BITS, Pilani-Dubai
Dubai International Academic City, Dubai

Comprehensive Examination – III year 2007-2008

Date: 05.01.08 **Total Marks: 40** **Weightage: 40%**
Course: Optimization **Course No. AAOC UC222**

Answer all questions
Use separate answer books for Part – A, Part – B and Part – C

Part – A

1. Solve the following LPP by Graphical method.

Maximize $z = 2x_1 + x_2$
Subject to

$$x_1 - x_2 \le 10$$
$$2x_1 \le 40$$
$$x_1, x_2 \ge 0$$

[2]

2. Solve the following game graphically and find the value of the game. The payoff is for player *A*.

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{pmatrix} 8 & 4 & -2 \\ -2 & -1 & 3 \end{pmatrix} \end{array}$$

[2]

3. Solve the dynamic programming problem.

Min $z = y_1^2 + y_2^2 + y_3^2$
Subject to

$$y_1 + y_2 + y_3 = 10$$
$$y_1, y_2, y_3 \ge 0$$

[3]

4. Solve the following LPP by revised simplex method.

Maximize $z = -x_1 + 3x_2 - 2x_3$
Subject to

$$3x_1 - x_2 + 2x_3 \le 7$$
$$-2x_1 + 4x_2 \le 12$$
$$-4x_1 + 3x_2 + 8x_3 \le 10$$
$$x_1, x_2, x_3 \ge 0$$

[3]

Part – B

5. Solve the following assignment problem.

Operators	Jobs				
	A	B	C	D	E
1	6	2	5	3	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

(a) Specify the minimum time required for the completion of the jobs.
(b) Also specify the allocation of jobs to the operators. **[3]**

6. For the following transportation problem:

Factory	W_1	W_2	W_3	W_4	Capacity
F_1	21	16	25	13	11
F_2	17	18	14	23	13
F_3	32	27	18	41	19
Requirement	6	10	12	15	

(a) Use the Vogel's approximation method to find the starting solution.
(b) Find the optimum solution. **[4]**

7. Solve using two-phase method.

Maximize $z = x_1 + 2x_2 + x_3$

Subject to

$$x_1 + x_2 + x_3 = 7$$
$$2x_1 - 5x_2 + x_3 \geq 10$$
$$x_1, x_2, x_3 \geq 0$$

[4]

8. Solve the following integer programming problem.

Maximize $z = 2x_1 + 3x_2$

Subject to

$$5x_1 + 7x_2 \leq 35$$
$$4x_1 + 9x_2 \leq 36$$
$$x_1, x_2 \geq 0 \text{ and integers.}$$

[4]

Part – C

9. A computer company is ready to make its annual purchase of computer chips. The company can purchase chips (in lots of 100) from three suppliers. Each chip is rated as being of excellent, good or mediocre quality. During the coming year the company will need 5000 excellent chips, 3000 good chips and 1000 mediocre chips. The characteristics of the chips purchased from each supplier are shown below:

	Characteristics of a lot of 100 chips			Price per
Supplier	Excellent	Good	Mediocre	100 chips
1	60	20	20	400
2	50	35	15	300
3	40	20	40	250

The company has budgeted $28,000 to spend on chips. If the company does not obtain enough chips of a given quality, then the company may special-order additional chips at $10 per excellent chip, $6 per good chip and $4 per mediocre chip. The company assesses a penalty of $1for each dollar by which the amount paid to suppliers 1 to 3 exceeds the annual budget. Formulate it as a goal programming model to minimize the penalty associated with meeting the annual chip requirements. **[3]**

10. Solve the following non-linear programming problem using Khun-Tucker conditions.

$$\text{Maximize } z = 2x_1^2 - 7x_2^2 + 12x_1x_2$$
$$\text{Subject to } 2x_1 + 5x_2 \le 98$$
$$x_1, x_2 \ge 0$$

[4]

11. A project consisting of twelve distinct activities is to be analyzed by using PERT. The following information is given (time estimates are in days)

Activity	Predecessor	Optimistic time	Normal time	Pessimistic time
A	--	2	2	2
B	--	1	3	7
C	A	4	7	8
D	A	3	5	7
E	B	2	6	9
F	B	5	9	11
G	C,D	3	6	8
H	E	2	6	9
I	C,D	3	5	8
J	G,H	1	3	4
K	F	4	8	11
L	J,K	2	5	7

Draw the network. Indicate the critical path, free float, total float and variance of all the activities. **[4]**

12. Consider the following LPP

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 460$$
$$x_1 + 4x_2 \leq 420$$
$$x_1, x_2, x_3 \geq 0$$

The associated optimum table for the primal is given as

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
Z	4	0	0	1	2	0	1350
x_2	$\frac{-1}{4}$	1	0	$\frac{1}{2}$	$\frac{-1}{4}$	0	100
x_3	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	230
x_6	2	0	0	–2	1	1	20

(a) Use post-optimal analysis to determine the optimum solution if the RHS of the three constraints are changed as $\begin{pmatrix} 500 \\ 400 \\ 600 \end{pmatrix}$ **[1]**

(b) Suppose a new constraint $3x_1 + 3x_2 + x_3 \leq 500$ is added determine the optimum solution. **[3]**

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1st Semester 2007 – 2008
Test-II (open book)

3rd year – All Discipline
16th December 2007

Course: Optimization (AAOC UC222)
Marks: 20 , Weightage 20%

Note: *1. Answer all Questions.*
2. All parts of the question should be done together

1. Determine the range of value of p and q that will make the payoff element a_{22} a saddle point for the game whose payoff (a_{ij}) is given below:

$$\text{Player A} \quad \begin{matrix} \\ \\ \\ \end{matrix} \quad \begin{matrix} \text{Player B} \\ \begin{pmatrix} 2 & 4 & 7 \\ 10 & 7 & q \\ 4 & p & 8 \end{pmatrix} \end{matrix}$$

[2]

2. Find the optimal assignment for the following:

$$\begin{matrix} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \text{A} & 33 & 40 & 42 & 32 \\ \text{B} & 45 & 28 & 31 & 23 \\ \text{C} & 42 & 29 & 36 & 29 \\ \text{D} & 27 & 42 & 44 & 38 \end{matrix}$$

[2]

3. A hospital needs to purchase 3 gallons of a perishable medicine for use during the current month and 4 gallons for use during the next month. Because the medicine is perishable, it can only be used during the month of purchase. Two companies sell the medicine. The medicine is in short supply. Thus, during the next two months, the hospital is limited to buying at most 5 gallons from each company. The two companies charge the prices as shown below.

Company	Current month's price per gallon	Next month's price per gallon
X	800	720
Y	710	750

Formulate as a balanced transportation problem and find the minimum cost of purchase. (Use Vogel's method to calculate the initial solution) **[4]**

4. The promoter of a rock concert must perform the following task before the concert can be held. Draw the network and find the minimum duration by CPM.

optimization

Activity	Description	Predecessor	Duration
A	Find site	--	2
B	Find engineers	A	4
C	Hire opening act	A	6
D	Set radio and TV ads	C	3
E	Set up ticket agents	A	3
F	Prepare electronics	B	4
G	Print advertising	C	5
H	Set up transportation	C	1
I	Rehearsal	F, H	2
J	Last minute details	I	2

[4]

5. Solve the following integer programming problem.

$$\text{Maximize } z = x_1 + x_2$$

$$\text{Subject to } 2x_1 + 5x_2 \le 16$$

$$6x_1 + 5x_2 \le 30$$

$$x_2 \ge 0$$

$$x_1 \ge 0 \text{ and integer}$$

[4]

6. Two products are manufactured on two sequential machines. The following table gives the machining times in minutes per unit for the two products.

	Machining time in min	
Machine	Product 1	Product 2
1	5	3
2	6	2

The daily production quotas for the two products are 80 and 60 units respectively. Each machine runs 8 hours a day. Overtime, though not desirable, may be used if necessary to meet the production quota. Formulate the problem as a goal programming model. **[4]**

(2)

BITS, PILANI – DUBAI
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TEST – 1 (Closed Book)

Course: AAOC UC222 Optimization **Max. Marks 25** **Date: 11.11.2007**

Answer all questions

1. Using Graphical method, find the maximum value of $z = 7x_1 + 10x_2$ **[5]**
   Subject to the constraints $x_1 + x_2 \le 30000$
   $x_2 \le 12000$
   $x_1 \ge 6000$
   $x_1 \ge x_2$
   $x_1, x_2 \ge 0$

2. Find the optimal solution of the following Linear Programming Problem: **[5]**
   Minimize $z = 3x_1 + 2x_2$
   Subject to $-2x_1 + 3x_2 = 3$
   $4x_1 + 8x_2 \ge 5$
   $x_1, x_2 \ge 0$

3. Consider the following LP: **[5]**
   Maximize $z = 5x_1 + 2x_2 + 3x_3$
   Subject to $x_1 + 5x_2 + 2x_3 = 30$
   $x_1 - 5x_2 - 6x_3 \le 40$
   $x_1, x_2, x_3 \ge 0$

   Given that the artificial variable x_4 and the slack variable x_5 form the starting basic variables and that M was set equal to 100 when solving the problem the following table is obtained.

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	23	7	105	0	?
x_1	1	5	2	1	0	30
x_5	0	−10	−8	−1	1	10

   (i) Is the above table optimal? If yes write the optimal solution.
   (ii) Write the associated dual problem and determine its optimal dual solution.

4. A firm produces three items A, B, C and requires two types of resources – man hours and raw material. The following LPP has been formulated to determine the optimum production schedule that maximizes the total profit:

Optimization

Maximize $z = 3x_1 + x_2 + 5x_3$

Subject to $6x_1 + 3x_2 + 5x_3 \leq 45$ (man hours)

$3x_1 + 4x_2 + 5x_3 \leq 30$ (raw material)

$x_1, x_2, x_3 \geq 0$

The optimum table is given below

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	3	0	0	1	30
x_1	1	$-\frac{1}{3}$	0	$\frac{1}{3}$	$-\frac{1}{3}$	5
x_3	0	1	1	$-\frac{1}{5}$	$\frac{2}{5}$	3

(i) If the unit profit of product A is increased to 5, find the optimal solution. **[3]**

(ii) If a supervision constraint $2x_1 + x_2 + 3x_3 \leq 20$ is added to the original problem, how is the optimal solution affected? **[2]**

5. In the above problem if the available raw material is increased to 50 units, find the optimal solution? **[5]**