

**BITS, PILANI – DUBAI CAMPUS**  
**INTERNATIONAL ACADEMIC CITY, DUBAI**  
**OPTIMISATION (AAOC C 222)**  
**Comprehensive Examination**  
(III Year – I Semester 2011-2012)

Date: 02.01.12  
Time: 03 Hours

Max. Marks: 120  
Weightage: 40%

*Attempt all the questions.*  
**Use Separate answer books for Part A, Part B and Part C**

**Part A**

1) For the following transportation problem

Plant	Warehouse				Supply
	W1	W2	W3	W4	
A	10	2	20	11	15
B	12	7	9	20	25
C	4	14	16	18	10
<b>Demand</b>	5	15	15	15	

find initial basic feasible solution by using North West corner method and then use UV method to find optimal solution. **12 Marks**

2) For the following LPP

$$\text{Maximize } Z = 3x_1 + 2x_2 + 5x_3$$

Subject to

$$x_1 + 2x_2 + x_3 + x_4 = 430$$

$$3x_1 + 2x_3 + x_5 = 460$$

$$x_1 + 4x_2 + x_6 = 420$$

where  $x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ .

The associated optimal table is given as

Basic Variable	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	Solution
Z-row	4	0	0	1	2	0	1350
$x_2$	-1/4	1	0	1/2	-1/4	0	100
$x_3$	3/2	0	1	0	1/2	0	230
$x_6$	2	0	0	-2	1	1	20

If a new constraint  $3x_1 + 3x_2 + x_3 \leq 500$  is added to above LPP, then find the new feasible and optimal solution. **10 Marks**

- 3) Solve the following nonlinear programming problem by using Kuhn-Tucker conditions  
**8 Marks**

$$\text{Minimize } f(x_1, x_2) = x_1^2 - x_1x_2 + 2x_2^2 - x_1 - x_2$$

subject to

$$2x_1 + x_2 \leq 2$$

where  $x_1, x_2 \geq 0$ .

- 4) A city parks department has been given a grant of €600million to expand its public recreational facilities. Four different types of facilities have been requested by the city council members: gymnasiums, athletic fields, tennis courts and swimming pools. Each facility costs a certain amount, requires a certain number of hectares and has an expected usage as shown in the following table:

Facility	Cost (€million)	Required hectares	Expected usage (people/week)
Gymnasium	80	4	1500
Athletic field	24	8	3000
Tennis court	15	3	500
Swimming pool	40	5	1000

The parks department has located a total of 50 hectares of land for construction (although more land could be located if necessary). The council has established the following list of goals:

- The parks department must spend the total grant (otherwise the amount not spent will be returned to the government).
- The facilities should be used by 20,000 or more people weekly.
- If more land is required, the additional amount should be limited to ten hectares.

Formulate the goal programming problem.

**10 Marks**

### Part B

5. Find the optimum solution using graphical method. [Graph sheet will not be provided]

$$\text{Max } z = 80x_1 + 120x_2$$

subject to

$$x_1 + x_2 \leq 9,$$

$$x_1 \geq 2, x_2 \geq 3,$$

$$20x_1 + 50x_2 \leq 360,$$

$$x_1 \geq 0, x_2 \geq 0$$

**6 Marks**

6. Develop branch and bound tree for the following problem. Use  $x_1$  as the branching variable at node 0.  
 Max  $z = x_1 + x_2$   
 Subject to  
 $2x_1 + 5x_2 \leq 16$   
 $6x_1 + 5x_2 \leq 27$   
 $x_1, x_2 \geq 0$  and integer

**10 Marks**

7. Solve the following LP by the revised simplex method:

Maximize  $z = 6x_1 - 2x_2$   
 subject to  
 $x_1 - x_2 \leq 2$   
 $-x_1 + x_2 \leq 4$   
 $x_1, x_2 \geq 0$

**10 Marks**

8. Max  $z = 5x_1 + 4x_2$   
 subject to  $6x_1 + 4x_2 \leq 24$   
 $x_1 + 2x_2 \leq 6$   
 $-x_1 + x_2 \leq 1$   
 $x_2 \leq 2$   
 $x_1, x_2 \geq 0$

The optimal solution of the above LPP is

	$X_1$	$X_2$	$S_1$	$S_2$	$S_3$	$S_4$	Solution
$z$	0	0	$3/4$	$1/2$	0	0	21
$X_1$	1	0	$1/4$	$-1/2$	0	0	3
$X_2$	0	1	$-1/8$	$3/4$	0	0	$3/2$
$S_3$	0	0	$3/8$	$-5/4$	1	0	$5/2$
$S_4$	0	0	$1/8$	$-3/4$	0	1	$1/2$

If a new economic activity  $x_3$  is introduced such that the new dual constraint is  $\frac{3}{4}y_1 + \frac{3}{4}y_2 + y_3 \geq 3.5$ . Determine the optimum solution using sensitivity analysis.

**14 Marks**

### Part C

9. Solve the following game between two players A and B, using graphical method

**6 Marks**

$$\begin{array}{c}
 \text{B} \\
 \left[ \begin{array}{cc} -6 & -2 \\ -1 & -7 \\ 2 & -9 \\ -7 & -1 \end{array} \right] \\
 \text{A}
 \end{array}$$

10. Solve the following dynamic problem

10 Marks

$$\text{Maximize } z = y_1 y_2 y_3$$

Subject to

$$y_1 + y_2 + y_3 = 10$$

$$\text{where } y_1, y_2, y_3 \geq 0$$

11. Given the flow of the activities of a project

12 Marks

Activities	Time	Time	Time
	Optimistic	Most Likely	Pessimistic
A:(1,2)	6	6	24
B:(1,3)	6	12	18
C:(1,4)	12	12	30
D:(2,5)	6	6	6
E:(3,5)	12	30	48
F:(4,6)	12	30	42
G:(5,6)	18	30	54

(i) Find the critical path and the project completion time.

(ii) Find the variance and the standard deviation of the project length.

12. Solve the following problem using two Phase method

12 Marks

$$\text{Minimize } z = 6x_1 + 4x_2 + 3x_3$$

Subject to the constraints

$$4x_1 + 5x_2 + 3x_3 \geq 40$$

$$2x_1 + x_2 + 6x_3 \geq 50$$

$$3x_1 + 4x_2 + 2x_3 \geq 60$$

$$\text{where } x_1, x_2, x_3 \geq 0$$

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**INTERNATIONAL ACADEMIC CITY, DUBAI**  
 (III Year – I Semester 2011-2012)  
**OPTIMISATION (AAOC C 222)**  
**TEST– II (Open Book)**

Date: 13.11.11  
 Time: 50 Minutes

Max. Marks: 60  
 Weightage: 20%

**Attempt all the questions.**

- 1) Solve the following integer programming problem by using Branch and Bound method **21 Marks**

*Maximize*  $Z = 8x_1 + 5x_2$

Subject to

$x_1 + x_2 \leq 6$

$9x_1 + 5x_2 \leq 45$

where  $x_1, x_2$  are non negative integers

- 2) A company has factories at  $A, B$  and  $C$  which supply warehouse at  $D, E, F, G$ . Monthly factory capacities are 7, 9, 18 respectively. Monthly warehouse requirements are 5, 8, 7, 14 units respectively. Units shipping costs (in Rs.) are as follows: **14 Marks**

	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>
<i>A</i>	19	30	50	10
<i>B</i>	70	30	40	60
<i>C</i>	40	8	70	20

By using Vogel Approximation method find initial basic feasible solution and then use UV method to find optimal solution.

- 3) A company has a team of four salesmen  $A, B, C$  and  $D$  and three districts  $X, Y$ , and  $Z$  where the company wants to start its business. After taking into account the capabilities of salesman and nature of districts, the company estimates that the profit per day in lakh Rs. for each salesman in each districts is as below:

	X	Y	Z
A	2	3	5
B	3	2	3
C	4	2	2
D	5	1	2

Find the assignment of salesmen and profit to various districts which yield maximum profit.

**10 Marks**

4) Consider the following LP model:

$$\text{Maximize } Z = 3x_1 + 2x_2 + 5x_3$$

Subject to

$$x_1 + 2x_2 + x_3 + x_4 = 30$$

$$3x_1 + 2x_3 + x_5 = 60$$

$$x_1 + 4x_2 + x_6 = 20$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

$$\text{Basic variables} = (x_2, x_3, x_1) \quad \text{Inverse} = \begin{pmatrix} 1/4 & -1/8 & 1/8 \\ 3/2 & -1/4 & -3/4 \\ -1 & 1/2 & 1/2 \end{pmatrix}$$

(i) Write the dual form.

(ii) Check the optimality and feasibility of given basic solution.

**15 Marks**

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OPTIMISATION (AAOC C 222)  
TEST– I (Closed Book)

Date: 29.09.11  
Time: 50 Minutes

Max. Marks: 75  
Weightage: 25%

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**Attempt all the questions.**

- 1) John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores. In store 1, he can work between 5 and 12 hours a week. In store 2, he is allowed between 6 and 10 hours. Both stores pay the same hourly wage. In deciding how many hours to work in each store, John wants to base his decision on work stress. Based on interviews with present employees, John estimates that, on an ascending scale of 1 to 10, the stress factors are 8 and 6 at stores 1 and 2, respectively. Because stress mounts by the hour, he assumes that the total stress for each store at the end of the week is proportional to the number of hours he works in the store. How many hours should John work in each store. Formulate it as a LPP. **10 Marks**

- 2) Solve the following LP by using graphical method

$$\text{Minimize } Z = 20x + 10y$$

**15 Marks**

Subject to

$$x + 2y \leq 40$$

$$3x + y \geq 30$$

$$4x + 3y \geq 60$$

$$\text{where } x \geq 0, y \geq 0$$

- 3) Use Big M method to prove that the following problem has infeasible solution

$$\text{Maximize } Z = 3x_1 + 2x_2$$

**25 Marks**

Subject to

$$2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$\text{where } x_1, x_2 \geq 0$$

4) Solve the following LPP by simplex method

25 Marks

$$\text{Maximize } Z = 4x_1 + x_2$$

Subject to

$$3x_1 + x_2 \leq 4$$

$$2x_1 - x_2 \leq 2$$

where  $x_1, x_2, \geq 0$



BITS PILANI – DUBAI CAMPUS  
Dubai International Academic City

III Year – I Semester

QUIZ-II (CB)

Course: OPTIMIZATION

Course No. AAOC C222

Max marks: 21  
Minutes

Weightage: 7%

Date: 21-12-2011

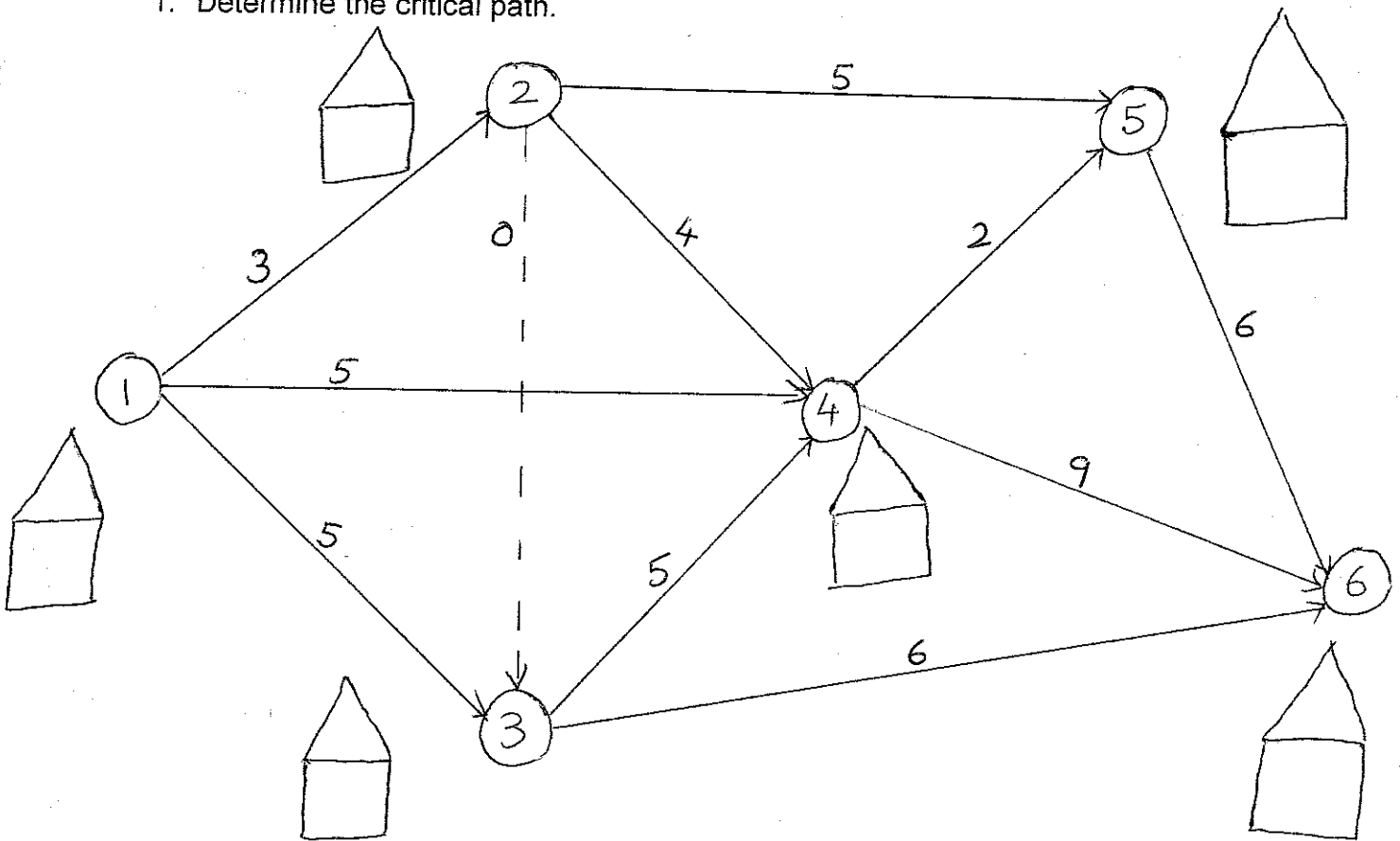
Time: 20

Name:

Id No.:

Sec:

1. Determine the critical path.



Ans. \_\_\_\_\_

2. For a minimization problem, use dual simplex to restore feasibility.

Basic	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	Solution
Z	0	-4/3	0	0	-4/3	8/3
$X_3$	0	4/3	1	0	1/3	1/3
$X_4$	0	-4/3	0	1	-1/3	-1/3
$X_1$	1	-1/3	0	0	-1/3	2/3

3. The following game has the saddle point at (A3,B2). Find p, q and value of game

	B1	B2	B3	B4
A1	-2	-1	0	5
A2	-3	-5	0	2
A3	p	5	6	8
A4	2	q	-8	2

Ans : \_\_\_\_\_

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III Year – I Semester

QUIZ-II (CB)

Course: OPTIMIZATION

Course No. AAOC C222

Max marks: 21  
Minutes

Weightage: 7%

Date: 21-12-2011

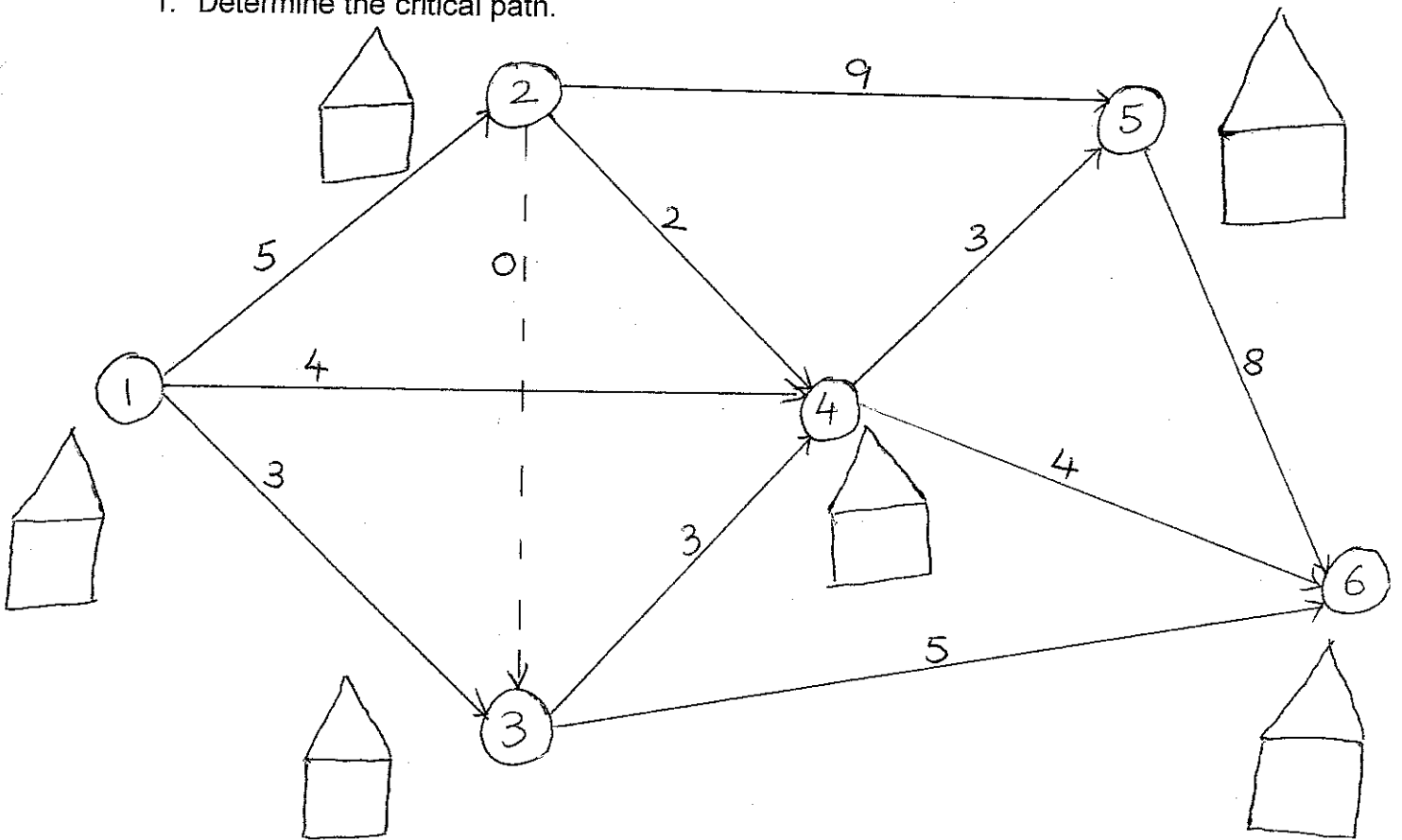
Time: 20

Name:

Id No.:

Sec:

1. Determine the critical path.



Ans. \_\_\_\_\_

2. For a minimization problem, use dual simplex to restore feasibility.

Basic	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	Solution
Z	0	-2	-1	0	0	3
$X_1$	1	1/2	-1/2	0	0	3/2
$X_4$	0	1/2	1/2	1	0	1/2
$X_5$	0	-1/2	-1/2	0	1	-1/2

3. The following game has the saddle point at (A2,B1). Find p, q and value of game

	B1	B2	B3	B4
A1	0	0	-2	5
A2	1	2	3	q
A3	p	-4	2	0
A4	1	3	-2	4
A5	0	1	-1	2

Ans : \_\_\_\_\_

**BITS, PILANI – DUBAI CAMPUS**  
**INTERNATIONAL ACADEMIC CITY, DUBAI**  
**(III Year – I Semester 2011-2012)**  
**OPTIMISATION (AAOC C 222)**  
**Quiz– I**

Date: 12.10.11  
Time: 20 Minutes

Max. Marks: 24  
Weightage: 8%

**Part A**

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**Attempt all the questions.**

- 1) Convert the following LP problem into *Phase I* form and indicate the starting basic feasible variables

$$\text{Maximize } Z = x_1 + 2x_2 - x_3$$

**4 Marks**

Subject to

$$x_1 - 2x_2 + 2x_3 \geq -6$$

$$2x_1 + x_2 \geq 4$$

$$-x_1 + 2x_2 - x_4 = -1$$

where  $x_1, x_2, x_3, x_4 \geq 0$

2) For the following LP problem

$$\text{Maximize } Z = 4x_1 + 2x_2$$

10 Marks

Subject to

$$x_1 + x_2 \leq 8$$

$$2x_1 + x_2 \leq 10$$

where  $x_1, x_2 \geq 0$ .

Starting Simplex table is given below

Basic Variable	$x_1$	$x_2$	$s_1$	$s_2$	Solution
Z-Row	-4	-2	0	0	0
$s_1$	1	1	1	0	8
$s_2$	2	1	0	1	10

Classify the nature of the solution and find the all possible optimal solutions.

3) Solve the following *integer programming* problem by using Branch and Bound method

$$\text{Maximize } Z = 2x_1 + x_2$$

**10 Marks**

Subject to

$$10x_1 + 10x_2 \leq 9$$

$$10x_1 + 5x_2 \geq 1$$

where  $x_1, x_2$  are non negative integers.

**BITS, PILANI – DUBAI CAMPUS**  
**INTERNATIONAL ACADEMIC CITY, DUBAI**  
**(III Year – I Semester 2011-2012)**  
**OPTIMISATION (AAOC C 222)**  
**Quiz– I**

Date: 12.10.11  
Time: 20 Minutes

Max. Marks: 24  
Weightage: 8%

**Part B**

NAME :

ID NO. :

**Attempt all the questions.**

- 1) Convert the following LP problem into *Phase I* form and indicate the starting basic feasible variables

$$\text{Maximize } Z = 2x_1 + 3x_2$$

**4 Marks**

Subject to

$$x_1 - 2x_2 + x_3 \geq 6$$

$$2x_1 + x_2 \geq 4$$

$$x_1 - 2x_2 + x_4 = -1$$

where  $x_1, x_2, x_3, x_4 \geq 0$



2) Solve the following *integer programming* problem by using Branch and Bound method

$$\text{Maximize } Z = 2x_1 + x_2$$

**10 Marks**

Subject to

$$10x_1 + 10x_2 \leq 9$$

$$10x_1 + 5x_2 \geq 1$$

where  $x_1, x_2$  are non negative integers.

3) For the following LP problem

$$\text{Maximize } Z = 2x_1 + 4x_2$$

10 Marks

Subject to

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

where  $x_1, x_2 \geq 0$ .

Starting Simplex table is given below

Basic Variable	$x_1$	$x_2$	$s_1$	$s_2$	Solution
Z-Row	-2	-4	0	0	0
$s_1$	1	2	1	0	5
$s_2$	1	1	0	1	4

Classify the nature of the solution and find the all possible optimal solutions.