BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: Test: II (OB) Course: Optimization
Course No. AAOCU222
Duration: 50 min Total Marks: 20 Weightage: 20

Answer ALL Questions

Q1. Listed in the table are the activities and sequencing necessary for a maintenance job on the heat exchangers in a refinery:

Activity	Description	Predecessors	Duration
A	Dismantle pipe connections	None	2
B	Dismantle header, floating front	A	3
C	Remove tube bundle	B	4
D	Clean bolts	B	2
E	Clean header & floating front	B	5
F	Clean tube bundle	C	3
G	Clean shell	C	4
H	Replace tube bundle	F,G	6
I	Prepare shell pressure test	D,E,H	7
J	Reassemble	I	8

(i) Draw the network. (2)
(ii) Identify critical path and find total project duration. (2)
(iii) Determine total and free floats for all the activities. (2)
(iv) Find the probability of completing the project in 30 days. (2)

Q2. Solve Maximize $f = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$ (2)

Q3. A company manufactures three types of lamps: a desk lamp, a bedside lamp and a floor lamp. The lamps are all solid brass and are produced in two distinct steps: turning

and finishing. The schedule for labor and material inputs and availability of each type is as follows:

	Desk lamp	Bedside lamp	Floor lamp	Availability
Turning labor	1 hour	3 hours	2 hours	1500/month
Finishing labor	2 hours	2 hours	4 hours	1000/month
Brass	2 kg	1.5 kg	3 kg	3000/month

Profit contribution of each type is Rs.400, Rs.500 and Rs.650 respectively. The company has two equally desirable goals: minimizing the ideal time in finishing and making a monthly profit of Rs.100,000. Set up the problem as a goal program. (4)

Q4. Solve the following quadratic programming problem

$$\text{Maximize } z = 8x_1 + 10x_2 - x_1^2 - x_2^2$$

$$\text{Subject to } \quad 3x_1 + 2x_2 \leq 6$$

$$x_1, x_2 \geq 0$$

(6)

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Comprehensive Examination – III year 2005-2006

Date: 29.12.05 **Total Marks: 40** **Weightage: 40%**
Course: Optimization **Course No. AAOCU222**

Answer all questions
Use separate answer books for Part – A and Part – B

Part – A

1. Find the optimal assignment for the following problem: (2)

5	7	11	6
8	5	9	6
4	7	10	7
10	4	8	3

2. Solve the game $\begin{pmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{pmatrix}$ graphically. (2)

3. An American University office is processing freshman applications for the upcoming academic year. The applications fall into three categories: in-state, out-of-state and international. The male-female ratios for in-state and out-of-state applications are 1:1 and 3:2 respectively. For the international students, the corresponding ratio is 8:1. The American College Test (ACT) score is an important factor in accepting new students. Statistics indicate the average ACT scores for in-state, out-of-state and international students are 27, 26 and 23 respectively. The committee on admissions has established the following desirable goals for the new freshman class:

   a) The incoming class is at least 1200 freshmen.
   b) The average ACT score for all incoming students is at least 25.
   c) International students constitute at least 10% of the incoming class.
   d) The female-male ratio is at least 3:4.
   e) Out-of-state students constitute at least 20% of the incoming class.

   Formulate the problem as a goal programming model. (4)

4. Write the Khun-Tucker necessary and sufficient conditions for the following problem. (4)

$$\text{Maximize } f(X) = x_1^3 - x_2^2 + x_1 x_3^2$$

$$\text{Subject to } x_1 + x_2^2 + x_3 = 5$$

$$5x_1^2 - x_2^2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0$$

5. Find the critical path, total duration, total float and free float for the following activities. (4)

Activity	Duration	Activity	Duration
(1,2)	4	(5,6)	4
(1,3)	1	(5,7)	8
(2,4)	1	(6,8)	1
(3,4)	1	(7,8)	2
(3,5)	6	(8,10)	5
(4,9)	5	(9,10)	7

6. Solve the following problem by dynamic programming. (4)

$$\text{Minimize } x_0 = y_1^2 + y_2^2 + y_3^2$$
$$\text{Subject to } y_1 y_2 y_3 = 27$$
$$y_i > 0, i = 1,2,3$$

Part – B

1. The ABC company has been a producer of picture tubes for television sets and certain printed circuits for radios. The company has just expanded into full scale production and marketing of AM and AM-FM radios. It has built a new plant that can operate 48 hours per week. Production of an AM radio in the new plant will require 2 hours and production of an AM-FM radio will require 3 hours. Each AM radio will contribute Rs.40 to profits, while an AM-FM radio will contribute Rs.80 to profits. The marketing department, after extensive research has determined that a maximum of 15 AM radios and 10 AM-FM radios can be sold each week. Determine the optimal production mix of AM and AM-FM radios that will maximize profits. (2)

2. Find the solution of the following LPP by revised simplex method at the end of first iteration. (2)

$$\text{Maximize } z = 6x_1 - 2x_2 + 3x_3$$
$$\text{Subject to } 2x_1 - x_2 + 2x_3 \le 2$$
$$x_1 + 4x_3 \le 4$$
$$x_1, x_2, x_3 \ge 0$$

3. Solve the following LPP by Big-M method. (4)

$$\text{Minimize } z = 3x_1 + 2.5x_2$$
$$\text{Subject to } 2x_1 + 4x_2 \ge 40$$
$$5x_1 + 2x_2 \ge 50$$
$$x_1, x_2 \ge 0$$

4. A company wants to produce three products A, B and C. The unit profits on these products are Rs.4, Rs.6 and Rs.2 respectively. These products require two types of resources – man power and material. The following LP model is formulated for determining the optimal product mix.

$$\text{Maximize } z = 4x_1 + 6x_2 + 2x_3$$
$$\text{Subject to } x_1 + x_2 + x_3 \le 3 \text{ (man power)}$$
$$x_1 + 4x_2 + 7x_3 \le 9 \text{ (Material)}$$

The optimal table is as follows:

Basic	x_1	x_2	x_3	s_1	s_2	Solution
z	0	0	6	$10/3$	$2/3$	16
x_1	1	0	−1	$4/3$	$-1/3$	1
x_2	0	1	2	$-1/3$	$1/3$	2

a) Discuss the effect of changing the resource vector (RHS) from $\begin{pmatrix}3\\9\end{pmatrix}$ to $\begin{pmatrix}6\\9\end{pmatrix}$ on the optimal solution and find the new optimal solution. (1)

b) Suppose an administrative constraint is added. Products A, B and C require 2, 3 and −2 hours of administrative services, while the total available administrative hours are 4. Find the optimal solution. (3)

5. Solve the following transportation problem and find the optimal allocation. (4)

21	16	25	13	11
17	18	14	23	13
32	27	18	41	19
6	10	12	15	

6. Find the optimal solution for the integer programming problem. (4)

$$\text{Maximize } z = 2x_1 + x_2$$
$$\text{Subject to } x_1 \le \frac{3}{2}$$
$$x_2 \le \frac{3}{2}$$
$$x_1, x_2 \ge 0 \text{ and are integers.}$$

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 18.12.05 Test: II (OB) Course: Optimization
Course No. AAOCU222
Duration: 50 min Total Marks: 20 Weightage: 20

Answer ALL Questions

Q1. A project consists of eight activities with the following relevant information:

Activity	Immediate Predecessor	Optimistic Time	Most likely	Pessimistic
A	None	4	7	10
B	None	3	6	9
C	A,B	3	5	7
D	A,B	3	5	7
E	C,D	3	4	11
F	C	2	4	12
G	E,F	5	7	9
H	E	2	6	16

(i) Draw the network. (2)

(ii) Find the critical path and project completion time. (2)

(iii) Find the total float and free float for all the activities. (2)

(iv) What is the probability that the project will be completed by 18 weeks? (2)

Given

Z	−2.96	−2.98	−3.1	−3.2	−3.4
P	.0015	.0014	.00135	.00069	.00034

Q2. Solve Maximize $f = x_1(x_2 - 1) + x_3(x_3^2 - 3)$. (2)

Q3. A manufacturing firm produces two types of product A and B. According to past experience production of either Product A or Product B requires an average of one hour in the plant. The plant has a normal production capacity of 400 hours a month. The marketing department of the firm reports that because of limited market, the maximum number of product A and product B that can be sold in a month are 240 and 300

respectively. The net profit from the sale of Product A and Product B are Rs.800/- and Rs.400/- respectively. The manager has set the following goals arranged in the order of importance

P1: He wants to avoid any underutilization of normal production capacity.

P2: He wants to sell maximum possible units of Product A and Product B. Since the net profit from the sale of Product A is twice the amount from Product B, the manager has twice as much desire to achieve sales for Product A as for product B.

P3: He wants to minimize the overtime operation.

Formulate as a goal programming problem. (6)

Q4. Solve the following Non-linear programming problem by Khun-Tucker conditions:

$$\text{Maximize } z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$$

$$\text{Subject to } 2x_1 + x_2 \leq 5$$

$$x_1, x_2 \geq 0$$

(4)

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date : 16.11.05

Course No. AAOCU222
Duration: 50 min

Test: I (CB)

Total Marks: 20

Course: Optimization

Weightage: 20

Answer ALL Questions

PART – A

Q1. Use Simplex method to solve the following problem (4)

Maximize $x_0 = 3x_1 + 4x_2$
Subject to the constraints
$x_1 + x_2 \le 450$
$2x_1 + x_2 \le 600$
$x_1, x_2 \ge 0$

Q2. (a) Explain the term artificial variable and its use in linear programming and when do you say that the solution is infeasible.

(b)Use Two Phase method or Big M method to solve

Maximize $x_0 = 5x_1 + 3x_2$
Subject to
$2x_1 + x_2 \le 1$
$x_1 + 4x_2 \ge 6$
$x_1, x_2 \ge 0$ (6)

PART – B

Q3. Solve the following linear programming problem by revised simplex method given the starting basic feasible vector $X_{B_0} = \begin{pmatrix} x_2 \\ x_4 \\ x_5 \end{pmatrix}$, $B^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$ and the constants $X_B = \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$

Minimize $Z = 7x_2 + 11x_3 - 10x_4 + 26x_6$

Subject to $x_2 - x_3 + x_5 + x_6 = 6$

$x_2 - x_3 + x_4 + 3x_6 = 8$

$x_1 + x_2 - 3x_3 + x_4 + x_5 = 12$

$x_1, \cdots, x_6 \geq 0$ (5)

Q4. The table gives the information regarding the quantity required by 4 markets and supply capacity of 3 warehouses. The unit transportation cost from a warehouse to a market is also given below:

	I	II	III	IV	
A	5	2	4	3	22
B	4	8	1	6	15
C	4	6	7	5	8
	7	12	17	9	

The present allocation is as follows:
12 units from A to II, 1unit from A to III, 9 units from A to IV, 15 units from B to III, 7 units from C to I and 1 unit from C to III.

(i) Check and see if the allocation is optimal. (1)

(ii) Find the optimal solution. (3)

(iii) What do you mean by degeneracy in transportation problem and how do you rectify that? (1)

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 10.11.05 **Test: I (CB)** **Course: Optimization**
Course No. AAOCU222
Duration: 50 min **Total Marks: 20** **Weightage: 20**

Note: Use separate answer books for Part – A and Part – B

Answer ALL Questions

PART – A

Q1. Use Simplex method to solve the following problem (4)

$$\text{Maximize } x_0 = 3x_1 + 2x_2$$
Subject to the constraints
$$x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Q2. Use Two Phase method or Big M method to solve

$$\text{Maximize } x_0 = 6x_1 + 4x_2$$
Subject to
$$2x_1 + 3x_2 \le 30$$
$$3x_1 + 2x_2 \le 24$$
$$x_1 + x_2 \ge 3$$
$$x_1, x_2 \ge 0$$

Show that there are alternative solutions and find them. (6)

PART – B

Q3. (a) Determine the solution of the following

$$x_1 + 3x_2 = 2$$
$$3x_1 + x_2 = 3.$$

State the general conditions for vector dependence-independence that lead to unique solution, infinity of solution and no solution. $\left(2\frac{1}{2}\right)$

b) Using revised simplex algorithm find the new solution column for the following linear programming problem at the end of first iteration.

$$\text{Minimize } z = 2x_1 + x_2$$
$$\text{Subject to } 3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

$\left(2\frac{1}{2}\right)$

Q4. Find the initial solution of the following transportation problem by Vogel's approximation method and also find the optimal solution after executing *u-v* method.

0	2	1	6
2	1	5	7
2	4	3	7
5	5	10	20

(5)

Set I

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 29/9/05 Test : Quiz I Course: Optimization
Duration :30 min Total Marks : 10 Weightage: 10

NOTE : (Answer all Questions)

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1. [2]

Production in a factory is going on round the clock. Factory workers report for duty at the end of every 4 hours. Each worker after reporting works for 8 hours continuously. The minimum number required during various periods is given in the table below.

Minimum number of workers to report for duty

Interval number	From	To	Minimum number of workers required
1	12midnight	4 am	20
2	4am	8am	25
3	8am	12noon	35
4	12noon	4pm	32
5	4pm	8pm	22
6	8pm	12 midnight	15

The LPP to minimize the number of workers to report at the beginning of each period for duty in a day is given by

2. [1]

Show the feasible region for the following problem on the graph

Max $x_0 = -3x_1 + 2x_2$

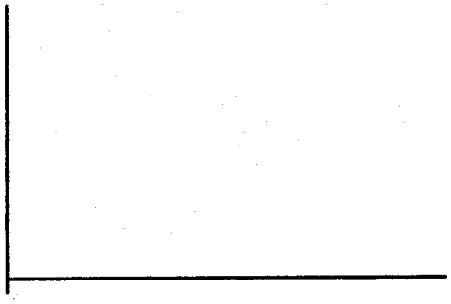
Subject to the constraints

$x_1 \leq 3$

$x_1 . x_2 \leq 0$

$x_1 . x_2 \leq 2$

$x_i \geq 0$, where $i = 1, 2$



3.What type of a solution do we have for the above problem? .....[1]

4.Which variable cannot be increased?.....[1]

5.Which constraint if removed does not change the feasible space?.....[1]

6.Can we find the range of optimality for the ratio $\frac{c_1}{c_2}$?.....[1]

For the problem

Max $x_0 = 6x_1 + 8x_2$

subject to the constraints

$5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

$x_i \geq 0$, where $i = 1, 2$

7.Complete the following tables [1]

	x_0	x_1	x_2	s_1	s_2	Sol^n
x_0	1			0	0	0
s_1	0					
s_2	0					

8. [1]

	x_0	x_1	x_2	s_1	s_2	Sol^n
x_0	1					
x_2	0					
s_2	0					

9.Optimality has been reached yes/no [1/2]

10. Basic variable .....leaves the basis and non-basic variable.....enters the basis [1/2]

Set II

BITS, Pilani-Dubai Campus
Knowledge Village, Dubai

Date: 29/9/05 Test : Quiz I Course: Optimization
Duration :30 min Total Marks : 10 Weightage: 10

NOTE : (Answer all Questions)

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1. [2]

Production in a factory is going on round the clock. Factory workers report for duty at the end of every 4 hours. Each worker after reporting works for 8 hours continuously. The minimum number required during various periods is given in the table below.

Minimum number of workers to report for duty

Interval number	From	To	Minimum number of workers required
1	12midnight	4 am	10
2	4am	8am	20
3	8am	12noon	35
4	12noon	4pm	32
5	4pm	8pm	22
6	8pm	12 midnight	10

The LPP to minimize the number of workers to report at the beginning of each period for duty in a day is given by

2. [1]

Show the feasible region for the following problem on the graph

Max $x_0 = -3x_1 + 2x_2$

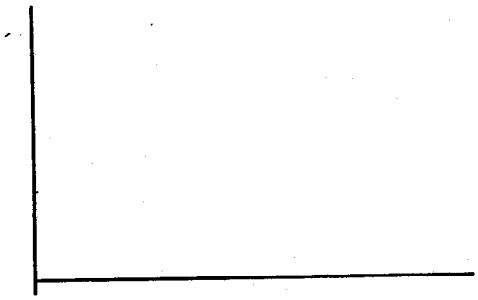
Subject to the constraints

$x_1 \leq 3$

$x_1 . x_2 \leq 0$

$x_1 . x_2 \leq 3$

$x_i \geq 0$, where $i = 1, 2$



3. What type of a solution do we have for the above problem? .....[1]

4. Which variable can be increased?.....[1]

5. Which constraint if removed does not change the feasible space?.....[1]

6. Can we find the range of optimality for the ratio $\frac{c_2}{c_1}$?.....[1]

For the problem

Max $x_0 = 6x_1 + 8x_2$

subject to the constraints

$5x_1 + 10x_2 \leq 60$

$4x_1 + 4x_2 \leq 40$

$x_i \geq 0$, where $i = 1, 2$

7.Complete the following tables [1]

	x_0	x_1	x_2	s_1	s_2	Sol"
x_0	1			0	0	0
s_1	0					
s_2	0					

8. [1]

	x_0	x_1	x_2	s_1	s_2	Sol"
x_0	1					
x_2	0					
s_2	0					

9.Optimality has been reached yes/no [1/2]

10. Basic variable .....leaves the basis and non-basic variable.....enters the basis [1/2]

BITS, Pilani - Dubai Campus
Knowledge village, Dubai

Date: 9.10.05 Test: Quiz I Make up Course: OPTIMISAT

Duration: 30 min Total marks: 10 Weitage: 10

ANSWER ALL QUESTIONS

1) (2)

A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given below.

Food type	Yield per unit			cost/ Unit
	Proteins	Fats	Carbohydrates	
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

The L.P model is given by

2) Show the feasible region for the following problem on the graph. (1)

Maximize $z = 40x_1 + 100x_2$

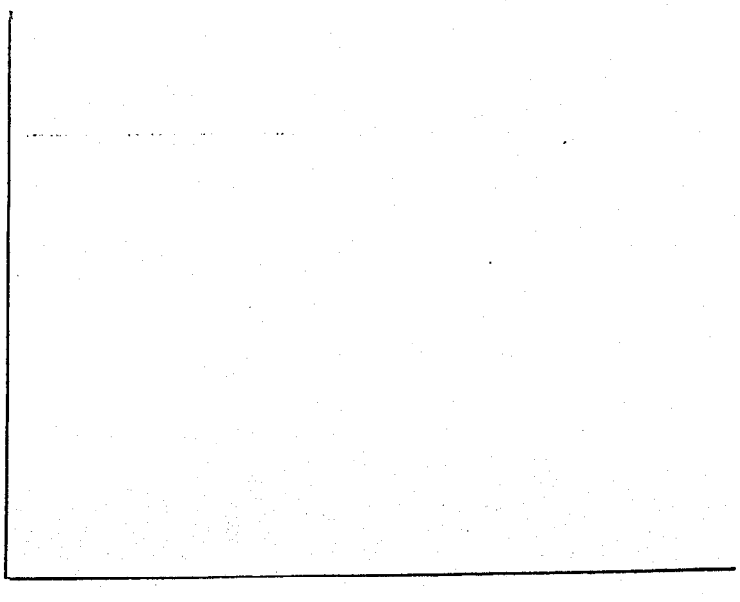
Subject to $12x_1 + 6x_2 \leq 3000$

$4x_1 + 10x_2 \leq 2000$

$2x_1 + 3x_2 \leq 900$

$x_1, x_2 \geq 0$

3) ~~What type of a solution we have for the problem?~~ (1)



3) What type of a solution we have for the problem? (

_____

4) Which constraint if removed does not change the feasible space? _____ (1

5) What is the optimal value? _____ (1)

6) Does this problem have a unique solution? why! (1)

For the problem

Maximize $z = 3x_1 + 4x_2$

Subject to $x_1 + x_2 \leq 450$

$2x_1 + x_2 \leq 600$

$x_1, x_2 \geq 0$

7) Complete the following tables (1)

Basic	z	x_1	x_2	s_1	s_2	solution
z	1			0	0	0
s_1						
s_2						

8) (1)

Basic	z	x_1	x_2	s_1	s_2	solution
z	1					
x_2	0					
s_2	0					

9) Based on the table (8) optimality has been reached Yes / No (½)

10) Basic variable _____ leaves the basis and non basic variable _____ enters the basis. (½)