.12 \end{pmatrix}$ and $\bar{p} = (1, 2, 3)$. For step 2 with $\bar{p} = (1, 2, 3)$ find

(a) $a_{p_3,2} =$

(b) $a_{p_3,3} =$

[1]

(c) If initially $b = (15, 37, -10)$

$\tilde{b}_1 =$

$\tilde{b}_2 =$

[1]

6. Consider the system of equation

$$\begin{aligned} 0.1x_1 + 7x_2 - 0.3x_3 &= -19.3 \\ 3x_1 - 0.1x_2 - 0.2x_3 &= 7.85 \\ 0.3x_1 - 0.2x_2 + 10x_3 &= 71.4 \end{aligned}$$

(a) Given the above system to be solved out of the two methods Jacobi and Gauss Seidel, which method would give a better estimate? [0.5]

(b) The system $x = Bx + c$ for the above equations is [1]

(c) $\|B\|_\infty =$ [0.5]

(d) To find whether the system has a convergent solution or not which condition is tested? [0.5]

(e) What will be the minimum number of iterations required if on using 7 digit arithmetic we want the solution to be correct up to 8 decimal digits if the starting solution is $(0, 0, 0)$ (Substitute in the formula and do not calculate) [0.5]

7. During an experiment the density of a liquid at the temperatures $0^\circ C$, $20^\circ C$ and $40^\circ C$ was observed to be 3.85, 0.800 and 0.212 respectively. If Lagrange's interpolating polynomials are used, write the interpolating polynomial to find the density of the liquid at $15^\circ C$. (Do not calculate) [1]

8. Given $\ln 1 = 0$, $\ln 4 = 1.386294$ and $\ln 6 = 1.791759$. If Newton's method of divided difference is used to find $\ln 2$, the following difference table is formed.

(a) Fill in the missing information [1]

x	$f(x)$	$f[\ ]$	$f^2[\ ]$
1	0		
4	1.386294		
6	1.791759		

(b) If a new point $(5, 1.609438)$ is added we would get a polynomial of degree__________ [0.5]

9. Backward formula for the Newton's interpolating polynomial is _____ [0.5]

10. Write the formula for finding minimum number of tabular points such that piecewise cubic interpolation of $f(x)$ defined on $[a,b]$ yields values correct up to 6 decimal places. [0.5]

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Test: I (CB)
Date: 25.03.07
Total Marks: 20

Course: Numerical Analysis – AAOCU341
Duration: 50 min
Weightage: 20

<u>Answer ALL Questions</u>

1. (a) Define a multiple root of order m and write the formula of Newton's method for multiple roots. **[1]**

   (b) For the system of non-linear equations $f(x, y) = 0$ and $g(x, y) = 0$ write the Newton's iteration scheme. **[2]**

2. Find the first approximate solution of $f(x) = x^3 - 2x^2 + x - 2 = 0$ by Muller's method with $x_0 = 1.5$, $x_1 = 3$, $x_2 = 0$ using 5 digit arithmetic. **[3]**

3. Rewrite the following system of equations in $x = f(x, y)$ and $y = g(x, y)$ form to apply fixed point iterations. Check the conditions for convergence over $0.5 \le x \le 1$ and $1 \le y \le 1.5$. Do one iteration starting with $x_0 = 1, y_0 = 1$ using 5 digit arithmetic $x^{\frac{1}{3}} + y^{\frac{1}{4}} - x - 1.175 = 0$ and $x^{\frac{1}{4}} + y^{\frac{1}{2}} - y - 0.8412 = 0$ **[3]**

4. Find the determinant of the matrix $A = \begin{pmatrix} 3.5 & 4.5 & 5.5 \\ 6.5 & 7.5 & 8.5 \\ 3.5 & 3.5 & 2.5 \end{pmatrix}$ by Gauss elimination with scaling and pivoting with 5 digit arithmetic and rounding. **[3]**

5. Find the inverse of the following matrix by Gauss elimination method with scaling and pivoting using 5 digit arithmetic. **[5]**

$$\begin{pmatrix} 4 & -10 & 5 \\ 5 & -4 & 10 \\ 10 & 5 & -4 \end{pmatrix}$$

6. Solve the following equations by Gauss Jordan method. **[3]**

$$x + y = 2$$
$$2x + 3y = 5$$

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III YEAR II SEMESTER

A

QUIZ – 1 (Closed Book)

Course Title: Numerical Analysis
Date: 15.03.2007
Time: 30 min

Course No: AAOC UC341
Max marks: 10
Weightage: 10%

Name of the Student: ______________________

ID No: ______________________

Branch: ______________________

Set: A

Recheck Request:

Answer all Questions

1. Name any 5 types of errors. [1]

2. During addition when will the error be least? Using 5 digit arithmetic with rounding add the following numbers such that the error is minimum 0.395688, 0.2089, 0.0023888, & 3.12768. [1]

3. Write the polynomial in the nested form and evaluate it at $x = 1.25$, use 5 digit arithmetic with rounding $0.725x^4 - 1.23x^3 + 0.5x^2 + 3$. If the true value is 3.148925781 what is the relative error and absolute error? [1]

4. Let $f(x)$ be computed for $0 \le x \le 9$. If x^* approximates x correct to n significant decimal digits then to how many significant digits does $f(x^*)$ approximate $f(x) = e^{\cos x}$. [1]

5. The condition number for $f(x) = \tan x$ at $x^* = \frac{\pi}{2} + (0.1)\frac{\pi}{2}$ is $k =$ ____________ [1]

6. Relative propagated error in addition and multiplication is given by

r_{x+y} and r_{xy} [1]

7. For the equation $f(x) = x\log_{10} x - 1.2$ the root lies between ________________ and the first two approximations correct to 2 digits by Bisection method are [1]

$x_1 =$

$x_2 =$

8. Write the Muller's formula $(a, b, c \text{ and } x)$ to solve the equation $f(x) = 0$. [1]

9. If an approximate value of the root of the equation $x^x = 1000$ is 4.5, find first two approximations x_1 and x_2 by Newton's method correct to 5 significant digits. [1]

10. The formula for finding an approximate root of an equation $f(x) = 0$ which lies in the interval $[a, b]$ in Regula Falsi method is ____________________ [0.5]

If successive approximations fall on the right of the root, the rate of convergence can be accelerated by ______________________ [0.5]