BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

I SEMESTER - 2013-2014

COMPREHENSIVE EXAMINATION (CB)

Course Title: Optimization **Course No. : MATH F212/AAOC C 222**

Max. Marks: 40 **Weightage: 40%** **Date: 02-01-2014** **Time: 3 hours**

Non-programmable calculator is permitted. Total no. of questions:10
Attempt all the questions.

1. Following is the cost matrix of an assignment problem. Find the assignments which will minimize the total cost. [4]

		Machines				
		M1	M2	M3	M4	M5
Jobs	J1	160	130	175	190	200
	J2	135	120	130	160	175
	J3	140	110	155	170	185
	J4	50	50	90	80	110
	J5	55	35	70	80	105

2. Solve the following transportation problem:

	Destination					
	A	B	C	D	E	Supply
Sources X	45	30	40	50	40	**40**
Y	35	30	100	45	60	**20**
Z	40	60	95	35	30	**40**
Demand	**25**	**10**	**20**	**30**	**15**	

[5]

3. A small project consists of 9 activities. G, H, I are terminal activities. Immediate predecessors and time estimates in days (optimistic, most likely, pessimistic) for all activities are listed below:

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	---	---	---	A	A	B	C	D	E, F
Optimistic time	5	18	26	16	15	6	7	7	3
Most likely time	8	20	33	18	20	9	10	8	4
Pessimistic time	10	22	40	20	25	12	12	9	5

a) Draw the network of the project.
b) Find expected durations and variances of all the activities.
c) What is the expected project length(T_E)? [5]
d) If the project completion due time is 41.5 days, what is the probability of meeting deadline? (Given $P(Z \leq -0.52) = 0.30$)

4. A firm produces two types of products A and B and sell them in the market. Profit per unit of product A is AED 80 and per unit of product B is AED 30. Following are the goals of the firm:

a) to earn a profit of exactly AED 8000 in the next day;
b) sale volumes of product A should be very closed to 10 but not exceeding 10;
c) sale volumes of product B should be very closed to 12 but not exceeding 12.
Formulate it as a goal programming problem. [2]

5. The following table represents the payoff matrix of player A in a two-person zero-sum game. Find the saddle point and the optimum pure strategies of both the players. Also find the value of the game. Is the game fair? [3]

		Player B			
		B1	B2	B3	B4
	A1	20	15	12	35
Player A	A2	25	14	8	10
	A3	40	2	10	5
	A4	-5	4	11	0

6. Write down KKT's conditions for the following NLPP;
Maximize $f = 12x_1 + 21x_2 + 2x_1x_2 - 2x_1^2 - 2x_2^2$
subject to $x_1 + x_2 \le 10, \quad x_1, x_2 \ge 0.$
Find the Hessian matrix of the objective function f.
Test whether it is negative definite. [4]

7. Use dynamic programming to solve:
Maximize $Z = y_1.y_2.y_3$ subject to $y_1 + y_2 + y_3 = 12, \quad y_1, y_2, y_3 \ge 0.$ [5]

8. A manufacturer produces two different models, X and Y of the same product. Both models require raw materials r1 and r2. At least 18 kg. of r1 and 12 kg. of r2 must be used daily. At most 34 hours of labour can be utilized daily. Each unit of X needs 2 kg. of r1 and 1 kg. of r2. Each unit of Y needs 1 kg. of r1 and 1 kg. of r2. It takes 3 labour-hours to manufacture each unit of X and 2 labour-hours for each unit of Y. The manufacturer earns a profit of Rs.50 by selling each unit of X and Rs.30 for each unit of Y. Formulate it as an LPP and solve it graphically to find number of each model that should be produced to maximize the daily profit. (*Draw the graph roughly on the answer book*) [5]

9. Consider the following LPP:
Maximize $Z = 3x_1 + 5x_2$ subject to $3x_1 + 2x_2 \le 18, \ x_1 \le 4, \ x_2 \le 6, \ x_1, x_2 \ge 0.$
The optimal simplex table is:

$C_j \rightarrow$		3	5	0	0	0	
	Basic	x_1	x_2	s_1	s_2	s_3	Solution
3	x_1	1	0	1/3	0	-2/3	2
0	s_2	0	0	-2/3	1	4/3	0
5	x_2	0	1	0	0	1	6
$Z_j \rightarrow$		3	5	1	0	3	$Z = 36$
C_j-$Z_j \rightarrow$		0	0	-1	0	-3	

A new constraint $2x_1 + x_2 \le 8$ is added to this LPP. Find the revised solution. [5]

10. Write down the dual of the following LPP:

Maximize $Z = 2x_1 + 3x_2$

subject to

$5x_1 + 2x_2 \leq 40,$

$3x_1 - x_2 \geq 20,$ [2]

$x_1 \geq 0, x_2$ is unrestricted.

BITS PILANI – DUBAI CAMPUS

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FIRST SEMESTER - 2013-2014

TEST – II (OB)

Course Title: Optimization **Course No. : MATH F212/AAOC C 222**

Max. Marks: 20 **Weightage: 20%** **Date: 20-11-2013** **Time: 50 min.**

Prescribed Textbook and hand-written class notes are allowed.

Attempt all the questions.

1. Find an initial basic feasible solution of the following TP by Vogel's approximation method and then find its optimum solution:

Destination

		D1	D2	D3	D4	Supply
Sources	S1	2	3	13	7	6
	S2	1	0	6	1	1
	S3	5	8	15	9	10
	Demand	7	5	3	2	

[5]

2. Solve graphically the following 2-person zero sum game and find the value of the game: (*find mixed strategies of both the players)* [6]

Player B

Player A	B1	B2	B3	B4
A1	2	2	3	-2
A2	4	3	2	6

3. Consider the following LPP:

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

subject to: $x_1 + x_2 + x_3 \le 3,\ x_1 + 4x_2 + 7x_3 \le 9,\quad x_1, x_2 \ge 0.$

The final simplex table is: [6]

	$C_j \rightarrow$	4	6	2	0	0	
	Basic	x_1	x_2	x_3	s_1	s_2	Solution
4	x_1	1	0	-1	4/3	-1/3	1
6	x_2	0	1	2	-1/3	1/3	2
	Z_j	4	6	8	10/3	2/3	**16**
	$C_j - Z_j$	0	0	-6	-10/3	-2/3	

The right-hand side constants of the constrains are changed from $\begin{bmatrix}3\\9\end{bmatrix}$ to $\begin{bmatrix}p\\q\end{bmatrix}$.

For this change, the revised right hand side (Solution)column in the above table

is changed from $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ to $\begin{bmatrix} 10 \\ -1 \end{bmatrix}$. What is the revised optimum solution? Find the values of p and q.

4. Write the dual of the following LPP:

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

subject to: $x_1 + x_2 + x_3 \leq 3,\ x_1 + 4x_2 + 7x_3 = 9,\quad x_1, x_2 \geq 0, x_3$ is unrestricted. [3]

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

FIRST SEMESTER - 2013-2014

TEST – I (CB)

Course Title: Optimization **Course No. : MATH F212/AAOC C 222**

Max. Marks: 25 **Weightage: 25%** **Date: 25-09-2013** **Time: 50 min.**

Attempt all the questions.

1. A firm manufactures three products X, Y and Z. Profit per unit of these products are AED 400.00, AED 300.00 and AED 500.00 respectively. These products need to process through two machines A and B. Processing times(in minute) per unit of the products are given in the following table:

	Product		
	X	Y	Z
Machine A	4	5	3
Machine B	2	1	4

The upper limits of production volumes for the products X, Y and Z are 100 units, 200 units and 80 units respectively. Machine-minutes available for A and B are 2000 minutes and 2500 minutes respectively. Total amount of products X and Y taken together should be at least equal to the amount of product Z. The manufacturer wants to find the amount of each of these products to be manufactured to maximize the profit. Formulate it as an LPP. [5]

2. Solve the following LPP by graphical method:

$$\begin{aligned} &\textit{Minimize } Z = 200x + 400y \\ &\textit{subject to } x + y \geq 200, \\ &\qquad x + 3y \geq 400, \\ &\qquad x + 2y \leq 350, \\ &\qquad x, y \geq 0. \end{aligned}$$

[5]

3. Solve the following LPP by big-M method:

$$\begin{aligned} &\textit{Minimize } Z = 600x_1 + 500x_2 \\ &\textit{subject to } 2x_1 + x_2 \geq 80, \\ &\qquad x_1 + 2x_2 \geq 60, \\ &\qquad x_1, x_2 \geq 0. \end{aligned}$$

[10]

4. The following table is the final simplex table of an LPP where x_1 and x_2 are decision variables:

C_j		1	1/2	0	0	
	Basic	x_1	x_2	s_1	s_2	**Solution**
1	x_1	1	½	½	0	2
0	s_2	0	3/2	-1/2	1	1
	$\mathbf{Z_j}$	1	1/2	1/2	0	2
	$\mathbf{C_j - Z_j}$	0	0	-1/2	0	

Find an alternative solution of this LPP. [5]

A

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

FIRST SEMESTER - 2013-2014

QUIZ – I (CB)

Course Title: Optimization **Course No. : MATH F212/AAOC C 222**

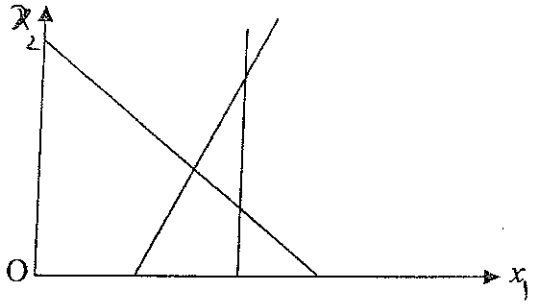
Max. Marks: 8 **Weightage: 8%** **Date: 09-10-2013** **Time: 20 min.**

NAME: ID. NO.:

Attempt all the questions. No extra sheet will be given for rough work.
Fill in the blanks with correct answers. Write your answer in proper place.

1. While solving pure integer programming problem using branch-and-bound method, subproblems SP_1 and SP_2 give us the following solutions:
   SP_1: $x_1 = 1.75$, $x_2 = 2$ and $Z = 2.75$;
   SP_2: $x_1 = 1$, $x_2 = 2$ and $Z = 3$.
   In the next step, the subproblems are
   SP_3: SP_1 with additional constraint _____;
   SP_4: SP_1 with additional constraint _____. [1]

2. The constraints of an LPP are:
   $2x_1 - x_2 \geq 6$, $x_1 + x_2 \geq 8$, $x_1 \leq 6$, $x_1, x_2 \geq 0$.
   Shade the feasible region of its graph given below.



[1]

3. Find an initial BFS of the following transportation problem by least cost method: [3]

	W1	W2	W3	W4	Capacity
F1	5	8	12	10	100
F2	7	4	11	13	150
F3	2	6	9	15	300
Demand	100	200	125	125	

4. Following is the incomplete u_i, v_j multipliers' table in which u_1 and v_2 are missing. These values are u_1 = _____ and v_2 = _____. '*' indicates occupied cell. [2]

	W1	W2	W3	W4	u_i
F1	7 *	10 *	12	10	---
F2	7	4 *	11 *	13 *	2
F3	2	6	9 *	15	0
v_j	-1	---	9	11	

5. If an additional constraint (not dummy) is added to an LPP, then the area of the feasible region will _____ (increase or decrease?) [1]

A

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NAME: ID. NO.:

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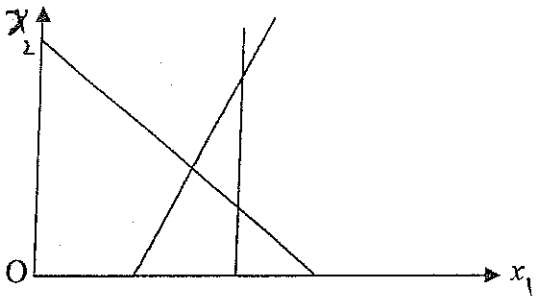
1. While solving pure integer programming problem using branch-and-bound method, subproblems SP_1 and SP_2 give us the following solutions:
   SP_1: $x_1 = 0.75$, $x_2 = 2$ and $Z = 2.75$;
   SP_2: $x_1 = 1$, $x_2 = 2$ and $Z = 3$.
   In the next step, the subproblems are
   SP_3: SP_1 with additional constraint _____;
   SP_4: SP_1 with additional constraint _____. [1]

2. The constraints of an LPP are:
   $2x_1 - x_2 \geq 6,\ x_1 + x_2 \leq 8,\ x_1 \leq 6,\ x_1, x_2 \geq 0.$
   Shade the feasible region of its graph given below.

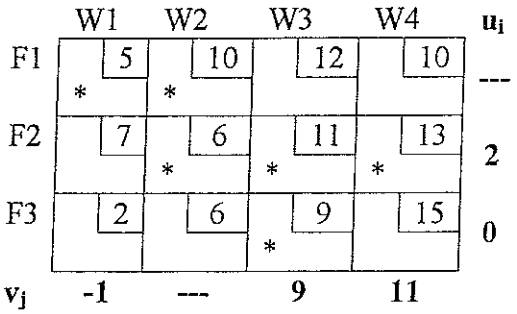


[1]

3. Find an initial BFS of the following transportation problem by least cost method: [3]

	W1	W2	W3	W4	Capacity
F1	5	8	12	10	150
F2	7	4	11	13	150
F3	2	6	9	15	300
Demand	100	200	125	175	

4. Following is the incomplete $u_i v_j$ multipliers' table in which u_1 and v_2 are missing. These values are u_1 = _____ and v_2 = _____. '*' indicates occupied cell. [2]



5. If an additional constraint (not dummy) is added to an LPP, then the area of the feasible region will _____ (increase or decrease?) [1]

B

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   SP_1: $x_1 = 1.25$, $x_2 = 1.50$ and $Z = 2.75$;
   SP_2: $x_1 = 1$, $x_2 = 2$and $Z = 3$.
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   SP_3: SP_1 with additional constraint _____;
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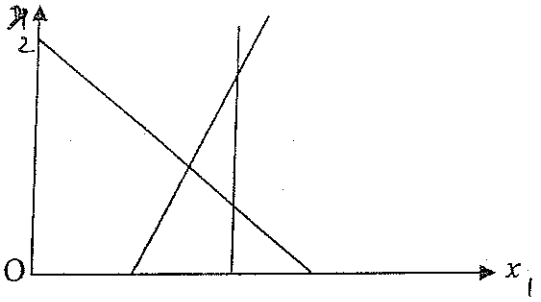

[1]

3. Find an initial BFS of the following transportation problem by least cost method: [3]

	W1	W2	W3	W4	Capacity
F1	15	8	12	10	**100**
F2	7	14	11	13	**150**
F3	20	6	9	15	**300**
Demand	**100**	**200**	**125**	**125**	

4. Following is the incomplete u_i, v_j multipliers' table in which u_1 and v_2 are missing. These values are u_1 = _____ and v_2 = _____. '*' indicates occupied cell. [2]

	W1	W2	W3	W4	u_i
F1	1 *	4 *	12	10	---
F2	7	4 *	11 *	13 *	2
F3	2	6	9 *	15	0
v_j	-1	---	9	11	

5. If an additional constraint (not dummy) is added to an LPP, then the area of the feasible region will _____ (increase or decrease?) [1]

B

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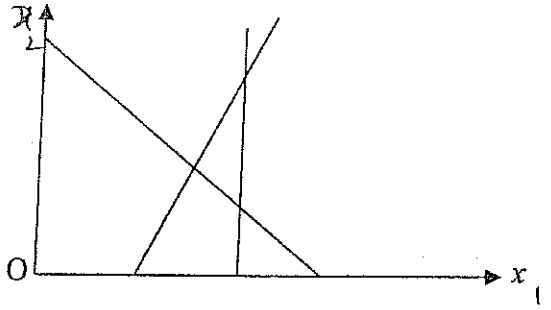

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	W1	W2	W3	W4	Capacity
F1	5	8	12	10	100
F2	7	14	11	13	150
F3	2	6	9	15	300
Demand	**100**	**200**	**125**	**125**	

4. Following is the incomplete u_i, v_j multipliers' table in which u_1 and v_2 are missing. These values are u_1 = _____ and v_2 = _____. '*' indicates occupied cell. [2]

	W1	W2	W3	W4	u_i
F1	9 *	12 *	12	10	---
F2	7	4 *	11 *	13 *	2
F3	2	6	9 *	15	0
v_j	-1	---	9	11	

5. If an additional constraint (not dummy) is added to an LPP, then the area of the feasible region will _____ (increase or decrease?) [1]