BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

COMPREHENSIVE EXAMINATION –III YEAR –II SEMESTER 2005-2006

Date: 28/05/06 Course: Numerical Analysis AAOC UC341
Duration: 3hours Total Marks: 40
Instructors: Dr Priti Bajpai, Dr A.Somasundaram

NOTE : ANSWER PART – A AND PART – B SEPARATELY

PART – A

Q1 **[2+2+2+2]**

(a) If $f(x)=x^{1/7}$ is computed for $0 \le x \le 7$ correct to n significant decimal digits then how many significant digits $f(x^*)$ approximates $f(x)$.

(b) Establish an iteration formula to find the reciprocal of a positive number N by Newton's method.

(c) Solve the following system of equation by Gauss elimination method with five digit arithmetic $3x+4y+5z=18$, $2x-y+8z=13$, $5x-2y+7z=20$.

(d) Solve the boundary value problem $4y''+y=0, y(0)=0, y(\pi)=2$ by replacing the derivative with a central difference approximation with $h=\frac{\pi}{3}$.

Q2 **[3+3]**

(a) Find by Regula- Falsi method the positive root of $x^2-\log_{10}x-12=0$ correct to 5 decimal places in 3 iterations.

(b) Solve the system of equation $28x+4y-z=32$, $x+3y+10z=24$, $2x+17y+4z=35$ by Gauss-Seidel method in 3 iterations with 5 digit arithmetic.

Q3 **[3+3]**

(a) Determine the largest Eigen value of the matrix $\begin{pmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{pmatrix}$ in three iterations with initial eigen vector $(1\ 1\ 1)^T$ and using five digit arithmetic.

(b) Find $\frac{dy}{dx}$ at $x=51$ from the following data with five digit arithmetic

x:	50	60	70	80	90
y:	19.96	36.65	58.81	77.21	94.61

PART – B

Q4 **[3+3]**

(a) Using 3-point Gauss- Chebyshev Quadrature, evaluate the integral

$$\int_0^1 \frac{\cos \pi x}{[x(1-x)]^{\frac{3}{2}}} dx$$

(b) Find $x_0, x_1, A_0, A_1, \alpha$ so that the following integration rule is exact for a polynomial of degree as high as possible

$$\int_{-1}^{1} \frac{f(x)}{1+x^2} dx = A_0 f(x_0) + A_1 f(x_1) + E \quad \xi \in (-1,1)$$

Q5 **[3+3]**

(a) Find the minimum number of equispaced tabular points required for piecewise cubic interpolation of the function $f(x) = \ln x$, on the interval [1, 2] to get 4 decimal place accuracy.

(b) Let $f(x) = x^3 - x^2 + 2x - 2$ interpolate $f(x)$ at $-2, 0, 1, 2$. Prepare the table of divided differences. Now $f(4) = -90$ is added to the data, to get the interpolating polynomial $p_4(x) = p_3(x) + g(x)$. Find $g(x)$ and hence interpolate $f(3)$.

Q6 **[4+4]**

(a) Use R-K method of order 4 to find the value of $y(0.1)$ and $y(0.2)$ with five digit arithmetic as a solution of $\frac{dy}{dx} = x + y^3, y(0) = 2$, where h=0.1

(b) Using Adam Moulton's Predictor corrector formula for the differential equation $\frac{d^2y}{dx^2} = y$ find the value of $y(0.4)$ using six digit arithmetic, given $y(0) = 2$, $y(0.1) = 2$, $y(0.2) = 2.04$, $y(0.3) = 2.09068$, $y'(0) = 1$, $y'(0.1) = 0.200334$, $y'(0.2) = 0.402677$, $y'(0.3) = 0.60904$.

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III YEAR II SEMESTER

B

QUIZ – 2 (Closed Book)

Course Title: Numerical Analysis | **Course No: AAOC UC341**
Date: 27.4.2006 | **Max marks: 10**
Time: 30 min | **Weightage: 10%**

Name of the Student: _____

ID No: _____

Branch: _____

Set: B

Recheck Request:

Answer all Questions

1. What is an ill conditioned system? [0.5]

2. The velocity of a particle which starts from rest is given by the following table:

t (sec)	0	2	4	6	8	10	12	14	16	18	20
v (ft/sec)	0	16	29	40	46	51	32	18	8	3	0

Composite Simpson's $\frac{3}{8}$ rule is .....[0.5]

and the total distance traveled is ..... [1]

3. Newton's forward interpolation polynomial is ..... [0.5]

4. If $f(x)=x^2e^{-\frac{x}{2}}$ then at $x=1.1, 2, 3.5, 5, 7.1$ the divided difference table with 4 digit arithmetic is

x	$f(x)$	$f[,]$
1.1		
2		
3.5		
5		
7.1		

[1.5]

5. If $x=(1.25, 0.02, -5.15, 0)$ then (i) $\|x\|_1 =$ .....

(ii) $\|x\|_2 =$ ..... (iii) $\|x\|_\infty =$ ..... [1.5]

6. The upper bound on error in piece-wise linear interpolation is

..... [0.5]

7. If $Ax=b$ is written as $x=Bx+c$ then $x=Bx+c$ has a unique solution if

..... [0.5]

8. The second derivative at x_i when the value of f are given with spacing

$x_i = x_0 + ih, i = 0, 1, \cdots n$ is ..... [0.5]

9. Using Jacobi's method find the following for the given system of equations with 4 digit arithmetic $3x+4y+15z=54.8;\ x+12y+3z=39.66;\ 10x+y-2z=7.74$

(i) $x=$

(ii) $y=$

(iii) $z=$

(iv) $x_1 =$ ..... when $x_0 =$ .....

(v) $y_1 =$ ..... when $y_0 =$ .....

(vi) $z_1 =$ ..... when $z_0 =$ ..... [1.5]

10. The interpolating polynomial in Lagrangian form for the data

x:	2	−1	4
f:	6	3	5

without expanding is given by $f(x) =$ ..... [1]

$$\frac{3h}{8}\left[(f_0+f_n)+3(f_1+f_2+f_4+\cdots)+2(f_3+f_6+f_9+\cdots)\right]$$

and the value of $f(1) =$ 490.5 [0.5]

6. If $f(x)=x^2e^{-\frac{x}{2}}$ then at $x=1.1, 2, 3.5, 5, 7.1$ the divided difference table is

x	$f(x)$	$f[,]$
1.1	0.6981	
		0.8593
2	1.4715	
		0.4381
3.5	2.1287	
		−0.0511
5	2.0521	
		−0.2877
7.1	1.4480	

[1.5]

7. Newton's forward interpolation polynomial is ..... [0.5]

$$p_n(x) = f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 + \cdots + \frac{s(s-1)\cdots(s-n+1)}{n!}\Delta^n f_0$$

where $s = \frac{x-x_0}{h}$

8. The upper bound on error in piece-wise linear interpolation is ..... $\frac{Mh^2}{8}$ [0.5]

9. The second derivative at x_i when the value of f are given with spacing $x_i = x_0 + ih, i = 0,1,\cdots n$ is ..... $f_i'' = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}$ [0.5]

10. The velocity of a particle which starts from rest is given by the following table:

t (sec)	0	2	4	6	8	10	12	14	16	18	20
v (ft/sec)	0	16	29	40	46	51	32	18	8	3	0

Composite Simpson's $\frac{3}{8}$ rule is .....[0.5]

$$\frac{3h}{8}\left[(f_0 + f_n) + 3(f_1 + f_2 + f_4 \cdots) + 2(f_3 + f_6 + f_9 + \cdots)\right]$$

and the total distance traveled is ..... 490.5 [1]

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Test: II (OB) Course: Numerical Analysis – AAOC UC341

Date: 07.05.06 Duration: 50 min
Total Marks: 20 Weightage: 20

Answer ALL Questions

1. Find A_0, A_1, x_1 so that the following rule is exact for all polynomial of degree as high as possible.

$$\int_0^1 \frac{f(x)}{\sqrt{1-x^2}}dx = A_0 f(0) + A_1 f(x_1) + \alpha f''(\xi) \qquad [5]$$

2. Calculate $\int_0^1 \frac{(\log x + e^x)}{[x(1-x)]^{3/2}}dx$ using Chebyshev 3 point quadrature formula. [5]

3. Solve $y_{n+2} + y_n = 5\,2^n$ given $y_0 = 1, y_1 = 0$. [2]

4. Using Runge-Kutta method of order four, solve $y'' = y + xy', y(0) = 1, y'(0) = 0$ to find $y(0.2)$ and $y'(0.2)$ with five digit arithmetic. [4]

5. Using 4th order Adams-Moulton predictor corrector and modifier find $y(1.4)$ given that $y' - x^2 y = x^2$, $y(1) = 1, y(1.1) = 1.2, y(1.2) = 1.4662, y(1.3) = 1.8213$ with five digit arithmetic. [4]

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Make up for Test: I (CB) Course: Numerical Analysis – AAOC UC341

Date: 04.04.06 **Duration: 50 min**
Total Marks: 20 **Weightage: 20**

Answer ALL Questions

PART – A

1. If $f(x) = x^3 - 2x^2 - 0.75x + 2.25 = 0$ has a double root use Newton's method for two iterations to find it. Use five digit arithmetic with rounding, given $x_0 = 1$. **[2]**

2. Solve the system of non linear equations using Newton's method with $x_0 = 1, y_0 = 1$ and five digit arithmetic with rounding in two iterations.

$$0.3\sqrt{x} - e^{y/3} + y = -0.469228$$
$$e^{x/3} + 0.5\sqrt{y} - x = 1.034917$$ **[4]**

3. Check the conditions of convergence over $0.5 \le x \le 1$ and $1 \le y \le 1.5$ and hence solve the system of non linear equations by fixed point iteration method in two iterations.

$$x^{1/3} + y^{1/4} - x - 1.175 = 0$$

$$x^{1/4} + y^{1/2} - y - 0.8412 = 0$$

Use five digit arithmetic with rounding and given $x_0 = 1, y_0 = 1$ **[4]**

PART – B

4. Solve the following system of equations by using Gauss-elimination algorithm

$$x_1 + x_2 - 2x_3 = 2.5$$
$$4x_1 - 2x_2 + x_3 = 5.5$$
$$3x_1 - x_2 + 3x_3 = 9$$ **[5]**

5. By Gauss elimination method find the inverse of the coefficient matrix of

$$4x_1 - 10x_2 + 5x_3 = 32$$
$$5x_1 - 4x_2 + 10x_3 = 39$$
$$10x_1 + 5x_2 - 4x_3 = 17$$ **[5]**

Also find the determinant value of the coefficient matrix

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Test: I (CB) Course: Numerical Analysis – AAOCU341

Date: 26.03.06 **Duration: 50 min**

Total Marks: 20 **Weightage: 20**

Answer ALL Questions

PART – A

1. If $f(x) = x^3 - 6.25x^2 + 12.5x - 7.8125 = 0$ has a double root use Newton's method to find it. Use five digit arithmetic with rounding, given $x_0 = 2$. **[2]**

2. Solve the system of non linear equations using Newton's method with $x_0 = 0, y_0 = 0$ and five digit arithmetic with rounding.

$$x^2 - y^3 - x + y - 1.125 = 0$$
$$x^3 + y^2 + x + y + 0.875 = 0$$ **[4]**

3. Check the conditions of convergence and hence solve the system of non linear equations by fixed point iteration method.

$$\frac{x^{3/2}}{9} + \frac{y^{2/3}}{4} - x + 0.64 = 0$$

$$\frac{x^{2/3}}{4} - \frac{y^{3/2}}{9} - y + 0.861 = 0$$

Use five digit arithmetic with rounding and given $x_0 = 0.8, y_0 = 1.2$,
$R: [0.8 \le x \le 1.2, 0.8 \le y \le 1.2]$ **[4]**

PART – B

4. Gauss elimination with scaled partial pivoting is performed on a 3x3 matrix A. After two steps of Gauss elimination working matrix with scaling factors 4,6,8 respectively, $\bar{p} = (p_1, p_2, p_3) = (2,1,3)$ and with multipliers is obtained as

$$\begin{pmatrix} 0.75 & -1.125 & 1.5 \\ 0.66667 & 0.16667 & -1 \\ 0.9375 & -0.75 & 2.5625 \end{pmatrix}$$

Using forward and backward substitution, solve the system of equation $AX = b$ where $b = (-16, 19, 21)^T$ using five digit arithmetic. **[3]**

5. Compute the determinant value of $\begin{pmatrix} 3 & 5 & 2 \\ 2 & -4 & -2 \\ 10 & 5 & 1 \end{pmatrix}$ by using Gauss elimination method. **[3]**

6. Find the inverse of $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & 1 \\ 1 & 0 & 2 \end{pmatrix}$ by Gauss elimination method. **[4]**

BITS, Pilani-Dubai Campus, Knowledge Village, Dubai
III YEAR II SEMESTER

MAKE UP FOR QUIZ – 1 (Closed Book)

Course Title: Numerical Analysis **Course No: AAOC UC341**
Date: 21.3.2006 **Max marks: 10**
Time: 30 min **Weightage: 10%**

Name of the Student: _____

ID No: _____

Branch: _____

Set: B

Recheck Request:

Answer all Questions

1. The error in adding 0.348, 0.1834, 345.4, 235.2, 11.75, 0.0849, 0.0214, 0.000354 will be the least when added in ..... order and the sum when 3 digit arithmetic used is ..... **[1]**

2. Name any 4 types of error. **[1]**

3. If e^x is to be evaluated correct to 8 significant digits for x lying between -8 and 9, what digit arithmetic should be used? **[2]**

4. Given $f(x) = 2.75x^3 - 2.95x^2 + 3.16x - 4.67$, the nested form of $f(x)$ is ..... and the value of $f(x)$ in nested form at $x = 1.07$ when 3 digit arithmetic used is ..... **[1]**

5. Given $f(x)=x^3-x-11=0$, on using bisection method the root lies between ..... and the first and second approximations are ..... and ..... **[1]**

6. Given $f(x)=x^2-\log x-12=0$ if the root lies in the interval (3,4) then the Regula Falsi formula is .....

and hence the value of x_1 is ..... **[1]**

7. Given $f(x)=x^3+3x-1=0$ if the root lies in the interval (0,1) then the Newton's formula is .....

and hence the value of x_1 is ..... **[1]**

8. Solve the function $\cos x=3x-1$ by the fixed point theorem in a suitable interval I.

   a. I = ...

   b. $g(x)=$ ..... and $|g'(x)|\leq$

   c. $x_1=$

   d. $x_2=$

[2]

BITS, Pilani-Dubai Campus, Knowledge Village, Dubai
III YEAR II SEMESTER

A

QUIZ – 1 (Closed Book)

Course Title: Numerical Analysis | **Course No: AAOC UC341**
Date: 9.3.2006 | **Max marks: 10**
Time: 30 min | **Weightage: 10%**

Name of the Student: _____

ID No: _____

Branch: _____

Set: A

Recheck Request:

Answer all Questions

1. If the approximate value of $\frac{8}{9}$ is 0.889 then the relative error is less than

[1]

2. Given $f(x)=2x^3-3x^2+5x-4$, the nested form of $f(x)$ is ..... and the value of $f(x)$ in nested form at $x=3.125$ when 3 digit arithmetic used is ..... [1]

3. The number of operations needed to find the value of $f(x)$ given above in the original form is .....and the number of operations needed if it is evaluated in the nested form is ..... [1]

4. If e^{5x} is to be evaluated correct to 8 significant digits for x lying between -10 and 8, what digit arithmetic should be used? [2]

5. Given $f(x)=x^3-9x+1=0$, on using bisection method the root lies between ..... and the first and second approximations are ..... and ..... [1]

6. Given $f(x)=xe^x-2=0$ if the root lies in the interval (0,1) then the Newton's formula is .....

and hence the value of x_1 is .....when x_0 = ... [1]

7. The formula for calculating a root of $f(x)$ by Regula-Falsi method in the interval (a,b) is

x_1 =

[1]

8. Solve the function $e^{-x}-10x$ by the fixed point theorem in a suitable interval I.

(i) I = ...

(ii) $g(x)$ = ... and $|g'(x)| \leq$

(iii) x_1 =

(iv) x_2 =

[2]