BITS PILANI, DUBAI CAMPUS

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

III YEAR – I SEMESTER 2013-14

GRAPHS AND NETWORKS (MATH F243)

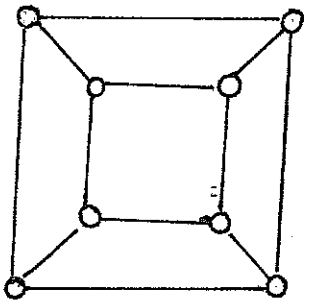
COMPREHENSIVE EXAMINATION (Closed Book)

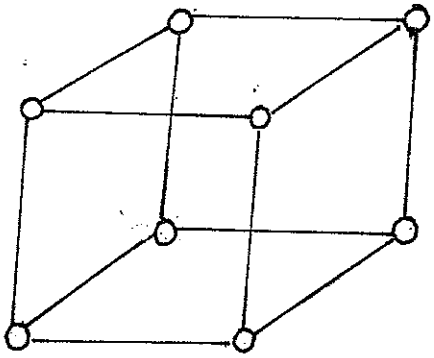
Time: 3 Hours | Max. Marks: 40

Date: January 02, 2014 | Weightage: 40%

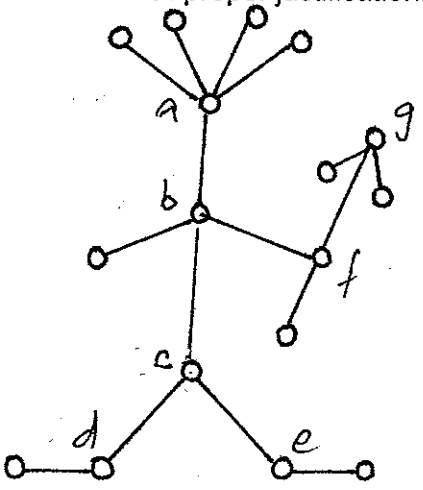
Answer all questions

1. How are N_n, P_n, C_n, K_n related via the subset relationship. **[1]**
2. A (p,q) graph has t vertices of degree m and all other vertices are of degree n. Show that $(m-n)t + pn = 2q$. **[2]**
3. A sequence $d = (d_1, d_2, \cdots, d_n)$ is graphic if there is a simple non directed graph with degree sequence d. Is the degree sequence $d = (6,6,5,4,3,3,1)$ graphic? Justify your answer. **[2]**
4. Check whether the following graphs are isomorphic. If they are isomorphic, construct an isomorphism between them. **[3]**



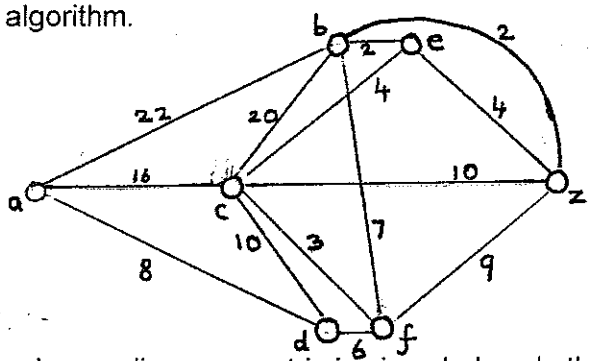


5. What is the length of strictly increasing subsequence of the sequence $(8,3,7,5,1,6,2,9,4)$. Use a data tree to determine all of the strictly increasing subsequence. Also find the strictly increasing subsequence of maximum length. **[4]**
6. For the graph given below find the eccentricity of all the internal vertices and find the centre of the tree with proper justification. **[4]**



7. Using tree to Prufer code construct a tree from its Prufer code $(4,4,2,5,4,5)$. **[2]**

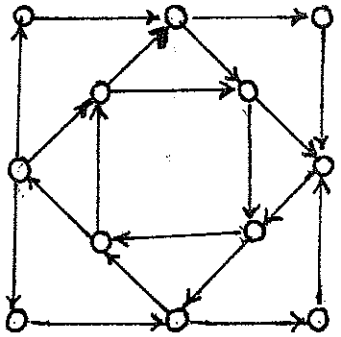
8. Find the shortest distance from vertex *a* to all vertices in the following weighted graph using Dijkstra's algorithm. **[3]**



9. Find the digraph whose adjacency matrix is given below. Is the resulting digraph strongly connected? Justify your answer. **[2]**

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

10. Check whether the following digraph and the underlying graph are both Eulerian. Justify your answer. **[2]**



11. Prove that any acylic digraph $\overline{G}$ yields a partial order $\leq$ on $V(\overline{G})$ given by $u \leq v$ iff there is a directed path from v to u in $\overline{G}$. **[3]**

12. Prove that every connected graph with n vertices, m edges and f faces satisfies the equation $n-m+f=2$. **[3]**

13. For $m, n \geq 2$ find the connectivity of the following: **[2]**

(i) For any tree T on 2 or more vertices.

(ii) For any cycle C_n on $n \geq 3$ vertices.

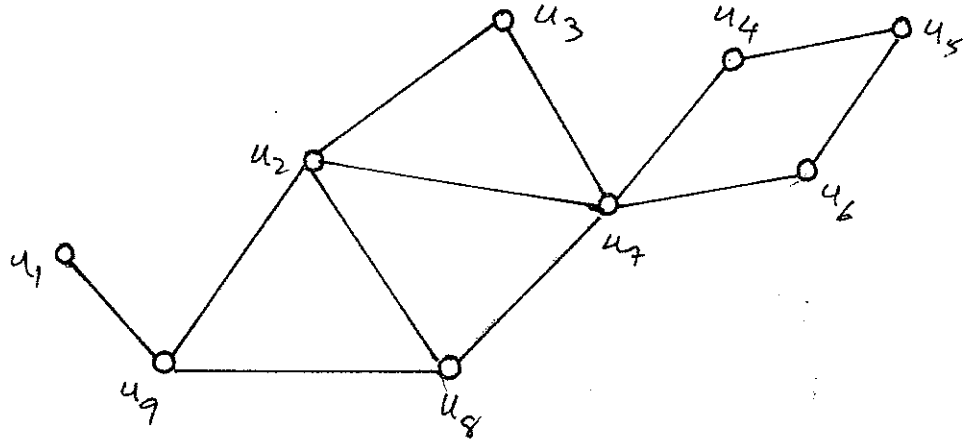
(iii) For the complete graph K_n.

(iv) For the complete bipartite graph $K_{m,n}$.

14. Suppose there are seven final exams at a university to be scheduled. Let the courses be numbered 1 through 7. Suppose that the following pairs of courses have common students: 1 and 2, 1 and 3, 1 and 4, 1 and 7, 2 and 3, 2 and 4, 2 and 5, 2 and 7, 3 and 4, 3 and 6, 3 and 7, 4 and 5, 4 and 6, 5 and 6, 5 and 7, 6 and 7. Using graph coloring schedule the final exams so that no student has two exams at the same time. **[3]**

15. (i) Define independent set.

(ii) A certain country wants to determine how many distinct radio stations it can have broadcasting within its borders. The terrain is hilly and mountainous. Each potential radio station must be assigned a frequency that is robust and differs substantially from the frequency assigned to any other station in the country. Nine different frequencies are available and some of them close or similar enough to be confused with each other is connected by an edge as given below. Determine the maximum number of non interfering radio stations the country can build. **[2]**



16. (i) Define dominating set.

(ii) In a state there are nine cities, each of which needs a police force to patrol the streets. The above given graph indicates the cities that are close to one another and linked via a good road system are connected by an edge. Determine the minimum number of police stations needed to be built to ensure that all streets are patrolled.

[2]

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GRAPHS AND NETWORKS (MATH F243)

TEST – II (Open Book)

Time: 50 Minutes Max. Marks: 20

Date: November 20, 2013 Weightage: 20%

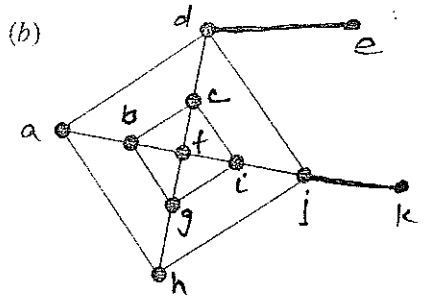
Answer all questions and each question carries 4 marks

1. Prove that every connected graph has a spanning tree.

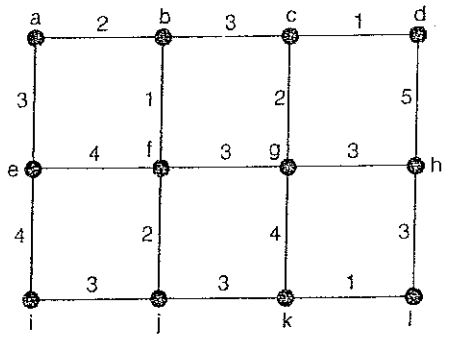
2. Let G be a graph with vertex set $V=\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ and edge set $E=\{(v_2, v_3),(v_2, v_1),(v_4, v_5),(v_4, v_6),(v_5, v_8),(v_6, v_7),(v_4, v_2),(v_7, v_9),(v_7, v_{10})\}$. Is this graph a rooted tree? Justify your answer.

3. Let T be a tree with 50 edges. The removal of certain edge from T results in two disjoint trees T_1 and T_2. Given that the number of vertices in T_1 is equal to the number of edges in T_2. Determine the number of vertices and number of edges in T_1 and T_2.

4. Find a spanning tree of the following graph by using Depth First Search Algorithm.



5. Using Prim's algorithm, find a minimal spanning tree for the following graph:



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GRAPHS AND NETWORKS (MATH F243)

TEST – I (Closed Book)

Time: 50 Minutes | Max. Marks: 25

Date: September 25, 2013 | Weightage: 25%

Answer all questions

1. Draw the digraph of the relation '$\subseteq$' on all the non-empty subsets of the set $\{0,1,2\}$. **[4]**

2. If *n* is a positive integer, let D_n denote the set of positive divisors of *n*. Is $(D_n, /)$ a Partially ordered set (Poset)? Justify your answer. **[3]**

3. Draw a picture of the following graph and state whether it is directed or non-directed and whether it is a simple graph. Also find the degree of each vertex if it is non-directed or find the in-degree and out-degree of each vertex if it is a digraph.
$G=(V,E)$ where $V=\{a,b,c,d,e\}$, $E=\{(a,b),(a,c),(a,d),(a,e),(e,c),(c,a)\}$ **[4]**

4. How many vertices will the following graph have if they contain 21 edges, 3 vertices of degree 4 and other vertices of degree 3. **[4]**

5. Find the number of vertices and edges for the graphs N_n, P_n, C_n, K_n. **[4]**

6. If a graph *G* contains a *u*, *v*-walk, then show that *G* contains a *u*, *v*-path. **[4]**

7. Define $N(u)$ and for what kind of vertices *u* in a graph *G* do we have $N(u)=N[u]$ **[2]**

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GRAPHS AND NETWORKS (MATH F243)

QUIZ – II (Closed Book)

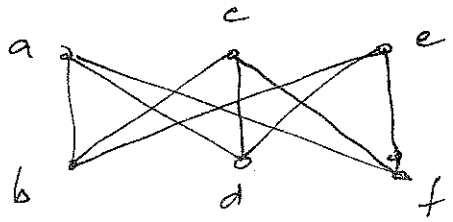
Time: 20 Minutes | Max. Marks: 7

Date: December 12, 2013 | Weightage: 7%

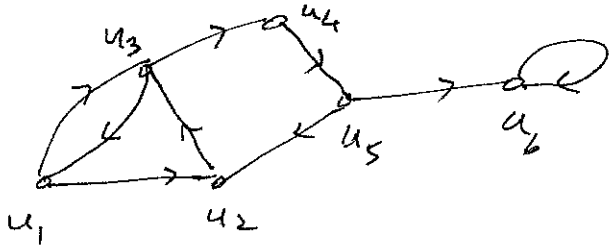
ID.No: Name:

Answer all questions

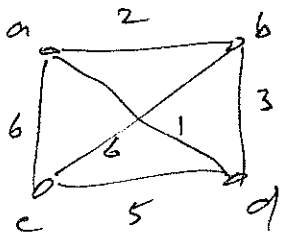
1. Check whether the following graph is Eulerian. Justify your answer. **[1]**



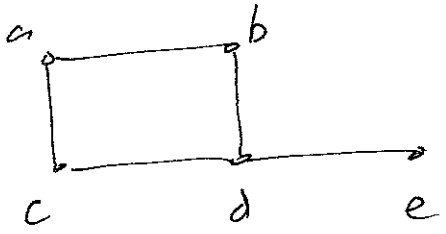
2. Find the adjacency matrix of the following digraph. **[2]**



3. Find three distinct Hamiltonian cycles in the following graph. Also find their weights. **[3]**



4. Find the edge connectivity $\kappa'(G)$ and connectivity $\kappa(G)$ for the following graph *G*. **[1]**



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GRAPHS AND NETWORKS (MATH F243)

QUIZ – I (Closed Book)

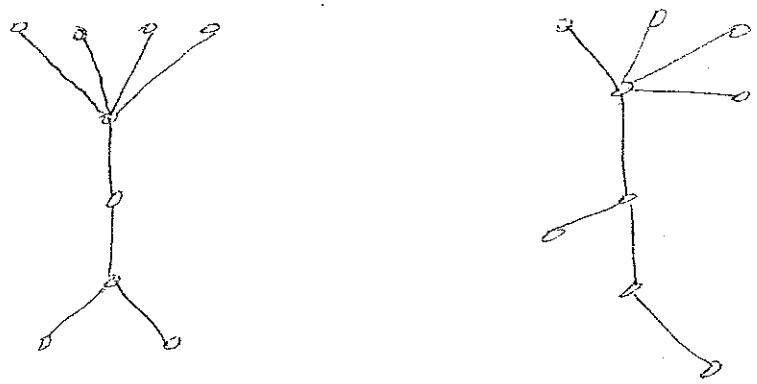
Time: 20 Minutes | Max. Marks: 8

Date: October 24, 2013 | Weightage: 8%

ID.No: Name:

Answer all questions

1. Check whether the following graphs are isomorphic. If they are isomorphic, construct an isomorphism between them. If they are not isomorphic state a graph-theoretic property satisfied by one and not the other. **[3]**



2. What is the complement of N_n? **[1]**

3. Let $n \in N$ be given. Which of the graphs N_n, P_n, C_n, K_n contain a cut-vertex? **[1]**

4. Draw all possible trees on six vertices. **[3]**