

**BITS, PILANI-DUBAI CAMPUS,
KNOWLEDGE VILLAGE, DUBAI.**

COURSE: CS UC 351

COURSE TITLE: THEORY OF COMPUTATION

DATE: 15.1.2004

DURATION: 3 HOURS

MAX. MARKS: 100

WEIGHTAGE: 40 MARKS.

COMPREHENSIVE EXAMINATION

PART – A (10 * 2.5 = 25)

1. Assume that $R = \{ (a,b), (b,c), (c,a) \}$ be a relation of $\{a,b,c\}$. Find R^4 ?
2. Show that for any $n \geq 0$, $1+2+\dots+n = (n^2 + n) / 2$.
3. What is $L(((a \cup b)^* a))$?
4. What is the reflexive, transitive closure R^* of $R = \{(a,b), (a,c), (a,d), (d,c), (d,e)\}$? Draw a directed graph representing R^* .
5. Draw Deterministic Finite Automata to accept the following set of string over the alphabet $\{0,1\}$ for all strings of length at most 5.
6. Explain why every NFA can be converted to an equivalent one that has a single final state.
7. Generate a meaningful language using the given $G=(W, \Sigma, R, S)$, where
 $W=\{S, A, N, V, P\} \cup \Sigma$,
 $\Sigma = \{Jim, big, green, cheese, ate\}$
 $R = \{P \rightarrow N, P \rightarrow AP, S \rightarrow PVP, A \rightarrow big, A \rightarrow green, N \rightarrow cheese, N \rightarrow Jim, V \rightarrow ate\}$.
8. Give the exact definition for Push-Down Automata, Turing machine.
9. Show that $S \Rightarrow^* aabbba$, considering G whose production are $S \rightarrow aAS \mid a, A \rightarrow SbA \mid SS \mid ba$.
And construct a derivation tree whose yield is $aabbba$.
10. Give the definition for Class P, Class NP computability.

PART - B (5 * 15 = 75)

1. a) Assume that Language $L = \{0,1\}^* \{10\}$ of all strings of 0's and 1's that end in 10. The transition table of the above language is: (8+7)

state	input	
	0	1
\wedge	0	1
0	00	01
1	10	11
00	00	01
01	10	11
10	00	01
11	10	11

Draw the Transition Diagram and give the Transition equation $\delta = ?$, for the above

- b) Design the transition table and transition diagram for the deterministic finite automaton M that accepts the language $L(M) = \{w : w \{a, b\}^* \text{ and } w \text{ does not contain three consecutive } b\text{'s}\}$ and is given by the

$$\delta = \{ (q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_1, b, q_0), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_0), (q_2, a, q_3), \\ (q_3, a, q_3), (q_3, b, q_3), (q_3, b, q_0) \} \text{ where } K = \{q_0, q_1, q_2, q_3\}, \Sigma = \{A, B\}, s = q_0, \\ F = \{q_0, q_1, q_2, q_3\}.$$

2. Construct a Non-deterministic finite automata and the Transition table from the transition equation given below: (10+5)

$$\delta = \{ (q_0, 0, q_1), (q_0, 1, q_5), (q_1, 0, q_6), (q_1, 1, q_2), (q_2, 0, q_0), (q_2, 1, q_2), (q_3, 0, q_2), (q_3, 1, q_6), \\ (q_4, 0, q_7), (q_4, 1, q_5), (q_5, 0, q_2), (q_5, 1, q_6), (q_6, 0, q_6), (q_6, 1, q_4), (q_7, 0, q_6), (q_7, 1, q_2) \},$$

where q_0 is the initial state and q_2 is the final state, and $\Sigma = \{0, 1\}$.

3. a) Consider the following productions:

$$S \rightarrow aB \mid bA$$

(4+4+4+3)

$$A \rightarrow aS \mid bAA \mid a$$

$$B \rightarrow bS \mid aBB \mid b$$

For the string *aaabbabbba*, find

- i) The leftmost derivation,
- ii) The rightmost derivation,
- iii) Parse tree.

b) Show that the grammar $S \rightarrow a \mid abSb \mid aAb$, $A \rightarrow bS \mid aAAb$ is Ambiguous.

4. a) Construct a Turing machine that perform the copying operation of the given string and so the Turing Machine is called as Copying Machine. Thus the given string is *#abc#*.

b) What is meant by Extension of the Turing Machine? (10 + 5)

5. a) What is called as Halting problem of Turing Machines? (8+7)

b) Give the four properties, that explain a L be an NP-complete Language.

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CS UC351, THEORY OF COMPUTATION
TEST – I (CLOSED BOOK)

MAXIMUM MARKS: 20

Time: 50 MINUTES

DATE: 02.11.2003

SUNDAY

ANSWER ALL QUESTIONS:

1. Give the proof for the statement $P(n)$ is $1+2+3+\dots+n = n(n+1)/2$ by induction. (2)
2. Language $L = \{0,1\}^* \{10\}$ of all strings of 0's and 1's that end in 10. The Finite Automaton recognized Transition table for the above language :

state	input	
	0	1
\wedge	0	1
0	00	01
1	10	11
00	00	01
01	10	11
10	00	01
11	10	11

Give the Transition Diagram for the above Language.

3. Design the deterministic finite automaton M that accepts the language (4)
 $L(M) = \{w : w \in \{a,b\}^* \text{ and } w \text{ does not contain three consecutive } b\text{'s}\}$ and is given by the
 $\delta = \{(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_1, b, q_0), (q_1, b, q_2), (q_2, a, q_2), (q_2, b, q_0), (q_2, a, q_3),$
 $(q_3, a, q_3), (q_3, b, q_3), (q_3, b, q_0)\}$
 where $K = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, $s = q_0$, $F = \{q_0, q_1, q_2, q_3\}$
4. Give the Proof for the Closure Properties of Context-Free Language. (4)
5. Give the derivation of the string $(x_1 * x_2 + x_1) * (x_1 + x_2)$ in G , Considering the grammar (4)
 $G = (V, \Sigma, R, E)$ where V , Σ , and R are as follow:
 $V = \{x_1, x_2, +, *, (,), T, F, E\}$
 $\Sigma = \{x_1, x_2, +, *, (,)\}$
 $R = \{E \rightarrow E + T, E \rightarrow T, T \rightarrow T * F, T \rightarrow F, F \rightarrow (E), F \rightarrow X_1, F \rightarrow X_2\}$
 The symbols E, T , and F are abbreviation for expression, Terms, and factor respectively.
6. Construct a Parse Tree for $((-((4x^2 - y^2) + 8xy))$, for the grammar $G = (V, \Sigma, R, S)$, (4)
 where $V = \{S, (,), 4, 8, +, -, x, y\}$, and $\Sigma = \{4, 8, x, y, (,)\}$ and
 $R = \{S \rightarrow -S, S \rightarrow ((S)), S \rightarrow (S), S \rightarrow 4S, S \rightarrow x S, S \rightarrow y S, S \rightarrow x, S \rightarrow y, S \rightarrow S-S,$
 $S \rightarrow 8S, S \rightarrow S+S\}$

9. Let L_1 and L_2 be two DCFL's. Then $L_1 \cup L_2$ is guaranteed to be:
- Empty
 - Regular
 - Deterministic Context-Free
 - context-free
 - none of the above.
10. M_1 is a DFA such that $L(M_1) = \{w : w \in \{a,b\}^* \text{ and } w \text{ contains an even number of } a\text{'s and an even number of } b\text{'s}\}$.
- ADFA = $\{\langle M, w \rangle : M \text{ is a DFA, } w \in \{a,b\}^*, \text{ and } M \text{ accepts } w\}$
 A DFA = $\{\langle M \rangle : M \text{ is a DFA and } M \text{ accepts } a\}$
 EDFA = $\{\langle M \rangle : M \text{ is a DFA and } L(M) = \Sigma^*\}$
 ALLDFA = $\{\langle M \rangle : M \text{ is a DFA and } L(M) = \{a,b\}^*\}$
 EQDFA = $\{\langle M_1, M_2 \rangle : M_1, M_2 \text{ are DFAs AND } L(M_1) = L(M_2)\}$
 Is $\langle M_1 \rangle \in \text{ALLDFA}$?
- Yes
 - No
11. Assuming the constraint of the above problem, is $\langle M_1, M_1 \rangle \in \text{EQDFA}$?
- Yes
 - No
12. Consider the Language $\text{FINITE DFA} = \{\langle M \rangle : M \text{ is a DFA and } L(M) \text{ is finite}\}$.
 FINITE DFA is:
- not co-recursively enumerable
 - recursively enumerable
 - co-recursively enumerable
 - recursive
 - none of the above.
13. A deterministic finite automaton is a simple Language Recognition Device
- Yes
 - No
14. A context free grammar G is a quadruple (V, Σ, R, S) where V is an alphabet, Σ is a subset of V , R is a finite subset of $(V - \Sigma)^+ \Sigma^*$, S is an element of $V - \Sigma$.
- Yes
 - No

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QUIZ (CLOSED BOOK)

MAXIMUM MARKS: 10

Time: 30 MINUTES

DATE: 22.12.2003

ANSWER ALL QUESTIONS:

1. Let $L1 = \{e, a\}$ and $L2 = \{a, b\}$. $M = (L2^*)(L1)$. M is :
i) $\{e, a\}$ ii) $\{a, b\}$ iii) $\{e, b\}$ iv) $\{a, b\}^*$ v) None of the above
2. The String **abbab** is a member of the Language represented by which of the following regular expressions? ($\Sigma = \{a, b\}$)
i) $(a \cup b)$ ii) $b(a \cup b)^*$ iii) **abbab** iv) ϕ v) $(ab)^+(bb)^+(ba)^+b$
3. Let $A = \{a, b, c, d, e\}$. How many elements are there in $A \times A$.
i) 1 ii) 25 iii) 5 iv) 32
4. Let $M = (K, \Sigma, \Delta, s, F)$ be an NFA, where
 $K = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b, c, d\}$,
 $\Delta = \{(q_0, a, q_0), (q_0, e, q_1), (q_1, b, q_1), (q_1, c, q_1), (q_1, e, q_2), (q_2, d, q_2)\}$
 $S = q_0$, $F = \{q_0, q_1, q_2\}$.
The regular language corresponds to the language accepted by M is :
i) $(abcd)^*$ ii) $a^*b^*c^*d^*$ iii) $(a \cup b \cup c \cup d)^*$ iv) $a^*(b \cup c)^*d^*$ v) none of above
5. Assume that symbols in upper case (S, A, B etc.) are non-terminals, and that symbols in lower case (a, b, c , etc.) are terminals. Which of the following is a context-free rule?
i) $SA \rightarrow B$ ii) $SA \rightarrow S$ iii) $b \rightarrow SA$ iv) $S \rightarrow aS$
6. Assume that symbols in upper case (S, A, B etc.) are non-terminals, and that symbols in lower case (a, b, c , etc.) are terminals. Which of the following is a NOT context-free rule?
i) $S \rightarrow B$ ii) $S \rightarrow S$ iii) $b \rightarrow SA$ iv) $S \rightarrow aS$
7. How many derivations does the string **ab** have in G
i) 0 ii) 2 iii) 1 iv) 3
8. Let $L1$ and $L2$ be two DCFL's. Then $L1L2$ is guaranteed to be:
i) Empty ii) Regular iii) Deterministic Context Free iv) context-free
v) none of the above.

15. The tabular representation of the transition function can be represented in a graphical model or diagram, called as :
- i) Device Diagram ii) Machine Diagram iii) State Diagram iv) Standard diagram .
16. A Non-Deterministic Finite automaton can be much more convenient device to design than a deterministic finite automata.
- i) Yes ii) No
17. In a non-deterministic finite automaton, Δ is :
- i) Deterministic state ii) Transition relation iii) Transfer function
18. For each Non-deterministic finite automata there is an equivalent deterministic finite automaton.
- i) Yes ii) No
19. The Class of language accepted by finite automata is closed under
- a) Union, b) concatenation, c) Kleene star, d) complementation, e) intersection.
- i) a, c, d, e ii) b, d, e iii) a, b, d, e iv) a, b, c, d, e
20. The Context free languages are closed under union, concatenation, Kleene star, Intersection.
- i) Yes ii) No


(M. MADHATJAGAM)