BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
(III YEAR – II SEMESTER 2005-2006)

COMPREHENSIVE EXAMINATION (CB)

Course Title: OPERATIONS RESEARCH **Course No.: AAOC UC312**

Max. Marks: 40 Weightage: 40% Date: 25-5-2006 Time: 3 hours

Instructions:

- Attempt all the questions.
- Write answers of ***SECTION-A*** and ***SECTION-B*** in two separate **Answer Books** and mark the corresponding Answer Books as " SECTION-A" and "SECTION-B".
- Write answers in the sequential order.
- Necessary *statistical table values* are given in the last page.

SECTION-A

1. In a railway yard, goods- trains arrive at a rate of 30 trains per day. Assuming that inter arrival time follows an exponential distribution and the service time distribution is also exponential with mean 36 minutes. Calculate
   (a) The mean number of trains in the yard.

   Assuming that the yard can admit only 9 trains, calculate
   (a) The probability that the yard is empty.
   (b) The mean number of goods- trains at the yard. [3]

2. Four varieties of wheat A, B, C, D were sown on a block of land by dividing the block into 16 plots of equal size and using 4x4 Latin square design (chosen at random), in order to take into account the possible fertility gradients in the soil. The resulting yields in kilograms were found to be as follows:

Row positions of the plot	Column positions of the plot			
	1	2	3	4
1	C=18	D=12	A=16	B=20
2	D=26	A=34	B=25	C=31
3	B=15	C=22	D=10	A=28
4	A=30	B=20	C=15	D=9

Test at 0.01 level of significance for the differences amongst the varieties.
(Necessary table value is given after Section-A)

[6]

3. A marketing firm producing detergents is interested in studying the consumer behavior in the context of purchase decision of detergents in a specific market. This company is a major player in the detergent market that is characterized by intense competition. It would like to know in particular whether the income level of the consumers influence their choice of the brand. Currently there are four brands in the market. Brand 1 and Brand 2 are the premium brands while Brand 3 and Brand 4 are the economy brands. A representative stratified random sampling procedure was adopted covering the entire market using income as the basis of selection. The categories that were used in classifying income level are: Lower, Middle, Upper Middle and High. A sample of 600 consumers participated in this study. The following data emerged from the study.

Cross Tabulation of Income versus Brand chosen (Figures in the cells represent number of consumers)

	Brands				
	Brand1	Brand2	Brand3	Brand4	Total
Income					
Lower	25	15	55	65	160
Middle	30	25	35	30	120
Upper Middle	50	55	20	22	147
Upper	60	80	15	18	173
Total	165	175	125	135	600

Analyze the cross-tabulation data above using chi-square test of independence and draw your conclusions at 5% level of significance. (Necessary table value is given after Section-A) [4]

4. Using the random numbers 0.75, 0.25, 0.55, 0.13, 0.65, 0.07, 0.90, 0.80 generate one observation of the binomial random variable X with n=8, p=0.3. [3]

5. There are two training centers- T_1 & T_2 for commando operations. After training 7 commandos for T_1 for commando operations & 8 commandos for T_2 commando operations, the efficiency scores in a commando test were recorded as under:
Score T_1: 20 17 19 19 21 30 50
Score T_2: 22 17 22 24 41 21 30 32
Use Wilcoxon rank-sum test at a level at most 0.06 to test the hypothesis that the two centers impart training which are alike against two sided alternative. [4]
(Necessary table value is given after Section-A)

Table values: As per standard notations

i) For $n=4,\ D_3 = 0, D_4 = 2.282$

ii) $\chi^2_{1,0.95} = 3.841$

iii) $t_{24,0.975} = 2.064$

iv) $\chi^2_{9,0.95} = 16.92$

v) $F_{3,6,0.99} = 9.78$

vi) *Wilcoxon Rank Sum table*

For $n=7$ & $m=8$

x	P	x^*
70	0.060	42
71	0.047	41
72	0.036	40
73	0.027	39

SECTION-B

6. Consider an infinite time horizon inventory system with zero lead time and finite delivery rate. Assume that the shortages are allowed and are completely backlogged. The following data are given:
Production(delivery) rate = 250 units per day;
Demand rate = 75 units per day;
Holding cost = Rs.2.50 per unit per day;
Shortage cost = Re.1.00 per unit per day;
Setup cost = Rs.50.00 per production run;
Unit (purchase) cost = Rs.1.50 per unit.
Find EOQ, EBO, OOC and the minimum total cost per day. [4]

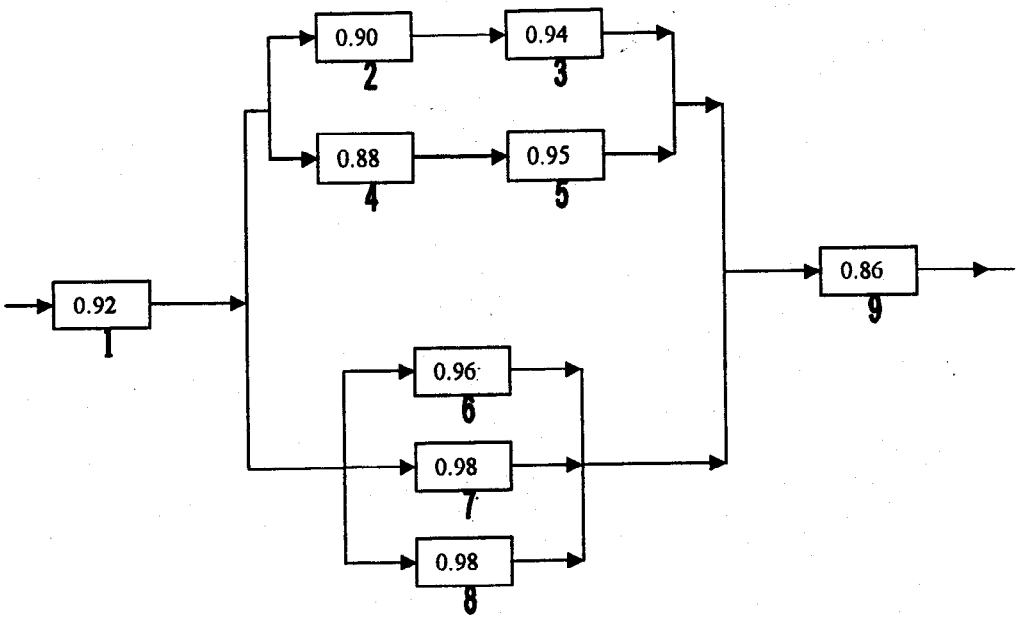
7. Consider a probabilistic, single period(short period) inventory system with no setup cost. The demand is assumed to be instantaneous, the holding cost is incurred on ending inventory. Let X denotes the demand which has uniform distribution on [a, b]. If C_0, C_1, C_2 denote purchase cost per unit, holding cost per unit per unit time and shortage cost per unit per unit time respectively, find the expression for the optimum reorder level R^*. [2]

8. The ranges R_j of 20 samples, each consisting of 4 readings are given below:

Sample No.	1	2	3	4	5	6	7	8	9	10	11	12
R_j	2.5	2.7	3.5	2.1	3.2	6.2	5.0	1.7	2.1	4.9	3.2	1.9
Sample No.	13	14	15	16	17	18	19	20				
R_j	2.8	3.6	3.3	2.1	2.3	2.4	2.0	1.5				

a) Use the above data to find CL, UCL, LCL for a $R-chart$.
b) Plot the points on the $R-chart$.
c) Is the process under control? [4]
(Necessary table values are given after Section A)

9. Find the reliability of a system connected as below in which the numbers outside the boxes denote the component numbers and the numbers inside the boxes denote reliability of the corresponding components. Assume that the components function independently.



[4]

10. A sample of 200 families with 3 children each gave the following results:

Male Children:	0	1	2	3
No. of families:	40	58	62	40.

Are the data consistent with the hypothesis that male and the female births are equally likely? Test the hypothesis at 5% level of significance. (Necessary table value is given after Section-A)

[3]

11. A random sample of size 25 is taken from a normal population. The sample mean and sample variance are found to be 500 and 16 respectively. Test the following hypothesis at 5% level of significance:

$H_0 : \mu = 490$

versus $H_1 : \mu \neq 490$ where μ denotes the population mean. [3]

(Necessary table value is given after Section-A)

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2005-06)

TEST – II (OB)

Course Title: Operations Research **Course No. : AAOC UC 312**

Max. Marks: 25 **Weightage: 25%** **Date: 14.05.2006** **Time: 50 min.**

NOTE

(i) ***Attempt all the questions.***
(ii) ***Attempt each question on a separate page.***
(iii) ***Attempt all questions of the same section together.***

SECTION – A

1. The following are the number of sales which a sample of 9 sales persons of industrial chemicals in California and a sample of 6 sales persons of industrial chemicals in Oregon made over a certain fixed period of time:
   California: 59 68 44 71 63 46 69 54 48
   Oregon: 50 36 62 52 70 41
   Assuming that the populations sampled can be approximated closely with normal distributions having the same variance, test the null hypothesis of equal means against two sided alternative at 1% level of significance. [4]

2. Four different, though supposedly equivalent, forms of a standardized reading achievement test were given to each of 5 students, and the following are the scores which they obtained:

	Student 1	Student 2	Student 3	Student 4	Student 5
Form A	14	6	11	0	9
Form B	14	10	16	9	16
Form C	12	7	-	9	12
Form D	12	9	11	6	7

   Test at 0.05 level of significance for the difference in the forms of a standardized reading achievement test. [6]

3. A die is rolled 180 times and the following results were obtained:

Number Showing	1	2	3	4	5	6	Total
Frequency	20	30	40	30	30	30	180

   Test at 1% level of significance the hypothesis H_0: The die is unbiased, against the suitable alternative. [3]

SECTION-B

4. Dubai Driving Licence issuing authority has conducted a road test and a written test for 12 people to judge their driving skill. Their scores in the tests are as follows:

Student No. :	1	2	3	4	5	6	7	8	9	10	11	12
Written Test:	25	30	50	32	40	70	75	80	60	65	90	55
Road Test :	25	35	45	50	35	90	75	60	55	75	50	80

Use *Wilcoxon's signed rank-sum test* at 5% level of significance to test the null hypothesis that scores of two test have no difference against the alternative that written test scores are lower. [4]

5. Use *Kolmogorov-Smirnov* test at 5% level of significance to test whether the following data could have come from normal distribution with mean 18 and variance 4:

14, 15, 16, 20, 24, 22, 12, 19. [4]

6. Three different coaching centers offer coaching to the students appearing in a competitive test. The results of the test are recorded below:

Coaching Center	No. selected	No. rejected
1	50	150
2	80	170
3	20	80

Assume that the inputs to the three centers are of same standard. Test the null hypothesis that the three centers are equally good. Take $\alpha = 0.05$. [4]

MAKE UP

BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
(III YEAR – II SEMESTER 2005-2006)

QUIZ – II (CB)

OPERATIONS RESEARCH
(AAOC UC 312)

Max. Marks: 10 Weightage: 10% Date:02-05-2006 Time: 30 Mins.

Name : Id. No.: Sec.:

Write your Name, Id. No. and Sec. in the space provided.
Attempt all the questions.
Each question carries 1 mark.
Overwriting/multiple answers will be treated as incorrect answer.

1. If Z is $N(0, 1)$ variate, then $Y = Z^2$ is a χ^2 variate with degrees of freedom
   a) 3 b) 2 c) 1 d) None of these.

2. In testing of statistical hypothesis, the error committed by accepting the null hypothesis when it is false is known as _____.

3. A population is distributed as $N(\mu, 16)$. A sample of 25 items has a mean 4. An observed value of the suitable test statistic for testing $H_0 : \mu = 5$ is
   _____

4. Let X be $N(\mu, 36)$. H_0: $\mu = 10$ versus H_1:$\mu = 12$. For a sample of size 16 & $\alpha = 0.05$
   (i) Type II error is _____
   (ii) The value of power Function when $\mu = 12$ is
   _____
   (iii) If the sample mean is 12, then the probability of Type I error is
   _____

5. If in 10 trials the number of success(X) is 4 with probability of success 0.4, then the moment generating function of the suitable distribution of (X) is
   _____

6. Suppose that the life length of a certain product obeys the exponential distribution with parameter λ. A random sample of 25 products yielded a mean life of 1068 hours. To test whether the data provide sufficient information to interpret that the mean life of products is more than 1000 hours & $\alpha = 0.01$

6.

(i) specify null & alternative hypothesis

(ii) specify the critical region

7. Consider H_0: $\mu = 10$ versus H_1:$\mu > 10$. The critical region will be termed as

(i) BCR
(ii) UMPCR
(iii) GLRT
(iv) None of these

TABLE VALUES:

$\chi^2_{50,0.99} = 76.154$

$Z_{0.95} = 1.645$

F(Z=1.33) = 0.9082

A

BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
(III YEAR – II SEMESTER 2005-2006)

QUIZ – II (CB)

OPERATIONS RESEARCH
(AAOC UC 312)

Max. Marks: 10 Weightage: 10% Date: 20-04-2006 Time: 30 Mins.

Name : Id. No.: Sec.:

Write your Name, Id. No. and Sec. in the space provided.
Attempt all the questions.
Each question carries 1 mark.
Overwriting/multiple answers will be treated as incorrect answer.

1. If X has Poisson distribution with parameter 0.5, then the mgf of X is
   a) $\frac{1}{1-2t}$ b) $\frac{2}{2-t}$ c) $\frac{5}{1-2t}$ d) None of these.

2. Let X be a Poisson random variable with parameter λ. An example of simple hypothesis is
   a) $\lambda = 5$ b) $\lambda < 4$ c) $\lambda \neq 3$ d) $\lambda \geq 4$.

3. Let X_1 be $N(\mu_1, \sigma_1^2)$ and X_2 be $N(\mu_2, \sigma_2^2)$ variates and are independent. If σ_1^2 and σ_2^2 are known and $\overline{X_1}$, $\overline{X_2}$ are the sample means of X_1 and X_2 of random samples of sizes n_1 and n_2 respectively, then the distribution of $Y = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$ is
   a) t-distribution b) $N(0,1)$ distribution c) χ^2 distribution
   d) None of these.

4. Let X be exponentially distributed with parameter $\lambda(>0)$ and s be the sum of the observations of a sample of size n taken from X. Then the critical region obtained by LR test of size α of the hypothesis $H_0 : \lambda = \lambda_0$ versus $H_1 : \lambda = \lambda_1$ when $\lambda_1 > \lambda_0$ is
   a) $C = \{s \geq \frac{1}{2\lambda_0}\chi^2_{2n,1-\alpha}\}$ b) $C = \{s \leq \frac{1}{2\lambda_0}\chi^2_{2n,\alpha}\}$ c) $C = \{s \geq \frac{1}{2\lambda_1}\chi^2_{2n,1-\alpha}\}$
   d) None of these.

5. Let X be $N(\mu,\sigma^2)$, where μ & σ^2 are both unknown. If
$H_0 : \sigma^2 = \sigma^2_0$, $H_1 : \sigma^2 < \sigma^2_0$
The critical region of the GLRT(α) is
(a) $C = \{ s^2 > \frac{1}{(n-1)} \sigma^2_0 \chi^2_{n-1,\alpha} \}$
(b) $C = \{ s^2 \leq \frac{1}{(n-1)} \sigma^2_0 \chi^2_{n-1,\alpha} \}$
(c) $C = \{ s^2 \leq \frac{1}{(n-1)} \sigma^2_0 \chi^2_{n-1,1-\alpha} \}$

(d) None of these.

6. A random sample of size 36 is used to test the null hypothesis H_0: $\mu = 40$ versus the alternative hypothesis H_1: $\mu = 36$, where μ is the mean of a normal distribution with variance 16. If the critical region is taken as $C = \{ \bar{x} \leq 38 \}$, the probability of Type I error is
(a) 0.05 (b) 0.9987 (c) 0.0013 (d) 0.95

7. Area of the critical region depends on:
(a) Size of Type I error
(b) Size of Type II error
(c) Number of observations
(d) value of the statistic

8. Neyman Pearson lemma is the test for simple null hypothesis versus simple alternative hypothesis giving the critical region for which
(a) Type II error is minimum.
(b) Type I error is minimum
(c) Power of the test is maximum
(d) Power of the test is minimum.

(i) a& c (ii) b & c (iii) a, b & c (iv) None of these.

9. Let p be the proportion of defectives in a large dichotomous population. Consider the hypothesis $H_0 : p = 0.3$ versus $H_0 : p < 0.3$. When the sample size is 50 and the level of significance is 0.05, the critical region obtained by normal approximation is _____.
[Given $z_{0.05} = -1.645$]

10. A sample of 12 specimen taken from a normal population is expected to have mean 50mg/cc. The sample has a mean 64 mg/cc with a variance of 25. To test H_0: μ=50 versus H_1: μ< 50. Specify the critical region.

_____

TABLE VALUES:

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

MAKEUP-3

BITS, PILANI – DUBAI CAMPUS
Dubai Knowledge Village

QUIZ-I (CB)

OPERATIONS RESEARCH
(AAOC UC 312)

Max. Marks : 10 **Weightage: 10%** **Date: 25-4-2006** **Time: 30 mins.**

Name : **Id. NO.:** **Section:**

Attempt all the questions.
Each question carries 1 mark.

Fill in the blanks with correct answers:

1. In a (M/M/1) : (FCFS/∞/∞) queueing model, mean arrival rate and the mean service rate are 4 and 6 respectively. The probability that a server is busy and the expected queue length at any time are respectively _____ and _____ .

2. Consider a single server queueing system with Poisson arrival and exponential service time. Assume that at most 10 customers are allowed in the system. Further, it is given that queue discipline is FCFS and size of population of prospective customers is infinite. If mean arrival rate and mean service rates are same, the expected number of customers in the system is _____ and the probability that there are 4 customers in the system at any time is _____ .

3. In a railway yard, goods train arrive at a rate of 20 trains per day. Assume that the interarrival time follows exponential distribution and the service time distribution is also exponential with mean 25 minutes. If the yard can admit only 9 trains, the probability that the yard is empty is _____ .

Tick the correct answer:

4. In Kendall's notation, the Machine Repair Model is
   a) $(M/M/s):(FCFS/K/K)$ b) $(M/M/s):(FCFS/\infty/\infty)$ c) None of these

5. If T has exponential distribution with parameter λ, then the inequation $P(a \le T \le b) > P(a+c \le T \le b+c)$; c>0, 0<a<b is
   a) true b) false

6. In a birth and death process, if the birth rate is λ_n when $N(T) = n$ then probability of two or more births in the interval $(T, T + \Delta T)$ is

a) $1 - \lambda_n T + o(\Delta T)$ b) $o(\Delta T)$ c) $\lambda_n T + o(\Delta T)$ d) None of these.

7. If there are s customers in a queueing system in which mean arrival rate is λ and the mean service rate μ, then the utilization factor is

a) $\frac{\lambda}{\mu}$ b) $1 - \frac{\lambda}{s\mu}$ c) $\frac{\lambda}{s\mu}$ d) None of these.

8. In Kendall's notation (a, b, c) : (d, e, f), c stands for

a) queue discipline b) max. allowable customers in the system c) number of servers in parallel d) None of these.

MAKE-UP

BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
III YEAR – II SEMESTER
(2005-2006)

TEST-I (CB)

OPERATIONS RESEARCH
(AAOC UC312)

Max. Marks : 15 Weightage : 15% Date : Time : 50 minutes

Answer all the questions.

1. The annual consumption rate of a product is 200 units. The ordering cost is Rs.250 per order, purchase cost is Rs.20 per unit, carrying cost is Rs.2 per unit per year. Shortages are allowed and are fully backlogged. The cost per unit of shortage is Rs.1.50 per year. Find the economic order quantity, optimum ordering cycle and the minimum total annual cost. Assume that delivery is instantaneous. [3]

2. The demand of an item is 225 units per day and the rate of production is 450 units per day. The cost of holding inventory is Re.0.50 per unit per day. The back ordering is allowed at the cost of Re.0.50 per unit per day. The cost of starting a production run is Rs.100 per run. The production cost per unit is as follows:
   Rs.2.00 if $Q < 200$;
   Rs.1.75 if $200 \le Q < 500$;
   Rs.1.50 if $500 \le Q < 1000$;
   Rs.1.25 if $1000 \le Q < 1500$;
   Rs.1.20 if $Q \ge 1500$
   where Q denotes the number of units produced in a single run. Find the EOQ, EBQ, optimum production cycle. [4]

3. Simulate two observations of a random variable having the normal distribution with $\mu = 50$ and $\sigma = 5$. Use the random numbers: $u_1 = 0.253$ and $u_2 = 0.531$. [3]

4. In a car repairing shop, there are two mechanics for repairing the cars. The cars which come for repairs in a day has a Poisson distribution with mean 3. The service time for any mechanic is exponentially distributed with mean rate of 2 cars per day. The day is of 8 hours duration.
   (a) Find the hours of the day for which all the mechanics are busy.
   (b) Find the probability that one mechanic is idle.
   (c) Find the expected number of cars in the queue. [3]

5. Generate a value x of a geometric random variable with parameter p = 0.3. Use the random numbers 0.37, 0.45, 0.67, 0.75, 0.8. [2]

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2005-06)

TEST – I (CB)

Course Title: Operations Research **Course No. : AAOC UC 312**

Max. Marks: 15 **Weightage: 15%** **Date: 02.04.2006** **Time: 50 min.**

NOTE

(i) ***Attempt all the questions.***
(ii) ***Attempt each question on a separate page.***
(iii) ***Attempt all questions of the same section together.***

SECTION – A

1. A typist at an office receives on the average 22 letters per day for typing. The typist works 8 hours a day and it takes on the average 20 minutes to type a letter. The company has determined that the cost of a letter waiting to be mailed is 80 paise per hour and the equipment operating cost plus the salary of the typist will be Rs. 40 per day.
   (a) What is the average number of letters waiting to be typed?
   (b) Forced to increase the letter typing service, the above company is planning to take lease of one of the two models of an automated typewriter available in the market. The daily costs and the results increase in the typist's efficiency are displayed in the table given below:

Model	Additional cost per day	Increase in typist's efficiency
I	Rs. 37	50%
II	Rs. 39	75%

   What action should the company take to minimize the total daily costs of waiting letters to be mailed? [1 + 4]

2. Simulate 2 Poisson variates with mean 3. Use the random numbers given below in a sequence:
   $u_1 = 0.26$, $u_2 = 0.49$, $u_3 = 0.93$, $u_4 = 0.64$, $u_5 = 0.85$, $u_6 = 0.54$, $u_7 = 0.01$, $u_8 = 0.37$, $u_9 = 0.05$, $u_{10} = 0.43$ [2]

3. Simulate an exponential variate with parameter $\lambda=0.3$ and the random number $u = 0.45$. [1]

PTO

SECTION – B

4. The weekly consumption rate of a commodity is 850 units and the delivery rate is 1200 units per week. Shortages are allowed and are backlogged. Assume that the ordering cost is Rs.80 per order, carrying cost is Rs.1.50 per unit per week, the purchase price is Rs.40 per unit and the shortage cost is Rs.2 per unit per week. Further assume that lead time is zero, and time horizon is infinite. Find EOQ, OOC and the minimum total weekly cost. [3]

5. The daily demand of an item is 100 units; production rate is 200 units per day. The setup cost is Rs.500 per production run, holding cost is Rs.1.25 per unit per day. Shortages are not allowed. The production cost per unit (in Rupees) is as follows:

$C_0 = 5.00$ if $Q < 100$

$= 4.50$ if $100 \leq Q < 200$

$= 4.00$ if $200 \leq Q < 500$

$= 3.50$ if $500 \leq Q < 1000$

$= 3.00$ if $Q \geq 1000$

where Q denotes the quantity produced in one run. Determine the EOQ and minimum daily cost. [4]

A/13

BITS, PILANI – DUBAI CAMPUS

Knowledge Village, Dubai

(III year – II semester 2005-06)

QUIZ – I (CB)

Course Title: Operations Research **Course No. : AAOC UC 312**

Max. Marks: 10 Weightage: 10% Date: 23 Feb. 2006 Time: 30 min.

Name: **Id. No. :** **Sec:**

Instructions: *i) Write your name, Id. No. and Section in the space provided.*
ii) All questions are compulsory.
iii) Each blank carries one mark.
iv) No marks will be given for incorrect, overwritten, multiple answers.

1. There is an increase in the population & consequently increase in the number of cars in Dubai. On a particular road with one lane, it is noticed that during the peak hours of the day (i.e.7 to 9am and 5 to 7pm) cars arrive on the road every 5 minutes. Assume that the inter arrival time follows an exponential distribution and the service time (i.e. the cars can cross that particular road) is also to be assumed as exponential with mean of 18 minutes. What is the minimum number of lanes needed to avoid an infinite queue, that is, in order that the system achieves steady- state situation

   _____

2. The exponential distribution has "no memory" is shown by the theorem stating : If T has exponential distribution with parameter λ, then
   $P[T > t_0 + t_1 / T > t_o] = P[T > t_1]$, $t_0, t_1 > 0$.
   This is called _____ property of exponential distribution.

3. People arrive at the ticket counter of Cinestar, City Centre, Dubai in a Poisson manner at the rate of 2.5 per hour to purchase the ticket for the show. Assume that the service time follows exponential distribution with mean 3 minutes.
   (a) The number of persons expected to wait in the queue to purchase the ticket is _____.
   (b) Utilization factor is_____.

4. In the queuing system (M/M/1): (FCFS/3/∞), $\lambda = 10$ per day and $\mu = 15$ per day.
   (a) The fraction of customers lost is _____
   (b) Proportion of idle time of the server is _____

5. There are two clerks in a university to receive dues from the students. If the service time for each student is exponential with mean 4 minutes and if the students arrive in Poisson manner at the counter at the rate 10 per hour. The students arriving at the counter wait in the queue and are served on first come first serve basis.
   (a) Identify the model & denote it using Kendall's notation _____.
   (b) The probability that the student will have to wait for service is _____.

6. In a booking counter, on the average customer arrives every 5 minutes and average service time for each customer is 4 minutes. If we assume Poisson arrival and exponential service, then the fraction of time the server is idle is _____.

7. In a queuing system, arrivals are in a Poisson process with mean rate $\lambda = 5$ and the service times are independent exponentially distributed with parameter $\mu = 5$. If the number of servers assigned to serve the customers at any instant is equal to the number of customers in the system, then the probability that the state of the system is 4, is _____.

As per the standard notation:

1) $(M/M/s):(FCFS/m/\infty)$

(i) $P_n = \frac{\rho^n}{n!} \cdot P_0 \; ; \; n = 0, 1, 2 \ldots (s-1)$

$= \frac{\rho^n}{s!\, s^{n-s}} \cdot P_0 \; ; \; n = s, s+1 \ldots m$

$= 0 \; ; \; n > m$

(ii) $P_0 = \left[1 + \frac{\rho^s \left\{ 1 + \left(\frac{\rho}{s}\right)^{m-s+1} \right\}}{s! \left(1 - \frac{\rho}{s}\right)} + \sum_{n=1}^{s-1} \frac{\rho^n}{n!} \right]^{-1} ; \frac{\rho}{s} \neq 1$

2) $(M/M/s):(FCFS/\infty/\infty)$

(i) $P_n = \frac{\rho^n}{n!} \cdot P_0 \; ; \; n = 0, 1, 2 \ldots s-1$

$= \frac{\rho^n}{s!\, s^{n-s}} \cdot P_0 \; ; \; n = s, s+1 \ldots$

(ii) $P_0 = \left[\frac{\rho^s}{s! \left(1 - \frac{\rho}{s}\right)} + \sum_{n=0}^{s-1} \frac{\rho^n}{n!} \right]^{-1}, \; \rho < s$

MAKE-UP

BITS, PILANI – DUBAI CAMPUS
DUBAI KNOWLEDGE VILLAGE
III YEAR – II SEMESTER
(2005-2006)

QUIZ-I (CB)

OPERATIONS RESEARCH
(AAOC UC312)

Max. Marks : 10 Weightage : 10% Date : Time : 30 minutes

Name : Id No. : Section :

- *Write your Name, Id. No., Sec. in the space provided.*
- *Attempt all the questions.*
- *Each blank carries 1 mark*

Fill in the blanks :

1. If $X_1, X_2, ..., X_n$ are independent exponential variates with parameters $\lambda_1, \lambda_2, ..., \lambda_n$ respectively, then $Min(X_1 + X + ... + X_n)$ is also an exponential variate with parameter _____.
2. In a (M/M/1):(FCFS/10/∞) queueing system, if $\lambda = \mu$ then the expected queue length is _____ and the probability that there are 6 customers in the system at any instant is _____ .
3. In a (M/M/1):(FCFS/∞/∞) queueing system, if mean arrival rate is 0.2 and the mean service rate is 0.25, then the expected number of customers in the system is _____ and the probability that the server is busy, is _____.
4. Consider a queueing system with Poisson arrival and exponential service in which number of servers is sufficiently large. At any time, the number of servers engaged in the system is equal to the number of customers in the system at that time. If customers arrive on average every 5 minutes and average service time per customer is 8 minutes, then the average number of customers in the system at any instant of time is _____ and the probability that there are 4 customers in the system is _____.

5. A factory has 20 machines and has its own workshop for repairing the breakdown machines. Five repairmen are working in parallel in the workshop. Considering Poisson breakdown rate and exponential service, express the queueing system by Kendall's notation. _____.

6. As per the axioms of Birth and Death process the Probability of exactly one birth in the interval (T, T+ΔT] given that N(T) = n is denoted by _____.

7. A barber shop has two barbers and three chairs for customers. Assume that the customers arrive in Poisson manner at a rate of 5 per hour and that each barber services customers according to an exponential distribution with mean 15 minutes. Further, if a customer arrives and there are no empty chairs in the shop, he will leave. The expected number of customers that will arrive in the shop are _____.

As per the standard notation:

1) $(M/M/s):(FCFS/m/\infty)$

(i) $P_n = \frac{\rho^n}{n!} \cdot P_0 \; ; \; n = 0, 1, 2 \ldots (s-1)$

$= \frac{\rho^n}{s!\, s^{n-s}} \cdot P_0 \; ; \; n = s, s+1 \ldots m$

$= 0 \; ; \; n > m$

(ii) $P_0 = \left[1 + \frac{\rho^s \left\{ 1 + \left(\frac{\rho}{s}\right)^{m-s+1} \right\}}{s! \left(1 - \frac{\rho}{s}\right)} + \sum_{n=1}^{s-1} \frac{\rho^n}{n!} \right]^{-1} ; \frac{\rho}{s} \neq 1$

2) $(M/M/s):(FCFS/\infty/\infty)$

(i) $P_n = \frac{\rho^n}{n!} \cdot P_0 \; ; \; n = 0, 1, 2 \ldots s-1$

$= \frac{\rho^n}{s!\, s^{n-s}} \cdot P_0 \; ; \; n = s, s+1 \ldots$

(ii) $P_0 = \left[\frac{\rho^s}{s! \left(1 - \frac{\rho}{s}\right)} + \sum_{n=0}^{s-1} \frac{\rho^n}{n!} \right]^{-1}, \; \rho < s$