BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year II - SEMESTER 2007-08

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH C222)

COMPREHENSIVE EXAMINATION - (Closed-Book)

Time: 03 Hours
Date: June 1, 2008

Max. Marks: 40
Weightage: 40 %

Note:- All questions are compulsory and should be answered sequentially. This question paper has two pages and four questions.

1. (a) *If it does not rain or if it is not foggy, then the sailing race will be held and the life saving demonstration will go on. If the sailing race is held, then the trophy will be awarded. The trophy was not awarded.* Can you conclude: It rained?

   (b) Consider 5 distinct points (x_i, y_i) with integer values, where $i = 1,\ 2,\ 3,\ 4,\ 5$. Show that the midpoint of at least one pair of these five points also has integer coordinates.

   (c) Solve the recurrence relation $a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 0$ for $n \geq 3$ and $a_0 = 1$ $a_1 = 4$ and $a_2 = 8$.

   (d) What is *a bipartite graph and a complete bipartite graph*? Give an example of each.

   (3+2.5+2.5+2)

2. (a) Use *generating function* to solve the recurrence relation:
   $a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$. for $n \geq 2$. and $a_0 = 2$ $a_1 = 1$

   (b) Find the general solution of the recurrence relation:
   $a_n - 7a_{n-1} + 12a_{n-2} = n4^n$ for $n \geq 2$.

   (c) Prove that the square of any integer is of the form $3n$ or $3n + 1$.

   (d) Show that every integer $n > 1$ can be written as a product of some series of s prime numbers.

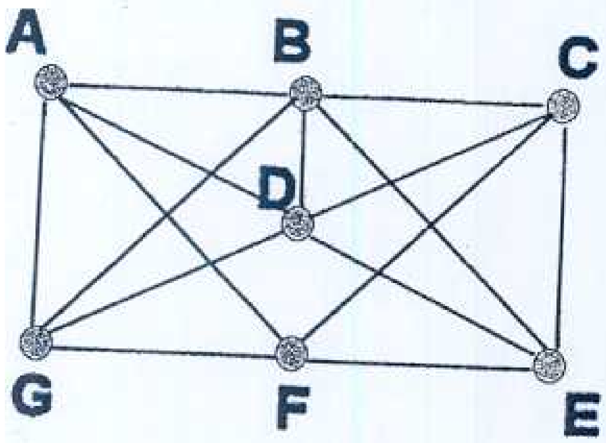
   (2.5 × 4)

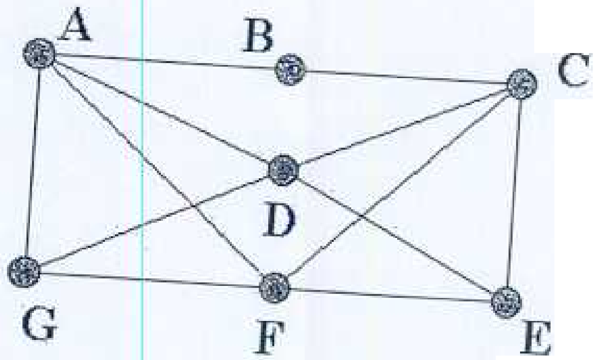
3. (a) Let $\{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$. Draw a *Hasse diagram* for the poset $(A; \mid)$. Determine all maximal and minimal elements and greatest and least elements if they exists. Is this relation on the set of integers a partial ordering? Explain.

   (b) Prove that for any polyhedral graph (a) $v \geq 2 + r/2$ (b) (a) $r \geq 2 + v/2$.

   (c) Given an example of a *totally ordered set* and a *well ordered set*.

(d) State the *Dirac's theorem* for *Hamilton's circuit*? Using Dirac's theorem, can you conclude that the following graphs have *Hamilton's circuit*? (3+2+2+3)





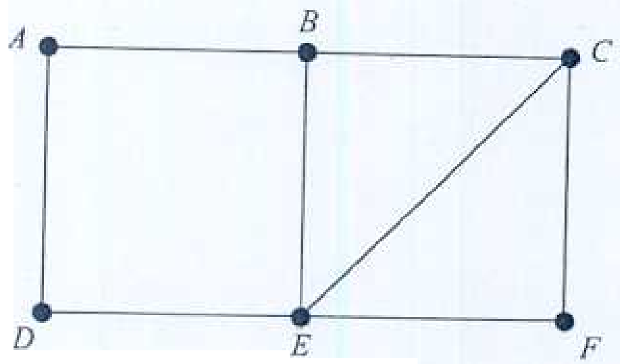
4. (a) Show that: (i) The *identity element* in a group G is unique. (ii) $(a^{-1})^{-1} = a$.

(b) Define a *cyclic group*. Prove that Z_7 with multiplication *modulo* 7 is an *abelian group* and which is a *cyclic* as well.

(c) State *Lagrange's theorem* for groups. Hence or otherwise prove that groups of prime order are *cyclic*.

(d) Let G be the graph as in the figure given below. Find all *simple paths* from vertex A to vertex F with *4 or more* edges and *k-cycle* that starts with vertex A for $k = 4, 5, 6$.

(2+ 3+2.5+2.5)



Good Luck . . .

BITS, PILANI - DUBAI CAMPUS

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2007-08)

DISCRETE STRUCTURES FOR COMPUTER SCIENCE

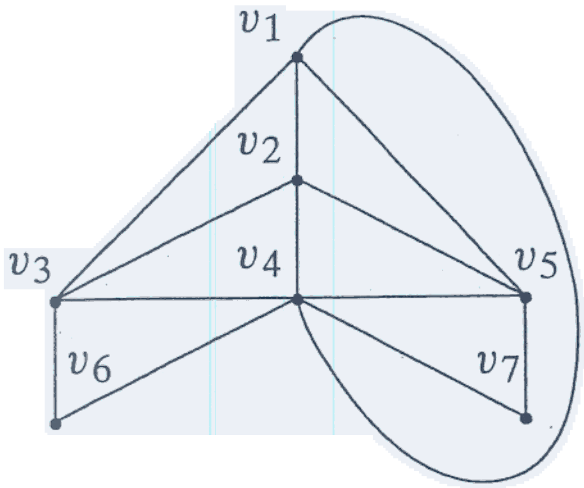
(MATH UC222) TEST - II (Open-Book)

Time: 50 Minutes
Date: May 04, 2008

Max.Marks: 20
Weighage: 20 %

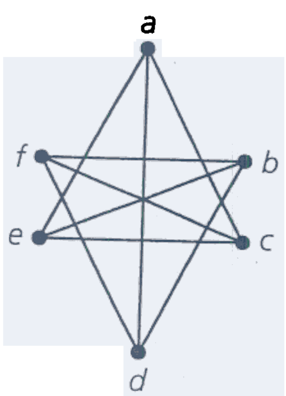
Note: 1. Text-book and class notes are allowed. 2.Solve all the four questions.

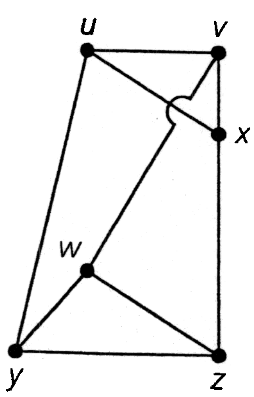
1. (a) Determine whether the following graph has an Euler circuit, an Euler path but no Euler circuit, or neither. Give reasons for your choice.



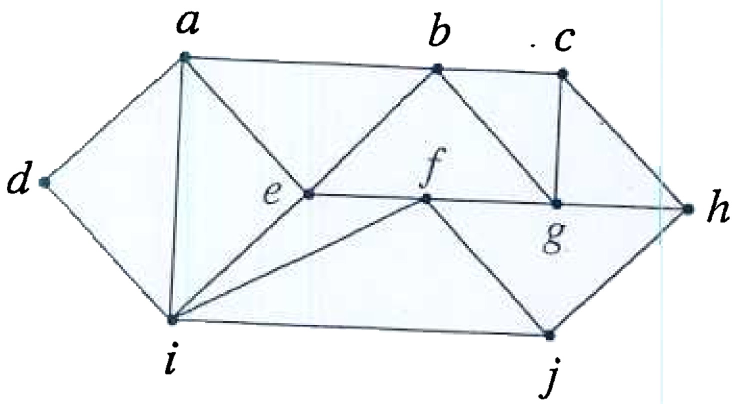
(b) Suppose that a connected planar simple graph with e edges and v vertices and each cycle of length at least k. Show that $e \leq \frac{k}{(k-2)}(v-2)$. (2.5 + 2.5)

2. (a) Determine whether the given pair of graphs is Isomorphic? Justify.





(b) Determine whether the following graph has an Hamiltonian circuit, an Hamiltonian path but no Hamiltonian circuit, or neither. Give reasons. (2.5 + 2.5)



3. (a) Let $A = \{1, 2, 3, 4, 8\}$. Let R be a relation on A such that aRb iff $a + b \leq 19$. Draw a diagraph of the relation. Also prove that R an equivalence relation.

   (b) Draw the complete graph on 7 vertices. Is it a planar graph? Explain. (3 + 2)

4. (a) Draw the Hasse diagram for the poset $(A, \subseteq)$, where $A = \mathcal{P}(\mathcal{U})$ with $\mathcal{U} = \{1, 2, 3, 4\}$. Let $B = \{\{1\}, \{2\}, \{3\}, \{1, 2\}\}$ be subset of A. Determine the lub and glb of B. Is $(A, \subseteq)$ a lattice? Explain.

   (b) Is the graph K_1 a complete bipartite? Justify. (3.5 + 1.5)

Good Luck .

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DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2007-08)

DISCRETE STRUCTURE FOR COMPUTER SCIENCE

(MATH UC222)

Test - I (Closed-Book)

Time: 50 Minutes

March 23, 2008

Max. Marks: 25

Weightage: 25 %

Note: 1. Answer all the questions sequentially. 2. Calculators are not allowed

1 (a) Is the compound statement $[p \to (q \to r)] \to [(p \to q) \to (p \to r)]$ a *tautology*? Justify.

(b) Let n be an integer. Prove that n is odd if and only if $7n + 8$ is odd. (2+3)

2. (a) Prove or disprove the following arguments and explain which *rules of inference* are used for each step. The universe is the set of living thing.
"Every living thing is a plant or an animal. David's dog is alive and it is not a plant. All animals have hearts. Hence, David's dog has a heart". (4)

3. (a) Prove that if any six numbers from 1 to 9 are chosen, then two of them will must add to 10.

(b) The nth term of the *Fibonacci series* would be $a_n = a_{n-1} + a_{n-2}$ for all n ≥ 3 (with $a_1 = a_2 = 1$). Prove that $a_n < (7/4)^n$ for all n ≥ 1. (2+3)

4. (a) Find the coefficient of x^{25} in $(X^2 + X^3 + \ldots + X^6)^7$.

(b) Write the *recurrence relation*, if it's general solution for t_n is $t_n = B_1 2^n + B_2 n 2^n + D_1 3^n + D_2 n 3^n$.

(2+3)

5. (a) Solve the *recurrence relation* by method of *generating function*: $a_n + a_{n-1}$ - $16a_{n-2} + 20a_{n-3} = 0$; $n \geq 3$ with initial conditions $a_0 = 0$, $a_1 = 1$ and $a_2 = -1$

(b) State any two *fallacies* of *logical inferences*.

(4+2)

Good Luck .