BITS, PILANI – DUBAI CAMPUS
INTERNATIONAL ACADEMIC CITY, DUBAI
OPTIMISATION (AAOC C222)
Comprehensive Examination
(III Year – I Semester 2012-2013)

Date: 10.01.13 Max. Marks: 120
Time: 03 Hours Weightage: 40%

Attempt all the questions.
Use Separate answer books for Part A, Part B and Part C

Part A

1) Solve the following dynamic problem **[10]**

Minimize Z= $y_1^2 + y_2^2 + y_3^2$

Subject to

$y_1 + y_2 + y_3 \geq 30$ where $y_1, y_2, y_3 \geq 0$

2) Given the flow of the activities of a project

Activities	*Optimistic Time (a)*	*Most Likely Time (m)*	*Pessimistic Time (b)*	*Activities*	*Optimistic Time (a)*	*Most Likely Time (m)*	*Pessimistic Time (b)*
A (1, 2)	6	7	8	G (4,5)	1	5	9
B (2,3)	0.5	1	1.5	H (4,7)	2	2	8
C (1,3)	1	2	9	I (5,6)	4	4	4
D (1,4)	1	4	7	J (5,7)	4	4	10
E (2,6)	1	2	3	K (6,8)	2	5	14
F (3,5)	1	2	9	L (7,8)	2	2	8

(i) Find the critical path and the expected project completion time.

(ii) Find the total float for all the non critical activities.

(iii) Find the variance of the project length. **[12]**

3) A department store plans to schedule its annual advertising. The total budget is set at $200,000. The store can purchase local radio spots at $100 per spot, local television spots at $500 per spot and local newspaper advertising at $200 per ad. The payoff from each advertising medium is a function of its audience size and audience characteristics. The generally accepted objective criterion for advertising is audience points, reflected in the following table:

Medium	*Points*
Radio	30 per spot
Television	150 per spot
Newspaper	150 per ad

The president of the firm has established the following goals for the campaign:

(i) The total budget should not exceed $200,000.
(ii) Meet the contract with the local television station that requires that the firm spend at least $30,000.
(iii) The corporate advertising policy prohibits annual newspaper ad expenditures in excess of $50,000.
(iv) Maximize the audience points for the advertising campaign.

The president has established unit weights on the goals of 10, 6, 3 and 1 for the goals 1 through 4, respectively. Formulate the above as a goal programming problem. **[8]**

Part B

1. Solve the following assignment model:

$$\begin{pmatrix} 3 & 9 & 2 & 7 & 5 \\ 6 & 1 & 5 & 6 & 6 \\ 9 & 4 & 10 & 7 & 3 \\ 9 & 6 & 2 & 4 & 5 \end{pmatrix}$$

[6]

2. A manufacturing firm produces three machine parts, A, B and C. The raw material costs of parts A, B and C are $5, $10, and $15 per unit and the corresponding prices of the finished parts are $50, $75 and $100 per unit respectively. Part A requires turning and drilling operations while part B needs milling and drilling operations. Part C requires turning and milling operations. The number of parts that can be produced on various machines per day and the daily costs of running the machine are given below.

	Number of parts that can be produced on		
Machine part	Turning Lathes	Drilling Machines	Milling Machines
A	15	15	-
B	-	20	30
C	25	-	10
Cost of running the Machine per day	$250	$200	$300

Formulate the problem as a LPP. **[6]**

3. A manufacturer produces four products A, B, C and D by using two types of machines lathes and milling machines. The times required on the two machines to manufacture 1 unit of each of the four products, the profit per unit of the product and the total time available on the two types of machines per day are given below.

	Time required per unit for product				
Machine	A	B	C	D	Total time available per day
Lathe machine	7	10	4	9	1200
Milling machine	3	40	1	1	800
Profit per unit	45	100	30	50	

The optimal table is as follows:

Basic	x_1	x_2	x_3	x_4	s_1	s_2	Solution
z	$\frac{25}{3}$	0	0	$\frac{50}{3}$	$\frac{22}{3}$	$\frac{2}{3}$	$\frac{28000}{3}$
x_3	$\frac{5}{3}$	0	1	$\frac{7}{3}$	$\frac{4}{15}$	$-\frac{1}{15}$	$\frac{800}{3}$
x_2	$\frac{1}{30}$	1	0	$-\frac{1}{30}$	$-\frac{1}{150}$	$\frac{2}{75}$	$\frac{40}{3}$

(i) If a new product E which requires 15 min of work on the lathe and 10 min on the milling machine per unit is available, will it be worthwhile to manufacture it if the profit per unit is 40? **[5]**

(ii) If each of the products A, B, C, and D require respectively 2, 5, 3 and 4 min of time per unit on grinding machine in addition to the operations specified, find the new optimal solution. Assume that the total time available on grinding machine per day is 600 min. **[8]**

(iii) Find the effect of changing the total time available per day on the two machines from 1200 and 800 min to 1500 and 1000 min. **[5]**

Part C

1. Find the dual of the following LPP:

Maximize $Z = 3x_1 + 4x_2 + 8x_3$

subject to $2x_1 + x_2 = 6,$

$-4x_1 + 5x_2 - 4x_3 \geq 10,$

$4x_1 + 7x_3 \leq 8,$

$x_1 \geq 0, x_2 \geq 0, x_3$ is unrestricted in sign. **[8]**

2. Write KKT's conditions and hence solve the following NLPP:

Maximize $z = 15x_1 + 30x_2 + 4x_1x_2 - 2x_1^2 - 4x_2^2$

subject to $x_1 + 2x_2 \leq 30,\ x_1, x_2 \geq 0.$ **[12]**

3. By using Branch and Bound method, solve the following integer programming

Minimize Z= $5x_1 + 4x_2$ **[10]**

Subject to $3x_1 + 2x_2 \geq 5$

$2x_1 + 3x_2 \geq 7$

where x_1, x_2 are non negative integers

4. Solve the following game between two players A and B, by using graphical method

$$\begin{array}{c} \\ A \end{array} \begin{array}{c} \begin{array}{cc} B & \end{array} \\ \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 5 & -1 \\ 6 & -6 \end{bmatrix} \end{array}$$

[10]

5. For the following transportation problem find initial basic feasible solution by using Vogel Approximation method and then use UV (method of multipliers) method to find optimal solution. **[10]**

Products	**Stores**			***Supply***
	S1	S2	S3	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Demand	75	20	50	

6. Solve the LPP by using big M-method: **[10]**

Maximize $Z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$

BITS, PILANI – DUBAI CAMPUS
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI
(III Year – I Semester 2012-2013)
OPTIMISATION (AAOC C222)
TEST– II (Open Book)

Date: 20.12.12
Time: 50 Minutes

Max. Marks: 60
Weightage: 20%

Attempt all the questions.

1) Solve the dynamic programming **10 Marks**

$Maximize\ Z = y_1\ y_2 y_3$

Subject to $y_1 + y_2 + y_3 = 5$

$y_1, y_2, y_3 \geq 0$

2) A production manager of a company wants to schedule a week's production run for two products A and B each of which requires the labor and raw materials as given below:

	Product	
	A	B
Labor hours	2	4
Raw material M1(in kg)	4	5
Raw material M2(in kg)	5	4

The weekly availability of resources is limited to 600 labor hrs, 1000kg of material M1 and 1200kg of M2. The unit profit for A and B is Rs.27 and Rs.55 respectively. Product A and B are in fact new models and are replacements of older once which have been discontinued very recently. The manager would like to maximize profit but he is equally concerned with maintaining work force of the division at nearly constant level in the interest of employee morale. The work force which consists of people engaged in production, sales distribution, peons and other general staff consisted of 108 persons in all. From detailed study it is known that production of one unit of A would maintain 0.3 person in the work force while one unit of B would maintain 0.75 person.

Had the production manager been considering only maximizing profit without regard to maintaining the work force, he would do so by producing 167.67 units of A and 66.67 units of B. On the basis of available capacity this would yield a profit equal to 167.67X27 + 66.67X55 = 8193.94. However this would maintain 100.3 persons in work force. The manager feels that probably he could increase the work force requirement to desired level by accepting somewhat lower profit. In keeping with this the following goals are established:

A profit of Rs.5400 per week and a work force of 108 person. Formulate this as a goal programming problem. **10 Marks**

3) A firm produces three items A, B, C and requires two types of resources – man hours and raw material. The following LPP has been formulated to determine the optimum production schedule that maximizes the total profit:

Maximize $z = 3x_1 + x_2 + 5x_3$

Subject to $6x_1 + 3x_2 + 5x_3 \leq 45$ (man hours)

$3x_1 + 4x_2 + 5x_3 \leq 30$ (raw material)

$x_1, x_2, x_3 \geq 0$

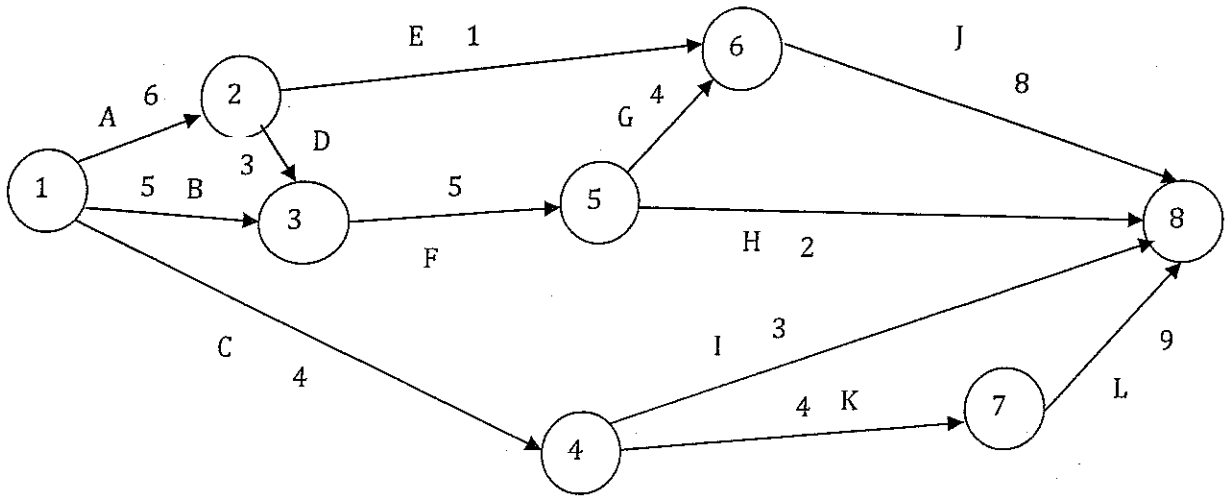
The associated optimal table for above primal is given below

Basic	x_1	x_2	x_3	s_1	s_2	Solution
z	0	3	0	0	1	30
x_1	1	–1/3	0	1/3	–1/3	5
x_3	0	1	1	–1/5	2/5	3

i) If the unit profit of product A is increased to 5, find the optimal solution. **[10]**

ii) In the above problem if the available raw material is increased to 50 units, find the optimal solution? **[10]**

4) Consider the following project network:



a) Find the critical path of this network by calculating earliest and latest occurrence times of all the events.

b) Find Total Floats and Free Floats of all the non-critical activities. **[20]**

BITS, PILANI – DUBAI CAMPUS
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(III Year – I Semester 2012-2013)
OPTIMISATION (AAOC C 222)
TEST– I (Closed Book)

Date: 04.11.12
Time: 50 Minutes

Max. Marks: 75
Weightage: 25%

Attempt all the questions.

1) For the following transportation problem

	Warehouse			
Plant	W1	W2	W3	***Supply***
A	5	8	4	10
B	7	2	8	5
C	3	7	9	10
Demand	6	8	9	

find initial basic feasible solution by using Vogel Approximation method and then use UV (method of multipliers) method to find optimal solution.

20 Marks

2) Solve the following assignment problem by using Hungarian method.

15 Marks

	I	II	III	IV	V
A	11	17	8	16	20
B	9	7	12	6	15
C	13	16	15	12	16
D	21	24	17	28	26
E	14	10	12	11	15

3) Solve the following integer programming problem by using Branch and Bound method **20 Marks**

Minimize $Z = 4x_1 + 3x_2$

Subject to

$5x_1 + 3x_2 \geq 30$

$x_1 \leq 4$

$x_2 \leq 6$

where x_1, x_2 are non negative integers

4)

(i) Solve the LPP by using big M-method: **8 Marks**

Maximize $z = x_1 + 3x_2$

Subject to $x_1 \leq 3$

$4x_1 + 3x_2 \geq 15$

$x_1, x_2 \geq 0$

(ii) Solve the LPP by using Two-phase method: **12 Marks**

Maximize $z = 3x_1 + 2x_2 + 3x_3$

Subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0$

INTERNATIONAL ACADEMIC CITY, DUBAI
(III Year – I Semester 2012-2013)
OPTIMISATION (AAOC C 222)
Quiz– II

Date: 29.11.12
Time: 20 Minutes

Max. Marks: 21
Weightage: 7%

Part A

Name : **Id No:**

Attempt all the questions.

1) Consider the following LPP: **6 Marks**

Maximize $Z = 4x_1 - 3x_2$

subject to $x_1 + x_2 \geq 4, \quad 4x_1 - x_2 \leq 20, \quad x_1 - 2x_2 = 10,$

x_1, x_2 are unrestricted.

a) How many variables are there in its dual _____
b) How many constraints are there in its dual _____
c) How many dual variables are unrestricted in sign _____
d) How many dual constraints are "=" type _____
e) Write down the dual objective function _____

2) Consider the LP problem

Maximize $Z = 35x_1 + 50x_2$ **10 Marks**

Subject to

$4x_1 + 6x_2 \leq 120$

$x_1 + x_2 \leq 20$

$2x_1 + 3x_2 \leq 40$

where $x_1, x_2 \geq 0$

The associated optimal table for above primal is given below

Basic Variable	x_1	x_2	s_1	s_2	s_3	Solution
Z-Row	---	---	---	---	---	---
s_1	0	0	1	0	-2	---
x_1	1	0	0	3	-1	---
x_2	0	1	0	-2	1	---

a) Without using simplex method, find Z-row elements
b) Without using simplex method, find the solution column.

3) Find the range of values for p and q which will render the entry (2, 2) a saddle point in the game with the following payoff matrix. **5 Marks**

$$\begin{pmatrix} 1 & q & 3 \\ p & 5 & 10 \\ 6 & 2 & 3 \end{pmatrix}$$

Value of p_____ Value of q_____

INTERNATIONAL ACADEMIC CITY, DUBAI
(III Year – I Semester 2012-2013)
OPTIMISATION (AAOC C 222)
Quiz– II

Date: 29.11.12 Max. Marks: 21
Time: 20 Minutes Weightage: 7%

Part B

Name : **Id No:**

Attempt all the questions.

1) Find the range of values for p and q which will render the entry (2, 2) a saddle point in the game with the following payoff matrix. **5 Marks**

$$\begin{pmatrix} 2 & 4 & 5 \\ 10 & 7 & q \\ 4 & p & 6 \end{pmatrix}$$

Value of p_____ Value of q_____

2) Consider the LP problem

Maximize $Z = 2x_1 - x_2 + x_3$ **10 Marks**

Subject to

$3x_1 + x_2 + x_3 \le 60$
$x_1 - x_2 + 2x_3 \le 10$
$x_1 + x_2 - x_3 \le 20$

where $x_1, x_2, x_3 \ge 0$

The associated optimal table for primal is given below

Basic Var.	x_1	x_2	x_3	s_1	s_2	s_3	Solution
Z-Row	---	---	---	---	---	---	----
s_1	0	0	1	1	-1	-2	---
x_1	1	0	1/2	0	1/2	1/2	---
x_2	0	1	-3/2	0	-1/2	1/2	---

a) Without using simplex method find Z-row elements.
b) Without using simplex method find the solution column.

3) Consider the following LPP: **6 Marks**

Minimize $Z = 3x_1 - 4x_2$

subject to $x_1 + x_2 \geq 4, \quad 4x_1 - x_2 \leq 20, \quad x_1 - 2x_2 = 10,$

x_1, x_2 are unrestricted.

a) How many constraints are there in its dual _____
b) How many dual constraints are "=" type _____
c) Write down the dual objective function _____
d) How many variables are there in its dual _____
e) How many dual variables are unrestricted in sign _____