BITS, Pilani – Dubai, Dubai International Academic City

III Year First Semester 2007-2008

Degree: B.E.(Hons.). Branch: C.S.

Comprehensive Examination Question Paper

Course No : CS UC351 Course Title: Theory Of Computation

Date: 06/01/08 Sunday Time: 10 a.m.- 1 Noon Total marks: 60

Data provided are complete. ***Closed Book.***

This question paper has 3 pages.

Answer **all** Questions.

1. Write down the following context free grammar [CFG] in CHOMSKY NORMAL FORM:

$E \rightarrow E+T \mid T*F \mid (E) \mid \mathbf{a} \mid \mathbf{b} \mid I\mathbf{a} \mid I\mathbf{b} \mid I\mathbf{0} \mid I\mathbf{1}$

$T \rightarrow T*F \mid (E) \mid \mathbf{a} \mid \mathbf{b} \mid I\mathbf{a} \mid I\mathbf{b} \mid I\mathbf{0} \mid I\mathbf{1}$

$F \rightarrow (E) \mid \mathbf{a} \mid \mathbf{b} \mid I\mathbf{a} \mid I\mathbf{b} \mid I\mathbf{0} \mid I\mathbf{1}$

$I \rightarrow \mathbf{a} \mid \mathbf{b} \mid I\mathbf{a} \mid I\mathbf{b} \mid I\mathbf{0} \mid I\mathbf{1}$

[5 marks]

2. It is required to construct a TURING MACHINE to compute the function **f(w) = ww** [i.e. concatenation] for an input string. **TRACE** through successive steps [at each step, show the contents of the tape] of the above TURING MACHINE in action, for the following initial configuration on the tape:

▶	#	t	a	r	#		
					↑		

(note: here # denotes blank symbol) [5 marks]

3. Convert the following NFA (as defined by Transition Table given below) to DFA:

input / state	0	1
→p	{ p, s }	{ p , q }
q	Φ	{ r }
r	{ r }	{ t , r }
s	{ t }	Φ
*t	{t}	{t}

[Note: Φ : empty set, → : start state, *: end state] [5 marks]

4. Prove that the CIRCUIT-SATISFIABILITY problem is NP-COMPLETE. [5 marks]

P.T.O.

5. Draw a DFA (deterministic finite automata) to accept the following sets of strings over the alphabet **{ 0,1 }** :

"All strings containing exactly 4 "0"s and at least 2 "1"s". [5 marks]

6.Design a TURING MACHINE to perform the following:
Read an input string in lower case alphabets (i.e. a-z) and covert it into its corresponding string comprising of uppercase characters (i.e. A-Z).
Example: INPUT OUTPUT

►#sample ►#SAMPLE

[5 marks]

7. Consider the following context free grammar (CFG) :

$S \rightarrow A1B$

$A \rightarrow 0A \mid \epsilon$

$B \rightarrow 0B \mid 1B \mid \epsilon$

You are given an input string as follows:
0010101
For the above input string,

a) Find the **leftmost derivation** and its corresponding **parse tree.**
b) Find the **rightmost derivation** and its corresponding **parse tree.** (5 marks)

8. Consider the following PUSH-DOWN AUTOMATA

$M = (K , \Sigma , \Gamma, \Delta , s , F)$ where $K = \{ s , f \}$, (set of states)
$F = \{ f \}$, (final state)
$\Sigma = \{ a , b \}$, (input symbols)
$\Gamma = \{ a \}$, (stack symbol)
initial state = { s }

Δ (transition relation) is given by the following five transitions :

(1) ((s,a,e) , (s,a))
(2) ((s,b,e) , (s,a))
(3) ((s,a,e) , (f,e))
(4) ((f,a,a) , (f,e))
(5) ((f,b,a) , (f,e))

Now, you are required to tabulate the sequence of steps in a table (as shown below) and see whether the following input strings are acceptable by the above PDA:
i) **aba** ii) **bab**

State	Unread Input	Stack	Transition used	Comments
.....	.....	.....	....	.....
.....	.....	.....	....	.....

(5 marks)

P.T.O.

9. Consider the following language : $\{ 0^n 1^n \mid n >= 1 \}$.
Apply PUMPING LEMMA to determine whether the above language is Regular.
[3 marks]

10. Draw a NFA to recognize the following set of strings : **abc**, **abd** and **aacd**.
Assume that the alphabet is {a, b, c, d}. [3 marks]

11. Use PROOF by INDUCTION to show that

$(7^n - 1)$ is divisible by 6 for n = 1,2,3, ...

[3 marks]

12. Represent the following sets by Regular Expressions:

i) $\{ a^2, a^5, a^8, a^{11}, \ldots\ldots \}$

[note : (a power n) denotes no of occurrences of a]

ii) { 1, 111, 11111, 1111111 , ... }

[2 marks]

13. Consider the following two regular expressions:

(a Ụ b)* and **a* (ba*)***

Are these two regular expressions equivalent ? Justify your reasoning. [2 marks]

14. State whether the following problems are DECIDABLE or UNDECIDABLE:

a) Given a Turing Machine M, does M have five states ?
b) Given a Turing Machine M, is there an INPUT string **w** such that M makes at least 5 moves ?
c) Given a Turing Machine M and an input string **w**, does M halt on input **w** ?
d) Given two Turing Machines M1 and M2, do they halt on the same input strings ?

[2 marks]

15. Illustrate Equivalence Relation with an example. [2 marks]

16. How does a Deterministic Turing Machine (M1) simulate Non-Deterministic Turing Machine (M) ? [2 marks]

17. Give a formal definition for an APPROXIMATION ALGORITHM. [1 mark]

BITS, Pilani – Dubai, Academic City, Dubai.

III Year FIRST SEMESTER 2007-2008

Degree: B.E. (Hons.) Branch: C.S.

TEST II Question Paper

Course No : CS UC351 Course Title: Theory Of Computation

Date: 6/12/07 Thursday Time: 50 minutes Total marks: 20

Data provided are complete. **OPEN BOOK**

This question paper has 2 pages.

Answer all Questions.

1. Write down the following context free grammar [CFG] in CHOMSKY NORMAL FORM:

   $S \rightarrow$ **a***A***b** | **a***B***b**
   $A \rightarrow C$ | **b**
   $B \rightarrow C$ | A
   $C \rightarrow CDE$ | **ϵ** (ϵ denotes null value)
   $D \rightarrow A$ | B | **ab**

   [5 marks]

2. It is required to construct a TURING MACHINE to perform COPY operation for an input string. **TRACE** through successive steps [at each step, show the contents of the tape] of the above TURING MACHINE in action, for the following initial configuration on the tape:

▶	#	u	n	i	X	#	#	#
						↑		

(note: here # denotes blank symbol) [5 marks]

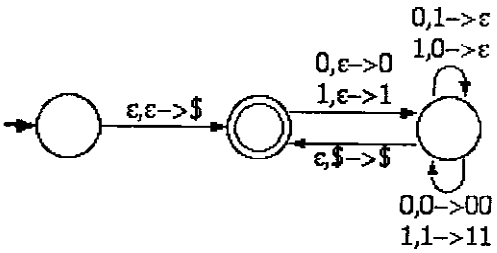
3. Distinguish between RECURSIVE and RECURSIVELY ENUMERABLE w.r.t. Turing Machines. Give an example in each category. [2 marks]

4. Design a TURING MACHINE to perform the following:
   READ an INPUT Binary String (alphabet is over 0, 1) and compute its 1's complement.
Example: INPUT : ▶#00010# OUTPUT: ▶#11101#

(note: here # denotes blank symbol) [3 amrks]

P.T.O.

5. Consider the following PDA shown below:



Which of the following input strings will be recognized / not recognized by the above PDA ?

a) 00011101
b) 010101
c) 100
d) 0001

[2 marks]

6. Consider the PDA M (given below) to accept the Language:

$L = \{ wcw^R : w \in \{a,b\}^* \}$

Let M be specified as follows:
Finite Set of states = {s,f} Input Symbols={a,b,c} Stack Symbols={a,b}
Initial state= {s} Final State={f} and the
Transition Relation is shown by the following five transitions:

(1) ((s,a,e), (s,a))
(2) ((s,b,e), (s,b))
(3) ((s,c,e), (f,e))
(4) ((f,a,a), (f,e))
(5) ((f,b,b), (f,e))

Now you are required to tabulate the sequence of steps in a table (as shown below) And see whether the following input string is acceptable by the above PDA:

baacaab

State	Unread Input	Stack	Transition Used
.....			

[3 marks]

BITS, Pilani – Dubai, Academic City, Dubai.

III Year First SEMESTER 2007-2008

Degree: B.E. (Hons.) Branch: C.S.

TEST I Answering / Marking Scheme

Course No : CSUC362 Course Title: Theory Of Computation

Date: 21st October 2007 Sunday Time: 50 minutes Total marks: 25

Data provided are complete. **Closed *Book*.**

Answer all Questions.

1. Write regular expressions for the following character sets [lanuages] whose alphabet is $\sum = \{0, 1\}$:
   1) All strings. (0|1)*
   2) All strings beginning with a **1**. 1(0|1)*
   3) All strings beginning with a **1** and ending with a **0**. 1(0|1)*0
   4) All strings that contain exactly *three* **1**'s. 0*10*10*10*

   [4 x 0.5 = 2 marks]

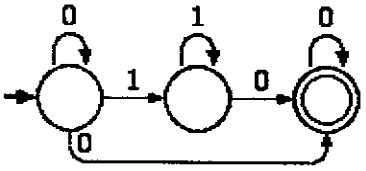
2. Correct NFA with States and Transitions [4*0.5=2 marks]

- 3. To show that language A is not regular
  - Approach: Proof by contradiction (0.5+1+1.5 marks]
    - Assume *A* is regular in order to obtain a contradiction
    - Use the pumping lemma to guarantee the existence of pumping length *p*
    - Find a string *s* of length p or more that cannot be pumped
      - Demonstrate that *s* cannot be pumped by considering all possible ways of dividing *s* into *x*, *y*, and *z*, and for each division find an *i* where *xyiz does not belong to A*

4. draw NFA, Table(Current state, input, New State), draw DFA

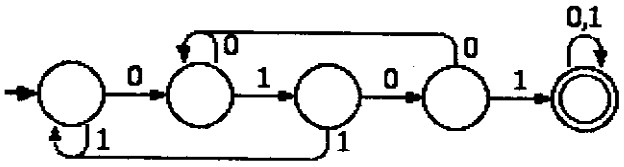
   0.5+3+1.5 mark

5. The language 0*1*0*0 (3 correct states and correct edges between them)

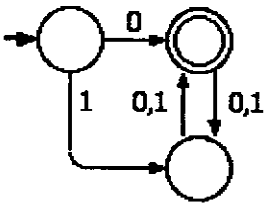


[States : 1.5 marks Edges: 2.5 marks]

6. All strings that contain the substring 0101.



All strings that start with 0 and has odd length or start with 1 and has even length.



[3+2 marks]

7. Give a formal proof for the following Theorem on Finite Automata:

"The Class of Regular Languages is closed under the **Concatenation** operation". Sequence of steps for each parameter in NFA definition and conclusion 6 * 0.5 = 3 marks

8. Regular Definition: Subsequent names defined using previous names in regular RE. Correct example : 1 mark. [like Real Number with mantissa and exponent]