BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Comprehensive Examination – III year 2006-2007

Date: 24.12.06 Total Marks: 40 Weightage: 40%
Course: Optimization Course No. AAOC UC222

Answer all questions
Use separate answer books for Part – A and Part – B

Part – A

1. A company produces two models of electronic gadgets that use resistors, capacitors and chips. The following table summarizes the data of the situation:

	Resource requirements per unit		
Resource	Model 1	Model 2	Maximum availability
Resistor	2	3	1200
Capacitor	2	1	1000
Chips	0	4	800
Unit Profit	3	4	

Formulate the problem as a linear programming problem. **(2)**

2. On a picnic outing, 2 two person teams are playing hide and seek. There are four hiding locations (A, B, C and D) and the two members of the hiding team can hide separately in any two of the four locations. The other team will then have the chance to search any two locations. The searching team gets a bonus point if they find both members of the hiding team. If they miss both, they lose a point. Otherwise, the game is a draw. Set up the problem as a two-person zero sum game. **(2)**

3. Solve question 1 given above and find the optimal solution. **(4)**

4. (a) In the question 1 given above in terms of the optimal profit, determine the worth of one resistor, one capacitor, one chip. **(1)**

   (b) If the available number of chips is reduced to 350 units determine the new optimum solution. **(3)**

5. Solve the following transportation problem and find the optimal solution. **(4)**

2	1	2	10
6	4	5	80
3	2	5	15
75	20	50	

6. City hospital plans the short-stay assignment of surplus beds (those that are not already occupied) 4 days in advance. During the 4-day planning period about 30, 25 and 20 patients will require 1, 2 or 3 day stays respectively. Surplus beds during the same period are estimated at 20, 30, 30 and 30. Use goal programming and formulate the problem to resolve over admission and under admission in the hospital.

(4)

Part – B

Q1.

(a) A company produces two types of hats .Type A requires as much labor time as a Type B. If all the available time is dedicated to Type B alone, the company can produce a total of 400 type B hats a day .The respective market limits for the two types are 150 and 200 hats per day .The profit is $8 per type A and $5 per type B hat. [2+2]

(i) Using graphical method find the number of hats of each type that would maximize the profit.

(ii) If the daily demand limit on type A is decreased to 115, how does it effect the optimal profit.

(b) Four jobs are assigned to four workers .The cost (in $)of performing the jobs is the function of skills of the workers. The following Table gives the costs of assignments. If the blanks denote the worker cannot do that particular job, find the optimal assignment

	JOBS 1	2	3	4
WORKERS 1	50	50	-	20
2	70	40	20	30
3	90	30	50	-
4	70	20	60	70

Q2.

(a) Use Branch and Bound method to solve the following problem starting with x_2 as the branching variable: [4+4]

Maximize $x_0 = 5x_1 + 4x_2$

Subject to the constraints

$x_1 + x_2 \le 5$

$10x_1 + 6x_2 \ge 45$

$x_1, x_2 \ge 0$

Also x_1, x_2 are integers.

(b) Given the following NLPP

$$\min f(x) = x_1^2 + x_2^2 + x_3^2$$

Subject to the constraints

$$g_1 = 2x_1 + x_2 \le 5$$

$$g_2 = x_1 + x_3 \le 2$$

$$g_3 = -x_1 \le -1$$

$$g_4 = -x_2 \le -2$$

$$g_5 = -x_3 \le 0$$

(i) Define a convex function. Show that the functions f(x) is convex for the above problem.

(ii) Use the Kuhn Tucker conditions to solve the given problem.

Q3

[4+4]

(a) Use the method of Dynamic Programming to solve the problem given below.

Find the minimum value of

$$F = y_1^2 + 2y_2^2 + 4y_3$$

Subject to the constraints

$$y_1 + 2y_2 + y_3 \ge 8$$

$$y_1, y_2, y_3 \ge 0$$

(b) Given

(1) The starting activity is A and K is the terminal activity
(2) A precedes B
(3) C and D are successor events to B
(4) D is the preceding event to G
(5) E and F occur after C
(6) G precedes H
(7) E, F precedes J
(8) F, H occur before J
(9) K succeeds J

Activity	Description	Time (weeks)
A	Market research	15
B	Planning	15
C	Deciding production policy	3
D	Preparing sales program	5
E	Preparing operation sheets	8
F	Buying material	12

G	Planning labor force	1
H	Making tools	14
J	Schedule production	3
K	Production	14

(i) Draw the network for the given project
(ii) Find the project completion time
(iii) The critical path
(iv) And show that the Total Float and Free Float for all activities on the critical path are zero

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Test: II (OB)
Course No. AAOC UC222
Duration: 50 min

Total Marks: 20

Date: 10.12.06
Course: Optimization
Weightage: 20

Answer ALL Questions

1. Use graphical method to solve the following game.

Player B

Player A $\begin{pmatrix} -3 & -6 & -2 & -4 & -1 \\ 2 & 1 & -3 & -2 & -5 \end{pmatrix}$

2. For what value of p the game with the given payoff is strictly determinable.

Player B

Player A $\begin{pmatrix} p & 5 & 1 \\ 0 & p & -6 \\ -1 & 3 & p \end{pmatrix}$

3. Use branch and bound method to solve the following LPP

$$\text{Maximize } z = 7x_1 + 9x_2$$
$$\text{Subject to } x_1 - 3x_2 \le 6$$
$$7x_1 + x_2 \le 35$$
$$x_2 \le 3.5$$
$$x_1, x_2 \ge 0$$

4. Four operators A, B, C, D are available to a manager who has to get four jobs P, Q, R, S done by assigning one job to each operator. Given the time needed by different operators for different jobs in the matrix below:

Jobs

Operators $\begin{pmatrix} 12 & 10 & 10 & 8 \\ 14 & 12 & 15 & 11 \\ 6 & 10 & 16 & 4 \\ 8 & 10 & 9 & 7 \end{pmatrix}$

(i) How should manager assign the jobs so that the total time needed for all four jobs minimum?

(ii) If job Q is not to be assigned to operator B, what should be the assignment and how much additional total time will be required?

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III YEAR I SEMESTER

B

QUIZ – 2 (Closed Book)

Course Title: Optimization
Date: 30.11.2006
Time: 30 min

Course No: AAOC UC222
Max marks: 10
Weightage: 10%

Name of the Student: ____________________

ID No: ____________________

Branch: ____________________

Set: B

Recheck Request:

Answer ALL Questions

1. A transportation problem is said to be balanced if

____

____ (0.5)

2. Find the initial basic feasible solution by Vogel's approximation method. Fill the allocations in the given question itself with out over writing. (2)

45	55	53	60	260	( )	( )	( )	( )
48	60	56	58	140	( )	( )	( )	( )
52	64	55	61	220	( )	( )	( )	( )
50	65	60	62	360	( )	( )	( )	( )
200	320	250	210					

( )	( )	( )	( )
( )	( )	( )	( )
( )	( )	( )	( )
( )	( )	( )	( )

3. For the following transportation problem

20	30	50	20	7
70	30	50	70	9
40	18	70	20	18
5	8	7	14	

the initial allocation is as follows: $x_{11} = 5, x_{14} = 2, x_{23} = 7, x_{24} = 2, x_{32} = 8, x_{34} = 10$

Is the given solution optimal? If not find the optimal solution.

(If you need extra space use the backside of this page) (3)

	$v_1 =$	$v_2 =$	$v_3 =$	$v_4 = 0$	
$u_1 =$	20			20	7
$u_2 =$			50	70	9
$u_3 =$		18		20	18
	5	8	7	14	

4. For the following LPP the optimal table is given below.

Maximize $z = -x_1 + 2x_2 - x_3$

Subject to $3x_1 + x_2 - x_3 + x_4 = 10$

$-x_1 + 4x_2 + x_3 - x_5 + A_1 = 6$

$x_2 + x_3 + x_6 = 4$

$x_1, x_2, x_3, x_4, x_5, x_6, A_1 \geq 0$

Basis	x_1	x_2	x_3	x_5	x_4	A_1	x_6	Solution
Z	1	0	3	0	0	M	2	8
x_4	3	0	-2	0	1	0	-1	6
x_2	0	1	1	0	0	0	1	4
x_5	1	0	3	1	0	-1	4	10

(a) Find the range for d_1 by changing the requirement vector from $\begin{pmatrix} 10 \\ 6 \\ 4 \end{pmatrix}$ to $\begin{pmatrix} 10 \\ 6+d_2 \\ 4 \end{pmatrix}$ consistent with the optimum solution. **(2)**

(b) If y_1, y_2, y_3 are the dual variables then the value of **(0.5)**

$(y_1, y_2, y_3) =$

© Suppose the objective function is changed to Maximize $z = -x_1 + 2x_2 + (-1 + d_3)x_3$ then the z-row value corresponding to x_3 is

(1)

(d) Suppose a new constraint $-2x_1 + 2x_2 + 9x_3 \leq 10$ is added, how does the optimal solution given in the table change?

(1)

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Test: I (CB)
Course No. AAOC UC222
Duration: 50 min

Total Marks: 20

Date: 29.10.06
Course: Optimization
Weightage: 20

Answer ALL Questions

PART – A

1. Solve the following problem by M method with x_3 and artificial variable as initial basic variables **(5)**

$$\text{Maximize } z = x_1 + 5x_2 + 3x_3$$
$$\text{Subject to } x_1 + 2x_2 + x_3 = 3$$
$$2x_1 - x_2 = 4$$
$$x_1, x_2, x_3 \geq 0$$

2. (a) What do you mean by an artificial variable and why do we need them? **(1)**

   (b) Consider the following LPP with a solution. **(3)**

$$\text{Maximize } z = 6x_1 + 4x_2$$
$$\text{Subject to } 2x_1 + 3x_2 \leq 30$$
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + x_2 \geq 3$$
$$x_1, x_2 \geq 0$$

Basic	x_1	x_2	x_3	x_4	x_5	Solution
Z	0	0	0	2	0	Z = 48
x_3	0	$\frac{5}{3}$	1	$-\frac{2}{3}$	0	14
x_5	0	$-\frac{1}{3}$	0	$\frac{1}{3}$	1	5
x_1	1	$\frac{2}{3}$	0	$\frac{1}{3}$	0	8

(a) Is the above solution optimal?

(b) Is the solution unique? If not find an alternative solution.

PART – B

3. Obtain the dual problem of the following L.P.P: **(2)**

$$\text{Maximize } z = 2x_1 + 5x_2 + 6x_3$$
$$\text{Subject to } 5x_1 + 6x_2 - x_3 \le 3$$
$$-2x_1 + x_2 + 4x_3 \le 4$$
$$x_1 - 5x_2 + 3x_3 \le 1$$
$$-3x_1 - 3x_2 + 7x_3 \ge 6$$
$$x_1, x_2, x_3 \ge 0$$

4. Consider the following L.P.P

$$\text{Maximize } z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$
$$\text{Subject to } x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \ge 0$$

The optimum objective row is given as $z + 2x_1 + 0x_2 + 0x_3 + 3x_4 = 16$. Use this information to determine the associated optimal dual solution. **(3)**

5. Use dual simplex method to solve the following L.P.P **(6)**

$$\text{Minimize } z = 3x_1 + x_2$$
$$\text{Subject to } x_1 + x_2 \ge 1$$
$$2x_1 + 3x_2 \ge 2$$
$$x_1, x_2 \ge 0$$

BITS, Pilani-Dubai Campus, Knowledge Village, Dubai
III YEAR I SEMESTER

A

QUIZ – 1 (Closed Book)

Course Title: Optimization
Date: 19.10.2006
Time: 30 min

Course No: AAOC UC222
Max marks: 10
Weightage: 10%

Name of the Student: ____________________

ID No: ____________________

Branch: ____________________

Set: A

Recheck Request:

Answer ALL Questions

1. Electra produces two types of electric motors each on separate assembly line. The respective daily capacities of the two lines are 600 and 750 motors. Type 1 motor uses 10 units of certain electronic component and type 2 motors uses only 8 units. The supplier of the component can provide 8000 pieces a day. The profits per motor for types 1 and 2 are 60 and 40 respectively.

   (a) The decision variables are

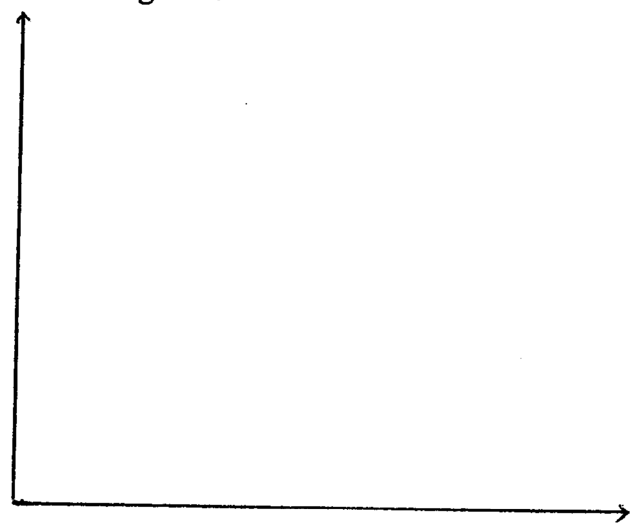
   ____

   ____ **(0.5)**

   (b) The objective function is ______________________ **(0.5)**

   (c) The constraints are **(1.5)**

   (d) The feasible region is **(1.5)**



   (e) The solution is **(0.5)**

   (f) The value of $z =$ **(0.5)**

   (g) The optimality range of the ratio of unit profits that will keep the solution in (e) unchanged is ______________________ **(0.5)**

   (h) If c_1 is not changed the range for c_2 is ______________ **(0.5)**

(i) What is the unit worth of resource for the component? **(0.5)**

2. For the following Linear Programming Problem

Maximize $z = 4x_1 + 5x_2 + 8x_3$

Subject to $x_1 + x_2 + x_3 \leq 100$

$3x_1 + 2x_2 + 4x_3 \leq 500$

$x_1, x_2, x_3 \geq 0$

(a) Complete the following table: **(0.5)**

Basic						Solution
z						

(b) The entering variable is ________ **(0.25)**

(c) The leaving variable is ________ **(0.25)**

(d) Complete the following table: **(1)**

Basic						Solution
z						
	1	1	1	1	0	100

(e) Is the solution feasible? Yes/No. If yes justify. **(0.5)**

(f) Is optimality reached? Yes /No **(0.5)**

(g) If yes then the optimal solution is ______________________ **(0.5)**