BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Comprehensive Examination –III Year –Sixth Semester 2004-2005
Date:5/06/05
Course: Numerical Analysis
Duration : 3hours
Total Marks : 40
Instructor:Priti Bajpai

Repeaters

NOTE : (Answer all Questions)

Q1 **[2+2+2+2]**

(a)

Let x=4.23, y=0.20598, z=0.2365 show that the distributive law does not hold, ie $x(y-z) \neq xy-xz$, on using 4 digit arithmetic with rounding.

(b)

Find the largest eigen value of

$A=\begin{bmatrix} 21 & 7 & -1 \\ 5 & 7 & 7 \\ 4 & -4 & 20 \end{bmatrix}$ take the starting vector as $x=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$,do 4 iterations .

(c)

Solve $y_{n+2} - 5y_{n+1} + 6y_n = 4$

(d)

Find sin25° using Newtons forward method to interpolate ,given

x	10	20	30	40	50
f(x)	0.1736	0.3420	0.5	0.6428	0.766

Q2. [3+3]

(a) Use the Bisection method to obtain the smallest root in (-2,-1.5) of the function

f(x)= $x^2 - 4x - 10 = 0$ by performing 5 iterations using 5 digit arithmetic with rounding .

(b) Using the Newtons method find the root of f(x) = $x^2 - 3x + 2 = 0$ by performing 5 iterations.Begin at x = 0. Using 4 digit arithmetic with rounding

Q3 [3+3]

(a) Solve the following system of equations

3x + y =5

x - 3y = 5

using Gauss Seidel's method .Do 5 iterations .

(b)

Solve the following equations

$2x + y = 25$

$2.001x + y = 25.01$

discuss the effect of illconditioning

Q4 [3+3]

(a) Using 3-point Gauss- Legendre quadrature , evaluate the integral

$$\int_{-1}^{1} \frac{x \sin x}{1+x^2} dx$$

(b) Find $x_0, x_1, A_0, A_1, \alpha$ so that the following integration rule is exact for a polynomial of degree as high as possible

$$\int_{-1}^{1} f(x)dx = A_0 f(x_0) + A_1 f(x_1) + \alpha f^{iv}(\xi), for \quad \xi \in (-1,1)$$

Q5

[3+3]

(a) Find the minimum number of equispaced tabular points required for piecewise quadratic interpolation of the function
f(x) = $\sqrt{x}$,on the interval [1,2] to get 7decimal place accuracy.

(b) Form the Lagranges interpolation polynomial for the following data

x	-1	2	4	5
f(x)	5	3	8	5

Find f(0)

Q6

[4+4]

(a) Using Adam Moultons Predictor corrector formula for the differential equation

$\frac{dy}{dx} = y + x^2$ find the value of y(0.4) given h=0.1 and

y(0) = 0
y(0.1) = 1.105513
y(0.2) = 1.224208
y(0.3) = 1.359576

(b) Do one step of R-K method of order 2 for the differential equation

$$\frac{d^2y}{dx_2} + 2x\frac{dy}{dx} - 3y = x^2 + 2,$$

$y(1) = 1, y'(1) = 2$

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Comprehensive Examination –III Year –Sixth Semester 2004-2005
Date: 2/06/05
Duration: 3hours
Instructor: Priti Bajpai
Course: Numerical Analysis
Total Marks: 40

NOTE: (Answer all Questions)

Q1

[2+2+2+2+2]

(a) If $x=0.456732\times10^{-2}$,$y=0.243451$,$z=-0.248000$ show associative law does not hold if 4 digit arithmetic is used with rounding .

(b) Find the approximate largest eigen value of

$A=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}$ by taking the starting vector as $x=\begin{bmatrix}0\\1\\0\end{bmatrix}$,Do 4 iterations .

(c) Solve the differential equation $y_{n+2}+4y_n=0$.

(d) Using composite Simpsons 3/8 th rule find the velocity after18 seconds if a rocket has acceleration as given in the table

t	0	2	4	6	8	10	12	14	16	18
a	40	60	70	75	80	83	85	87	88	88

Where time t is in seconds and a is the acceleration in meter per second ^{2}.

(e)
Given the boundary value problem

$$\frac{d^2u}{dx^2}+\frac{1}{4}u=0, \quad u(0)=0 \quad \text{and} \quad u(\pi\}=2$$

form the system of equations(take h=π/4) ,using finite difference quotients (no need to solve the system).

Q2. [3+3]

(a) Use the method of Regula Falsi to find a real root of $x^2 - x - 2 = 0$, perform 2 iterations and use 5 digit arithmetic with rounding.

(b) Given f(x) = $8x^3 - x + 3 = 0$, find an appropriate interval containing a real root and a corresponding iteration function g(x) so that the fixed point iteration converges .Justify the conditions of the fixed point theorem and find the minimum number of iterations required to get an accuracy up to 8 significant numbers.

Q3 [3+3]

(a) Solve the following system of equations

$$x + 2y - z = 6$$
$$2x - y + z = -1$$
$$x - y + 2z = -3$$

using Jacobi's method .Do 2 iterations starting with (0,0,0) .

(b) On using the Gauss algorithm method to solve Ax = b, where

$$A=\begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix} \text{ and } b=\begin{bmatrix} 15 \\ 37 \\ -10 \end{bmatrix} \text{ and true solution is } x=\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

The first working matrix after step 1 is

$$\begin{bmatrix} 1 & 0.3 & 0.4 \\ 0.2 & -1.06 & 0.22 \\ 0.3 & 0.11 & -1.12 \end{bmatrix} \text{ with } \bar{p}=(1,2,3)$$

Give the matrix after step 2 and find the solution using 5 digit arithmetic.

Q4

(a) Using 3-point Gauss- Chebyshev quadrature , evaluate the following integral, use 5 digit arithmetic [3+3]

$$\int_{-1}^{1} \frac{(1+x)e^{x}}{\sqrt{1-x^2}} dx$$

(b)Find x_1, A_0, A_1, α so that the following integration rule is exact for a polynomial of degree as high as possible

$$\int_{0}^{1} \sqrt{x} f(x) dx = A_0 f(\frac{1}{3}) + A_1 f(x_1) + \alpha f'''(\xi), for \ \xi \in (0,1)$$

Q5

(a) Find the minimum number of equispaced tabular points required for piecewise linear interpolation of the function [3+3]

f(x) = $e^{-2x} + \frac{22}{3}x^4$,on the interval [0,4] to get 4decimal place accuracy.

(b) Use Newtons backward method to interpolate the function at x=7, given

x	0	2	4	6	8
f(x)	1	5	31	121	341

Q6

(a) Use R-K method of order 4 to find approximate value of the pair of differential equations [3+3]

$$\frac{dy}{dx} = x + z, y(0) = 1,$$

$$\frac{dz}{dx} = y - x, z(0) = -1$$

at y(0.1), and z(0.1).

(b) Using Milnes Predictor corrector formula for the differential equation $\frac{dy}{dx} = xy + x^2 - 1$ find the value of y(1.4)

given

y(1) $= 0.649$

y(1.1) $= 0.731$

y(1.2) $= 0.854$

y(1.3) $= 1.028$

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 12/05/05
Duration: 30 min

Quiz: II
Total Marks:10

Course: Numerical Analysis
Weightage: 10

NOTE: (Answer all Questions)

Q1.The Taylors series for solving differential equations is given by

.... [1]

Q2,. Taylors series of order 3 when used for the differential equation $y'(x) = x^2 + y^2$,where $y(0) = 1, h = 0.25$ gives the solution at x = 0.25 as [1]

....

Q3.If first two terms of the Taylors series are taken then the formula is known as

.... [1/2]

Q4. Which Quadrature formula can be used for solving

$$\int_{-\infty}^{\infty} e^{-x^2} \cos x dx$$

[1/2]

....

Q5. The 3 point formula for solving Q4 with error is given by

[1]

....

Q6. The value of the Integral for Q4 on using the above formula is

[2]

....

Q7. The Adam Moulton predictor corrector formulas are given by [1]

....

....

....

Q8. The solution of the differential equation

$$y'(x) = \frac{2y}{x}, \quad \text{given } y(1) = 2, h = 0.25$$ [3]

at y=2 ,using the above formula where $y_1 = 3.13, y_2 = 4.50, y_3 = 6.13$ is

....

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 17/04/05
Course:Numerical Analysis
Duration :50 min
Test :OPEN BOOK(III rd yr)
Instructor:Priti Bajpai
Total Marks : 20
Weightage: 20

NOTE : (Answer all Questions)

Q1.

For a system [4]

$3x + 6y + 2z = 15$
$2x + y + 5z = 8$
$3x + y + z = 5$

use Jacobi's method to find the solution and also give the minimum number of iterations to get the solution correct to 4 significant digits .Do 2 iterations ,taking (0,0,0) as the starting values

Q2.

Form a Lagranges Interpolation polynomial from the following table and find $\log_{10} 4.15$ [4]

x	4.1	4.2	4.3	4.4
$\log_{10} x$	0.61278	0.62324	0.63347	0.64345

(Do not expand the polynomial)

Q3.

[4]

Given $f(x) = e^{\frac{x}{2}}$, form a Newtons backward difference table for [0,0.3] keeping the length of the interval as 0.1 and find $P_4(x)$ at x=0.31.Add a new point (0.4,1.2214) how does the table change ?Find the 3rd derivative at 0.35 .

Q4.

Based on nodes (0,0), (1,-2), (2,0), (3,12), using Newtons divided difference method, form the interpolating polynomial and interpolate the polynomial at x=4.What is the error? [4]

Q5.

[4]

Given $f(x)=\frac{2}{2+x}$, [a,b] =[0,1]and h =0.2 .Apply Composite Trapezoidal rule and Composite Simpsons $\frac{1}{3}$rd rule , to find the integral of the above function. Also give the errors in both the cases.

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 31/03/05
Duration: 30 min

Quiz: I
Total Marks:10

Course: Numerical Analysis
Weightage: 10

NOTE: (Answer all Questions)

Q1.
Fill up the following blanks [4]

a. If Propagated relative error in an arithmetic operation of multiplication of two numbers x and y is given by r_{xy} then

$r_{xy} =$ ....

b. The error in adding positive numbers is minimum when they are added in

....

c. Error in evaluation of a function of a single variable x is given by

....

d. x* approximates x correct to n digits after the dot if ....

Q2. [2]

The true value of the root is 1.0515 for the given function $f(x) = e^{-x} + 4x^3 - 5$, using Bisection method and making use of 5digit arithmetic with rounding, if 4 iterations are performed the relative error is

....

Q3. [2]

Which arrangement of $e^x - 3x^2 = 0$, will give roots -0.5 and 1 if we start with $x_0 = 0$.

....

Q4. [2]

For the matrix

$$A=\begin{bmatrix} 4 & -0.5 & 2 \\ 0.6 & -3 & 4 \\ -5 & 2 & 0.8 \end{bmatrix}$$

Gauss Elimination is used

The inverse of A =

And $|A|=$

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 6/03/05 **Test: I** **Course: Numerical Analysis**
Duration: 50 min **Total Marks: 20** **Weightage: 20**

NOTE: (Answer all Questions)

Q1. **[2+2]**

(a) Evaluate the value of f (x) = $x^3 - 3.966x^2 + 5.2431x - 2.3105$ at x = 1.372, in nested form, using 5 digit arithmetic with rounding.

(b) Let e^{5x} be evaluated correct to 8 significant digits for x lying between -10 and 8, what digit arithmetic should be used?

Q2. **[4]**

Find one real root of f (x) = $x^3 - 3x^2 + 1$, using Bisection method. Make use of 5digit arithmetic with rounding and do 5 iterations.

Q3. **[4]**

Find a numerical approximation to $\sqrt{\frac{3}{4}}$, using 3digit arithmetic. Apply Newtons Method and do 3 iterations.

Q4. **[4]**

Do two iterations of Mullers Method starting with 0.5, 1 , 1.5 to solve f(x) = $x^6 - 2$.Use 5 digit arithmetic with rounding.

Q5. **[4]**

Check the conditions of convergence and solve the following system of equations using Fixed Point Iteration method.Given $x_0 = 0.8, y_0 = 1.2$

$$\frac{1}{9}x^{\frac{3}{2}} + \frac{1}{4}y^{\frac{2}{3}} - x + 0.64 = 0$$

$$\frac{1}{4}x^{\frac{2}{3}} - \frac{1}{9}y^{\frac{3}{2}} - y + 0.861 = 0$$

R:$0.5 \le x \le 1$, $0.5 \le y \le 1$.Use 4digit arithmeticand do 4 iterations.