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**BITS PILANI, DUBAI CAMPUS**  
**II Semester (2013-2014)**  
**MATHEMATICS - III (MATH F211)**  
**Comprehensive examination (Closed book)**

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Time: 3 Hours  
Date: 26-05- 2014

Max.Marks: 120  
Weightage: 40%

Answer all the questions

**SECTION-A (40 Marks)**

1. Solve the following linear equation  $xdy + ydx = x \cos(x)dx$  (6M)
2. The equation  $(1 - x^2)y'' - 2xy' + 2y = 0$  has  $y_1 = x$  as an obvious solution. Find the general solution (8M)
3. Find the general solution of  $y''' - 3y'' + 4y' - 2y = 0$  (6M)
4. Using operator method find only the particular solution of  $y'' - 7y' + 12y = e^{2x}(x^3 - 5x^2)$  (10M)
5. Transform the following differential equation into a differential equation with constant coefficient and find the general solution.  
 $4x^2y'' - 3y = 0$  (10M)

**SECTION-B (40 Marks)**

1. Solve  $y'' + xy = 0$  using power series solution (10M)
2. Find two independent Frobenius series solutions at the regular singular point  $x=0$  for  $x^2y'' + xy' + \left(x^2 - \frac{1}{4}\right)y = 0$  (12M)
3. Show that  $J_{-\left(\frac{1}{2}\right)}(x) = \sqrt{\frac{2}{\pi x}} \cos(x)$  where  $J_p(x)$  denotes Bessel function of order p (8M)
4. Show that  $T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$  and hence find  $T_4(x)$  in terms of  $x$  where  $T_n(x)$  is  $n^{th}$  degree Chebyshev polynomial. (10M)

**SECTION-C** (40 Marks)

1. Find the Fourier series of the function defined as  $f(x) = |x|$  for  $-2 \leq x \leq 2$  and hence deduce the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$  (12M)

2. Find eigen values and eigen functions for the following Sturm Liouville's differential equation

$$y'' + \lambda y \text{ with the boundary conditions } y(0) = 0 \text{ and } y\left(\frac{\pi}{4}\right) = 0 \quad (8M)$$

3. By using Laplace transformation solve the differential equation

$$\frac{d^2 y}{dx^2} + 4y = 4x \text{ with initial conditions } y(0) = 1 \text{ and } y'(0) = 5. \quad (10M)$$

4. Find the general solution of the following liner system of differential equations

$$\frac{dx}{dt} = x - 2y$$

$$\frac{dy}{dt} = 4x + 5y$$

(10M)

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**BITS PILANI, DUBAI CAMPUS**

**II Semester (2013-2014)**

**MATHEMATICS - III (MATH F211)**

**Test II (Open book)**

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Time: 50 Minutes  
Date: 23-04- 2014

Max. Marks: 60  
Weightage: 20%

Answer all the questions

1. Solve  $3y' - y = y^4 \cos(3x)$  (10M)
  2. Solve  $x^2 y'' + 2xy' = \log(x)$  by the method of reduction of order (10M)
  3. Find the particular solution of the differential equation  $y'' - 2y' + y = \frac{e^x}{(1+x^2)}$  using the method of variation of parameters (10M)
  4. Find the complete solution of the differential equation  $y'' + 3y' - 28y = e^{-7x}$  by the method of undetermined coefficients (10M)
  5. Solve by operator method  $(2D^2 - D - 3)y = x^3 + x + 1$  (10M)
  6. Solve by power series method  $(1 - x^2)y' = 2xy$  (10M)
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**BITS PILANI, DUBAI CAMPUS****II Semester (2013-2014)****MATHEMATICS - III (MATH F211)****Test I (Closed book)**

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Time: 50 Minutes

Date: 26-02- 2014

Max. Marks: 75

Weightage: 25%

Answer all questions

1. Find the Laplace transform of  $f(x) = \begin{cases} \cos(x) & \text{in } 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{in } x > \frac{\pi}{2} \end{cases}$  (8M)

2. Find  $L\left[\int_0^x e^{-x} \cos(2x) dx\right]$  (7M)

3. Find  $L^{-1}\left[\frac{p+7}{p^2+10p+41}\right]$  (10M)

4. Find convolution of  $e^{-3t}$  and  $t^2$ .  
Also verify convolution theorem for the above given pair of functions (15M)

5. Solve  $y'' + 3y' + 2y = e^{-3t}$  given  $y(0) = 0, y'(0) = 0$  (15M)

6. Find the Fourier series of  $f(x) = \begin{cases} 0 & ; -\pi \leq x < 0 \\ \cos(x) & ; 0 \leq x \leq \pi \end{cases}$  where  $f(x) = f(x + 2\pi)$ .

Also sketch the graph of  $f(x)$  between  $(-3\pi, 3\pi)$  (20M)

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Date: 07.04.2014  
MAXIMUM: 21 MARKS

MATHEMATICS III (MATH F211) QUIZ-II

DURATION: 20 MINUTES

NAME: \_\_\_\_\_ ID: \_\_\_\_\_ Instructor's Name: \_\_\_\_\_

Answer all the questions:

1. Find indicial equation and roots for the following differential equation  $x^2 y'' + 3 \sin(x) y' + y = 0$
2. Classify the nature of singular points (with justification) for the following differential equation  $(x - 1)^2(x - 2)^2 y'' + (x - 1)y' + (x - 2)y = 0$

3. Find the general solution for the following differential equation near  $x = 0$  in terms of Hypergeometric function.

$$4x(1-x)y'' + (6-9x)y' - y = 0$$

4. Find the recurrence relation by the power series method for  $y'' - (1+x^2)y = 0$  near the ordinary point  $x = 0$

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SECOND SEMESTER 2013 – 2014

B

Date: 7.05.2014

MATHEMATICS III (MATH F211) QUIZ-II

MAXIMUM: 21 MARKS

DURATION: 20 MINUTES

NAME: _____	ID: _____	Instructor's Name: _____
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Answer all the questions

- 1) Find indicial equation and roots for the following differential equation  $x^2 y'' - 3 \{\cos(x) - 1\} y' - 6y = 0$
2. Classify the nature of singular points (with justification) for the following differential equation  
$$x^2(x - 4)^2 y'' + xy' + 2y = 0$$

3. Find the general solution for the following differential equation near  $x=0$  in terms of Hypergeometric function.

$$8x(1-x)y'' + (4-14x)y' - y = 0$$

4. Find the recurrence relation by the power series method for  $y'' + (1-x^2)y = 0$  near the ordinary point  $x=0$



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SECOND SEMESTER 2013 – 2014

A

Date: 19.03.2013  
MAXIMUM: 24 MARKS

MATHEMATICS III (MATH F211) QUIZ-I

DURATION: 20 MINUTES

NAME: _____	ID: _____	Instructor's Name: _____
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Answer all the questions:

1. Given  $f(x) = \begin{cases} -\pi & ; -\pi \leq x < 0 \\ x & ; 0 < x \leq \pi \end{cases}$  (5M)

Sketch the graph of  $f(x)$  in the interval  $(-\pi, \pi)$  and also find the value to which the Fourier series of  $f(x)$  converges at  $x=0$

2. Given  $f(x) = \begin{cases} 2 & ; -\pi \leq x < 0 \\ -2 & ; 0 \leq x \leq \pi \end{cases}$  (5M)

Find the value of  $b_n$  in the Fourier series of  $f(x)$

3. Find the Integrating factor which converts the following differential equation into an exact one.

$$(2x^3y^2 + 4x^2y + 2xy^2 + xy^4 + 2y)dx + (2y^3 + 2x^2y + 2x)dy = 0 \quad (5M)$$

4. Solve  $xdy + ydx = (1 - x^2)dx$

(5M)

5. A tightly stretched elastic string, tied at both ends, is distorted at time  $t=0$  to the shape  $g(x)$ ;  $0 \leq x < 2\pi$  and then released from rest, to start vibratory movement in the string. Write all the boundary and initial conditions of the corresponding one dimensional wave equation. (4M)