BITS, Pilani-Dubai
Dubai International Academic City, Dubai

Comprehensive Examination – III year I Semester 2009-2010

Date: 22.12.09 **Total Marks: 120** **Weightage: 40%**
Course: Optimization **Course No. AAOC C222**

Answer all questions
Use separate answer books for Part – A, Part – B and Part – C

Part – A

1. Use Graphical method to solve

$$\text{Maximize } z = 6x_1 + 4x_2$$
$$\text{Subject to } \quad -2x_1 + x_2 \le 2$$
$$x_1 - x_2 \le 2$$
$$3x_1 + 2x_2 \le 9$$
$$x_1, x_2 \ge 0$$

[10]

2. The following table gives the costs of transportation of goods from sources A,B,C to destinations P,Q,R,S. Using Vogel's method find the initial solution. Use u-v method to find the optimal solution. [10]

	P	Q	R	S	Supply
A	21	16	25	13	11
B	17	18	14	23	13
C	32	17	18	41	19
Demand	6	10	12	15	

3. Use the Branch and Bound technique to solve the following problem

$$\text{Maximize } z = 3x_1 + 2x_2$$
$$\text{Subject to } \quad 2x_1 + 2x_2 \le 7$$
$$x_1 \le 2$$
$$x_2 \le 2$$
$$x_1, x_2 \ge 0 \text{ and integers}$$

[10]

4. Solve the following LPP by Simplex method and find an alternate solution if exists.

$$\text{Maximize } z = 2x_1 + 4x_2$$
$$\text{Subject to } \quad x_1 + 2x_2 \le 5$$
$$x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0$$

[10]

Part – B

5. Solve the following problem using Big-M method:

$$\text{Maximize } z = 2x_1 + 3x_2 - 5x_3$$
$$\text{Subject to } x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

[10]

6. Solve the following 2 x 4 game by the graphical method:

A \ B	i	ii	iii	iv
i	3	3	4	0
ii	5	4	3	6

[10]

7. Solve the following using dynamic programming:

$$\text{Minimize } F = y_1^2 + y_2^2 + y_3^2$$
Subject to
$$y_1 + y_2 + y_3 \geq 15$$
$$y_1, y_2, y_3 \geq 0$$

[10]

8. Solve the following LP by the revised simplex method:

$$\text{Maximize } z = 6x_1 - 2x_2$$
$$\text{Subject to } x_1 - x_2 \leq 2$$
$$-x_1 + x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

[10]

Part – C

9. A company produces a toy carriage, whose final assembly must include four wheels and two seats. The factory producing the parts operates three shifts a day. The following table provides the amounts produced of each part in three shifts.

	Units produced per run	
Shift	Wheels	Seats
1	500	300
2	600	280
3	640	360

Ideally, the number of wheels produced is exactly twice that of the number of seats. However, because production rates vary from shift to shift, exact balance in production may not be possible. The company is interested in determining the number of production runs in each shift that minimizes the imbalance in the production of the parts. The capacity limitations restrict the number of runs to between 4 and 5 for shift 1, 10 and 20 for shift 2, and 3 and 5 for shift 3. Formulate the problem as a goal programming model. **[8]**

10. Write the Khun-Tucker conditions and using that solve the following non-linear programming problem.

Maximize $z = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$

Subject to $2x_1 + x_2 \leq 10$

$x_1, x_2 \geq 0$ **[8]**

11. The following table gives the activities for buying a new car.

Activity	Predecessor(s)	Duration
A: Conduct feasibility study	--	3
B: Find potential buyer for present car	A	14
C: List possible models	A	1
D: Research all possible models	C	3
E: Conduct interview with mechanic	C	1
F: Collect dealer propaganda	C	2
G: Compile pertinent data	D, E, F	1
H: Choose top three models	G	1
I: Test drive all three choices	H	3
J: Gather warranty and financing data	H	2
K: Choose one car	I, J	2
L; Choose dealer	K	2
M: Search for desired color and options	L	4
N: Test drive chosen model once again	L	1
O: Purchase new car	B, M, N	3

Construct the project network; find the critical path, total duration and total floats and free floats for the non critical activities. **[12]**

12. Consider the following LPP:

Maximize $z = 5x_1 + 4x_2$

Subject to $6x_1 + 4x_2 \leq 24$ (Raw material 1)

$x_1 + 2x_2 \leq 6$ (Raw material 2)

$-x_1 + x_2 \leq 1$ (Market limit)

$x_2 \leq 2$ (Demand limit)

$x_1, x_2 \geq 0$

The optimal table of the above problem is given below:

Basic	x_1	x_2	s_1	s_2	s_3	s_4	Solution
z	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21
x_1	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	3
x_2	0	1	$-\frac{1}{8}$	$\frac{3}{4}$	0	0	$\frac{3}{2}$
s_3	0	0	$\frac{3}{8}$	$-\frac{5}{4}$	1	0	$\frac{5}{2}$
s_4	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1	$\frac{1}{2}$

(i) Suppose the maximum daily availability of raw materials 1 and 2 are 28 and 8 tons respectively, find the new optimal solution using sensitivity analysis. **[8]**

(ii) Investigate the optimality for the new objective function $z = 3x_1 + 2x_2$ **[4]**

BITS, PILANI – DUBAI
Dubai International Academic City, Dubai, UAE

I Semester 2009 – 2010
Test-II (Open book)

3rd year – All Discipline | Course: Optimization (AAOC C222)
15th November 2009 | Marks: 60 | Weightage 20%

Answer all Questions.

1. Solve the following 2 x 5 game by graphical method:

$$\begin{array}{c} \\ A1 \\ A2 \end{array}\begin{array}{c} \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \begin{pmatrix} -5 & 5 & 0 & -1 & 8 \\ 8 & -4 & -1 & 6 & -5 \end{pmatrix} \end{array}$$
[8]

2. Solve the following integer programming problem by branch and bound method:

$$\text{Maximize } z = x_1 + 4x_2$$
$$\text{Subject to } 2x_1 + 4x_2 \le 7$$
$$5x_1 + 3x_2 \le 15$$
$$x_1, x_2 \ge 0 \text{ and integers}$$
[12]

3. A company produces two products. Relevant information for each product is shown in the table below:

	Product 1	Product 2
Labor required	5 hours	3 hours
Profit	AED 6	AED 3

The company has a goal of AED 68 in profits and incurs an AED 1 penalty for each AED it falls short of this goal. A total of 35 hours of labor are available. An AED 2 penalty is incurred for each hour of overtime (labor over 35 hours) used, and an AED 1 penalty is incurred for each hour of available labor that is unused. Marketing considerations require that at least 9 units of product 1 be produced and at least 12 units of product 2 be produced. For each unit of either product by which production falls short of demand, a penalty of AED 5 is assessed. Formulate an LP that can be used to minimize the total penalty incurred by the company. **[12]**

4. Consider the LPP:

$$\text{Minimize } z = -x_1 + 2x_2 - x_3$$
$$\text{Subject to } 3x_1 + x_2 - x_3 \le 10$$
$$x_2 + x_3 \le 4$$
$$-x_1 + 4x_2 + x_3 \ge 6$$
$$x_1, x_2, x_3 \ge 0$$

The optimal table is as follows:

Basic	x_1	x_2	x_3	s_1	s_2	s_3	a_1	Solution
Z	-1	0	-3	0	-2	0	$1-M$	8
s_1	3	0	-2	1	-1	0	0	6
s_3	1	0	3	0	4	1	-1	10
x_2	0	1	1	0	1	0	0	4

(i) Discuss the effect of changing the RHS value to $\begin{pmatrix} 3 \\ 10 \\ 10 \end{pmatrix}$ **[10]**

(ii) Suppose the objective function is changed to Minimize $z = 2x_1 - x_2 + 2x_3$ what will be the new optimal solution. **[6]**

(iii) Suppose a new activity x_4 is added with the corresponding column values $(1 \quad 1 \quad 2)^T$ with objective function coefficient as 1 what will be the new optimal solution. **[12]**

BITS, PILANI – DUBAI
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI
III – Year – Semester – I (2009-10)
OPTIMIZATION (AAOC C222)
TEST – 1 (Closed Book)

Time: 50 Minutes | Max. Marks 75
Date: September 27, 2009 | Weightage: 25%

Answer all questions

1. A company has two products Sofa and Chair. To produce one unit of Sofa, 2 units of material X and 4 units of material Y are required. To produce one unit of Chair, 3 units of material X and 2 units of material Y are required. As the raw material is in short supply not more than 16 units of each material can be used. The cost per unit of material X and material Y are 2.50 and 0.25 respectively. At least 2 units of Sofa must be produced and sold. Formulate it as a linear programming problem to minimize the cost. **[10]**

2. Solve the following LPP by graphical method:

   Maximize $z = 2x_1 + x_2$
   Subject to $x_1 + 2x_2 \le 10$
   $x_1 + x_2 \le 6$
   $x_1 - x_2 \le 2$
   $x_1 - 2x_2 \le 1$
   $x_1, x_2 \ge 0$ **[15]**

3. Solve the following LPP by Simplex method:

   Maximize $z = 200x_1 + 400x_2$
   Subject to $\frac{1}{40}x_1 + \frac{1}{60}x_2 \le 1$
   $\frac{1}{50}x_1 + \frac{1}{50}x_2 \le 1$
   $x_1, x_2 \ge 0$ **[15]**

4. Solve the following LPP by Simplex method:

   Minimize $z = 8x_1 - 2x_2$
   Subject to $-4x_1 + 2x_2 \le 1$
   $5x_1 - 4x_2 \le 3$
   $x_1, x_2 \ge 0$ **[10]**

5. Solve the following LPP by Big M method:

   Maximize $z = 10x_1 + 12x_2$
   Subject to $x_1 + x_2 = 5$
   $x_1 \ge 2$
   $x_2 \le 4$
   $x_1, x_2 \ge 0$ **[25]**

BITS, Pilani-Dubai, DIAC, Dubai
III YEAR I SEMESTER 2009-2010

QUIZ – 1 (Closed Book)

Course Title: Optimization **Course No: AAOC C222**
Date: 08.10.2009 **Max marks: 24**
Time: 20 min **Weightage: 8%**

Name of the Student: ______________________

ID No:______________________ **Section:**

Answer ALL Questions

1. Consider the following LPP:

$$\text{Maximize } z = 3x_1 + 2x_2$$
$$\text{Subject to } x_1 + x_2 \geq 2$$
$$x_1 + 3x_2 \leq 3$$
$$x_1 - x_2 = 1$$
$$x_1, x_2 \geq 0$$

The optimal table of Phase I is given below:

Basic	x_1	x_2	s_1	a_1	s_2	a_2	Solution
Z	0	0	0		0		0
x_2	0	1	$-\frac{1}{2}$		0		$\frac{1}{2}$
s_2	0	0	2		1		0
x_1	1	0	$\frac{1}{2}$		0		$\frac{3}{2}$

Form the optimal table of Phase II and find the optimal solution in the space provided below: **[4]**

Basic	x_1	x_2	s_1	s_2	Solution
Z					

2. Find an alternative optimal solution from the following optimum table in the space provided below the optimal table: **[12]**

Basic	x_1	x_2	x_3	x_4	Solution
z	0	0	0	0	15
x_2	0	1	0	$\frac{1}{6}$	$\frac{5}{2}$
x_3	0	0	1	$-\frac{1}{2}$	$\frac{5}{2}$
x_1	1	0	0	$\frac{7}{6}$	$\frac{5}{2}$

Basic	x_1	x_2	x_3	x_4	Solution
Z					

3. (i) Put a tick to the correct answer(s) related to Degeneracy in the Simplex Method. [2]

   (a) Degeneracy does not give rise to cycling
   (b) When minimum ratio for more than one variable is same it gives rise to Degeneracy
   (c) Degeneracy is always harmful
   (d) Practically Degeneracy means one or more redundant constraints are there in the mathematical model

(ii) Put a tick to the correct answer(s) related to Unbounded Solution in the Simplex Method. [2]

   (a) A model will have unbounded solution if the value of one or more variables can be increased or decreased indefinitely
   (b) We can have a mathematical model which has an unbounded region but bounded solution.
   (c) By inspecting a simplex table we cannot say the problem has unbounded solution.
   (d) Practically in such a mathematical model one or more constraints are redundant

(iii) Put a tick to the correct answer related to Alternate Solutions in the Simplex Method. [2]

   (a) A model will have Alternate Solutions when objective function assumes same value at more than one solution point
   (b) In case of Alternate Solutions the number of solutions is always finite
   (c) By inspecting a simplex table we cannot say the problem has Alternate Solutions
   (d) Practically in such a mathematical model one or more constraints are redundant

(iv) Put a tick to the correct answer related to Infeasible Solution in the Simplex Method. [2]

   (a) A model will have Infeasible Solution if all the constraints in the model are of less than equal to type.
   (b) We get Infeasible Solution if one or more artificial variables will be present in the basis with positive value even though optimality is reached.
   (c) By inspecting a simplex table we cannot say the problem has Infeasible Solution.
   (d) Practically in such a mathematical model one or more constraints are redundant.

BITS, Pilani-Dubai, DIAC, Dubai
III YEAR I SEMESTER 2009-2010

QUIZ – 2 (Closed Book)

Course Title: Optimization
Date: 21.10.2009
Time: 25 min

Course No: AAOC C222
Max marks: 21
Weightage: 7%

Name of the Student: ____________________

ID No: ____________________ Section:

Answer ALL Questions

1. A transportation corporation has three vehicles in three cities each of vehicles can be assigned to any of the four other cities. The distance differs from one city to another as given below:

	W	X	Y	Z
A	33	40	43	32
B	45	28	31	23
C	42	29	36	29

Assign a vehicle to a city in such a way that the total distance traveled is minimized and indicate the minimum distance. [7]

2. Find the basic feasible solution of the following transportation problem by Vogel's approximation method: [7]

	I	J	K	L	M	Supply
A	2	11	10	3	7	4
B	1	4	7	2	1	8
C	3	9	4	8	12	9
Demand	3	3	4	5	6	