

**BITS, PILANI – DUBAI CAMPUS  
KNOWLEDGE VILLAGE, DUBAI  
CS UC351, THEORY OF COMPUTATION  
TEST – I (CLOSED BOOK)**

**MAXIMUM MARKS: 20**

**Time: 50 MINUTES**

**DATE: 24.10.2004, SUNDAY**

**Weightage: 20%**

**ANSWER ALL QUESTIONS:**

1. Show that  $1^3+2^3+\dots+n^3 = [n(n+1)/2]^2$  by Mathematical Induction, where  $n \geq 0$ . (2)

2. Language  $L = \{0, 1\}^* \{10\}$  of all strings of 0's and 1's that end in 10. The Finite Automaton recognized Transition table for the above language:

	State	input	
		0	1
→	^	0	1
	0	00	01
	1	10	11
	00	00	01
	01	10	11
✳	10	00	01
	11	10	11

Give the Transition Diagram for the above Language.

3. If  $R = \{(a, b), (b, c), (c, a)\}$  is a relation in  $\{a, b, c\}$ , find  $R^*$ . (3)

4. Design the deterministic finite automaton  $M$  that accepts the language  $L(M) = \{w : w \in \{a, b\}^* \text{ and } w \text{ does not contain three consecutive } b\text{'s}\}$  and is given by the

$$\delta = \{(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_1, b, q_0), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_0), (q_2, a, q_3), (q_3, a, q_3), (q_3, b, q_3), (q_3, b, q_0)\}$$

$$\text{where } K = \{q_0, q_1, q_2, q_3\}, \Sigma = \{a, b\}, s = q_0, F = \{q_0, q_1, q_2, q_3\}$$

5. Give the Finite State Automaton for Automated Teller Machine considering all possible transition parameters as states, like Enter PIN Number, Eject card, Enter selection, Cash withdraw, Cash deposit, Accept envelop etc. . State whether the design is DFA or NFA. (5)

6. Give the derivation of the string  $(x_1 * x_2 + x_1) * (x_1 + x_2)$  in  $G$ , Considering the grammar  $G = (V, \Sigma, R, S)$  where  $V, \Sigma$ , and  $R$  are as follow: (3)

$$V = \{x_1, x_2, +, *, (, ), T, F, E\}$$

$$\Sigma = \{x_1, x_2, +, *, (, )\}$$

$$R = \{E \rightarrow E + T, E \rightarrow T, T \rightarrow T * F, T \rightarrow F, F \rightarrow (E), F \rightarrow X_1, F \rightarrow X_2\}$$

The symbols  $E, T$ , and  $F$  are abbreviation for expression, Terms, and factor respectively.

7. Let  $G = (\{S, A_1, A_2\}, \{a, b\}, P, S)$ , where  $P$  consists of  $S \rightarrow a A_1 A_2 a, A_1 \rightarrow b a A_1 A_2 b, A_2 \rightarrow A_1 a b, a A_1 \rightarrow b a a, b A_2 b \rightarrow a b a b$ . Test whether  $w = b a a b b a b a a a b b a b a$  is in  $L(G)$ . (3)

( Only Prescribed Text Book and Class notes are allowed )

Course: CS UC 351 – Theory of Computation – III Year (Computer Science )

Max. Marks: 20

Duration : 50 mins

Weightage : 20%

Date:12.12.2004, Sun.

1. Give the different types of Languages and their corresponding Automata. (1)
2. Show that  $S \rightarrow x^2y^2x^2$  and construct a derivation tree whose yield is  $x^2y^2x^2$ . The production rules are given:  $S \rightarrow xXS \mid x$ ,  $X \rightarrow SyX \mid SS \mid yx$  (2)
3. Do the leftmost derivation and rightmost derivation for the string 00110101 and also draw the derivation tree for the string with help of the production rules  $S \rightarrow 0B \mid 1A$ ,  $a \rightarrow 0 \mid 0S \mid 1AA$ ,  $B \rightarrow 1 \mid 1S \mid 0BB$ . (3)
4. Design a Push-down Automata corresponding to the grammar  $G = (V, \Sigma, P, S)$  where  $V = \{ S, (, ), *, \cup, \emptyset, a, b \}$   
 $\Sigma = \{ (, ), *, \cup, \emptyset, a, b \}$   
 $P = \{ S \rightarrow (SS), S \rightarrow S^*, S \rightarrow (S \cup S), S \rightarrow \emptyset, S \rightarrow a, S \rightarrow b \}$  (7)  
 { Note:- for this problem you have give the formal definition of the push-down automata based on the grammar and its variables and alphabets given above, and also find the transition(delta), based on transition give the Transition Table, and finally the Transition System contains three states. }
5. Construct the context-free grammar accepting the  $\{ a^n b^m a^n \mid m, n \geq 1 \}$ , from the PDA. The PDA is defined as follows:

$$A = ( \{ q_0, q_1 \}, \{ a, b \}, \{ a, Z_0 \}, \delta, q_0, Z_0, \emptyset ) \quad (7)$$

Where delta is define by:

$$R1: \delta (q_0, a, Z_0) = \{ (q_0, a, Z_0) \}$$

$$R2: \delta (q_0, a, a) = \{ (q_0, aa) \}$$

$$R3: \delta (q_0, b, a) = \{ (q_1, a) \}$$

$$R4: \delta (q_1, b, a) = \{ (q_1, a) \}$$

$$R5: \delta (q_1, a, a) = \{ (q_1, e) \}$$

$$R6: \delta (q_1, e, Z_0) = \{ (q_1, e) \}$$

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BITS, Pilani – Dubai Campus  
 Knowledge Village, Dubai  
 First Semester 2004 – 2005  
 Comprehensive Examination

CS UC 351 – Theory of Computation  
 Weightage: 40%  
 Max. Marks: 40

Date: 13.01.2005  
 Time: 3 Hrs.  
 Closed Book

Please Note:

1. All answer should be analytical, precise and yet, complete.
2. Start answering each question from a fresh page.

1. Prove the following by principle of induction: 3 marks

$$\sum_{k=1}^n k^2 = (n(n+1)(n+2))/6.$$

2. Give the Non-Deterministic Finite Automata design for the following language

$$L = (ab \cup aba)^*$$

4 marks

3. Construct the transition table and transition system for the given transition equation:

$$\Delta = \{ (q_0, a, q_1), (q_0, b, q_0), (q_1, a, q_0), (q_1, b, q_2), (q_2, a, q_3), (q_2, b, q_1), (q_3, a, q_3), (q_3, b, q_0), (q_4, a, q_3), (q_4, b, q_5), (q_5, a, q_6), (q_5, b, q_4), (q_6, a, q_5), (q_6, b, q_6), (q_7, a, q_6), (q_7, b, q_3) \}$$

$$\Sigma = \{a, b\}, K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}, s = q_0, F = \{q_3\}.$$

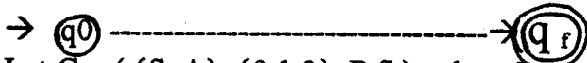
2 marks

4. Show the Finite State Automata for a system that scans for the bits sequence 01111110 in a string of bits (the sequence is allowed to appear anywhere in a bit string). This bit sequence is standard used to denote packets boundaries in serial communication. 4 marks

5. Give the Nondeterministic Machine for the FSA given below:

$$10 + (0+11)0^*1$$

4 marks



6. Let  $G = (\{S, A\}, \{0, 1, 2\}, P, S)$ , where  $P$  consist of  $S \rightarrow 0SA2$ ,  $S \rightarrow 012$ ,  $2A \rightarrow A2$ ,  $1A \rightarrow 11$ .

$$\text{Show that } L(G) = \{ 0^n 1^n 2^n \mid n \geq 1 \}$$

3 marks

7. Give the transition diagram and the transition equation for the Transition table given below:

State / Sigma	0	1
$\rightarrow [q_0, q_4]$	$[q_1, q_7]$	$[q_3, q_5]$
$[q_1, q_7]$	$[q_6]$	$[q_2]$
* $[q_2]$	$[q_0, q_4]$	$[q_2]$
$[q_3, q_5]$	$[q_2]$	$[q_6]$
$[q_6]$	$[q_6]$	$[q_0, q_4]$

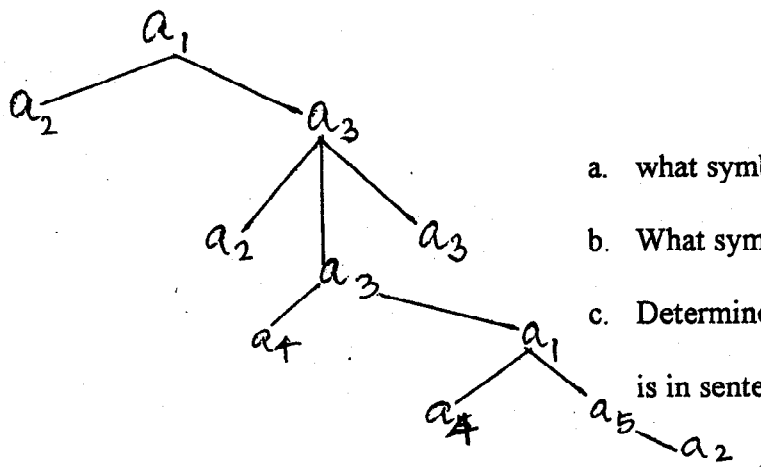
where  $s = [q_0, q_4]$ , and  $F = [q_2]$ .

3 marks

8. Give the symbolic definition of PDA, and explain the symbols used in it. 2 marks
9. Construct a PDA, A equivalent to the following context-free grammar:  $S \rightarrow 00BBBB, B \rightarrow 0S|1S|0$ .  
Test whether  $0^2 1 0^9$  is in  $N(A)$ . Also give the Transition rules for the PDA from the grammar. 4 marks
10. Consider the transition table given below describes the Turing Machine; give the computation sequence with respect to the Turing Machine description. 2marks

Present state	Tape symbol		
	b	0	1
$\rightarrow q1$	1Lq2	0Rq1	
q2	bRq3	0Lq2	1Lq2
q3		bRq4	bRq5
q4	0Rq5	0Rq4	1Rq4
* q5	0Lq2		

11. A derivation tree of a sentential form of a grammar G is given below: 2 marks



- what symbol are necessarily in  $V_n$ ?
- What symbols are likely to be in  $\Sigma$ ?
- Determine if the following string,  $a_1a_2a_3a_4a_1$  is in sentential form or not?

12. Design a Turing Machine that performs the copying operation of a given string, equivalent to that of the string copy operation in any programming Language. Justify your Turing machine design with a valid example. 4 marks

13. Convert  $[00 + 11 + (01 + 10)(00 + 11)^*(01 + 10)]^*$  to a Finite Automaton 3 marks