BITS, PILANI – DUBAI CAMPUS,
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI
SECOND SEMESTER 2012 – 2013
EEE / INSTR / ECE F243 Signals & Systems
COMPREHENSIVE EXAMINATION (CLOSED BOOK)
MAXIMUM MARKS: 80 WEIGHTAGE: 40%
DATE: 08/06/13 DURATION: 3 Hours

1. Determine the power of the signal given by the expression $f(t) = 10 \sin 5t \cdot \cos 10t$ **[4 marks]**

2. Sketch the signals (i) $f(t)= t^2 [u(t-6)-u(t-7)]$
(ii) $f(t)= (t-3)[u(t-4)-u(t-5)]$ **[4 marks]**

3. Find the convolution integral for the functions given by $f_1(t) = e^{-t}$ and $f_2(t)= \sin 3t$, for $t>0$ **[10 marks]**

4. Find the impulse response across the capacitor voltage (V_c (t)) for the circuit shown in Figure.1, with unit step function as an input to the circuit. **[8 marks]**

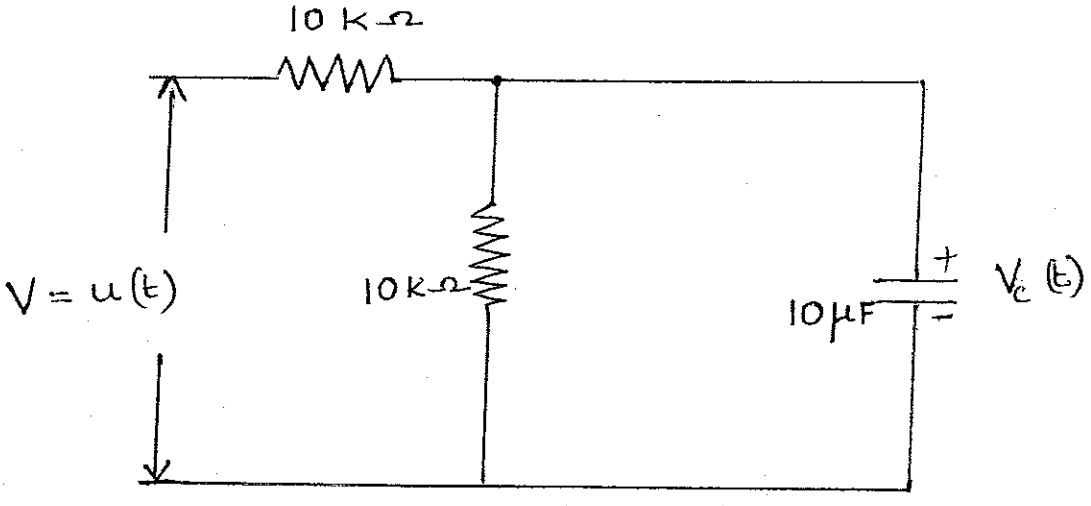


Figure.1

5. Find the Fourier transform of unit step function u(t) **[6 marks]**

6. Find the zero-input component of the response, y (t) for a LTIC system specified by the equation $(D^2+4D+4) = Df(t)$ for $t \geq 0$, if the initial conditions are $y(0)=3$ and $\frac{dy(0)}{dt} = -4$. **[6 marks]**

7. Transform the circuit shown in Figure 2 to its equivalent Laplace domain and obtain the expression for current $I_L(t)$. Assume initial energy stored in the circuit is zero. **[10 marks]**

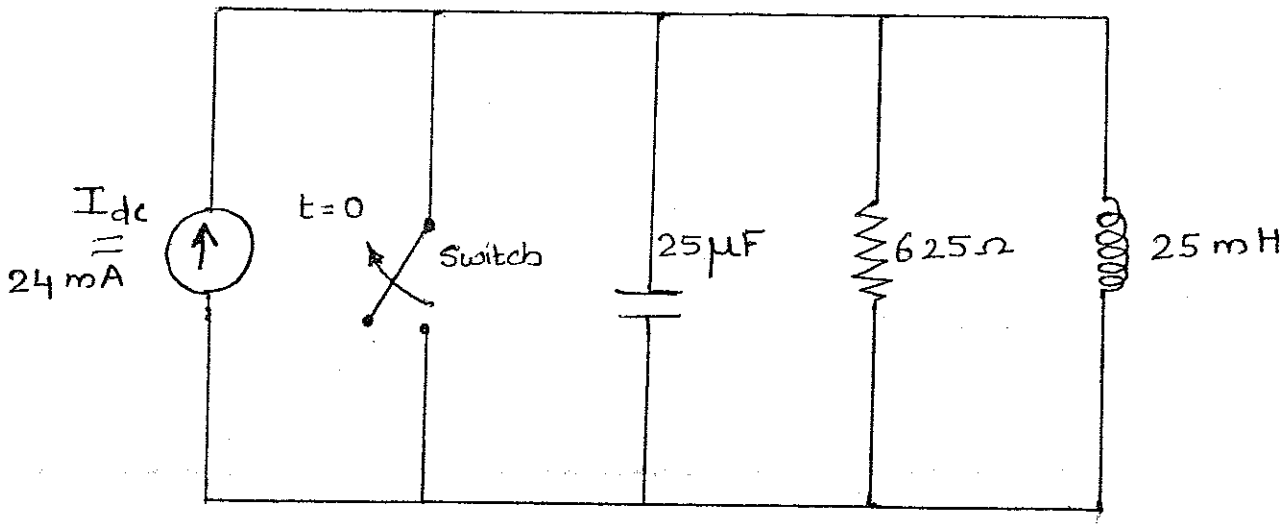


Figure.2

8. Find the inverse z-transform of $F[z] = \frac{z(2z^2-11z+12)}{(z-1)(z-2)^3}$ **[8 marks]**

(Trigonometric)

9. Find the Fourier series for the function given in Figure 3. **[8 marks]**

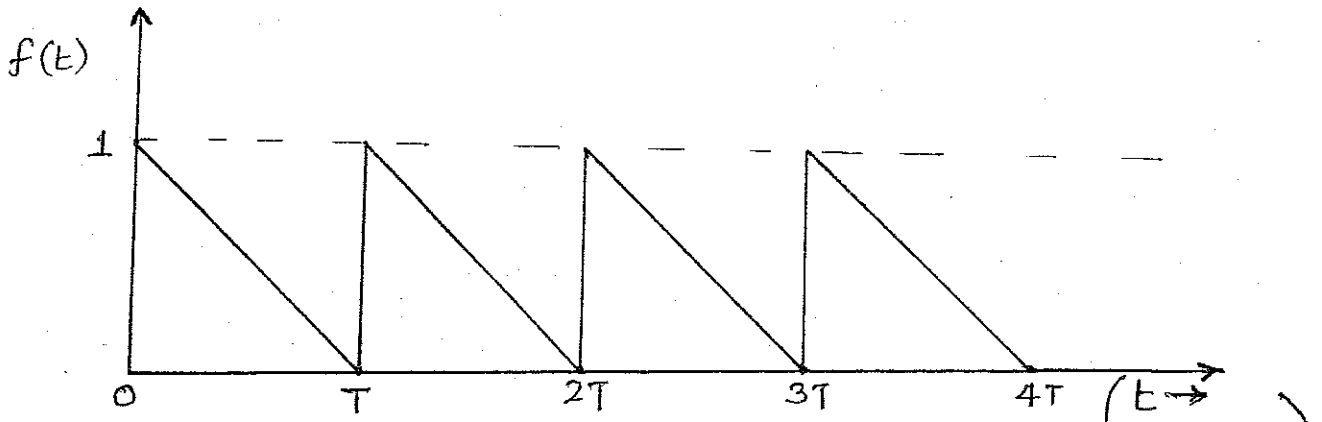


Figure.3 where, T = 1 ;

10. Obtain the canonical realization for the following transfer function:

a) H(s)= $\frac{4s-28}{s^2-6s-5}$ b) H(s) = $\frac{s-5}{s+7}$ **[4 marks]**

11. Find the inverse Laplace transform of $F(s)=\frac{s+3+5e^{-2s}}{(s+1)(s+2)}$ **[6 marks]**

12. Write short notes on:
    a) Sampling Theorem
    b) Butterworth and Chebychev Filters
    c) Deterministic and Random signals **[2+2+2 marks]**

08/06/2013

II - SEMESTER 2012-13.

EEE F243 SIGNALS & SYSTEMS

COMPREHENSIVE EXAMINATION (CLOSED BOOK)

ANSWER KEY.

1) $f(t) = 10 \sin 5t \cos 10t$.

$= \frac{10}{2} [\sin(5+10)t + \sin(5t-10t)]$

$= 5 [\sin 15t - \sin 5t]$

Power of a signal. = $\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} f^2(t)\, dt$ ⟶ [2 MARKS]

$\text{Limit}_{T\to\infty} 25 \int_{-T/2}^{T/2} (\sin 15t - \sin 5t)^2\, dt$

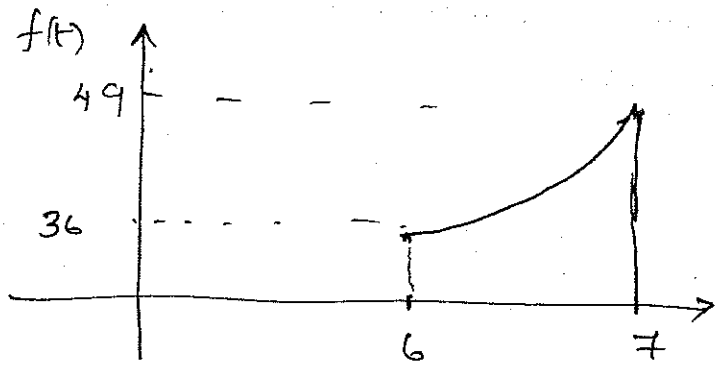
$\text{Limit}_{T\to\infty} 25 \int_{-T/2}^{T/2} (\sin^2 15t + \sin^2 5t + 2 \sin 15t \cdot \sin 5t)\, dt$

$= \lim_{T\to\infty} \frac{25}{T} \int_{-T/2}^{T/2} \left(\frac{1}{2} + \frac{1}{2}\right) dt + 0$

$= \lim_{T\to\infty} \frac{25}{T} \left[t\right]_{-T/2}^{T/2}$

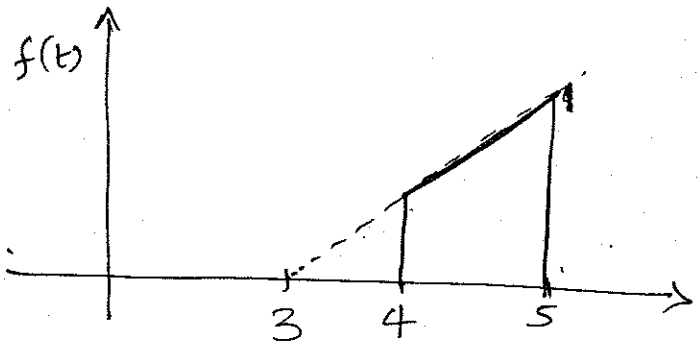
$= \lim_{T\to\infty} \frac{25}{T} \cdot T = \underline{\underline{25}}$ ⟶ [2 MARKS]

2] (i) $t^2 [u(t-6) - u(t-7)]$



→ [2 MARKS].

(ii) $(t-3)[u(t-4) - u(t-5)]$



→ [2 MARKS]

3] $I = \int_0^t e^{-\tau} \sin 3(t-\tau)\, d\tau$

$= \left\{ \sin 3(t-\tau)(-1)e^{-\tau} \right\} \Big|_{\tau=0}^{\tau=t}$

$- \int_0^t \left\{ (3) \cos 3(t-\tau) \int e^{-\tau} d\tau \right\} d\tau.$

$$= \sin 3t * (-3) \int_0^t e^{-\tau} \cos 3(t-\tau)\, d\tau$$

$$= \sin 3t - 3\left[-e^{-\tau}\cos 3(t-\tau)\right]_0^t$$

$$- \left\{\left\{\int_0^t 3\sin 3(t-\tau) \int e^{-\tau} d\tau\right\} d\tau\right.$$

$$= \sin 3t - 3\left[-\left\{e^{-t} - \cos 3t\right\} + 3\int_0^t \sin 3(t-\tau) e^{-\tau} d\tau\right.$$

$$= \sin 3t + 3e^{-t} - 3\cos 3t - 9I. \longrightarrow \text{(5 MARKS)}$$

put $t-\tau = z$.

$$\frac{d}{d\tau}\left\{\sin 3(t-\tau)\right\} = \frac{d}{d(t-\tau)} \sin 3(t-\tau)\,(-1)$$

$$= \left\{\frac{d}{dz}\sin 3z\right\}(-1) = -3\cos 3(t-\tau)$$

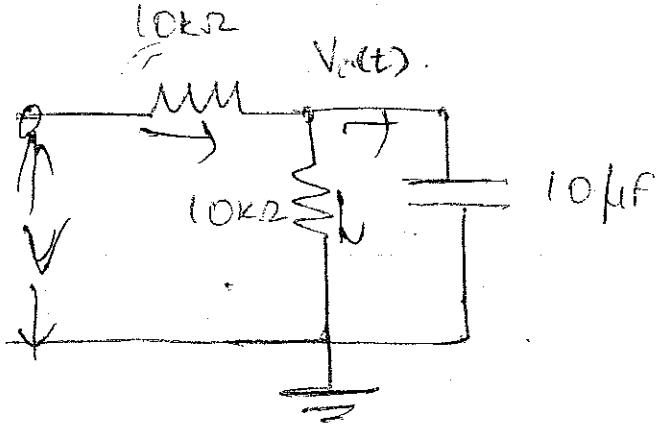
$$I = \sin 3t - 3\cos 3t + 3e^{-t} - 9I.$$

$$10I = \sin 3t - 3\cos 3t + 3e^{-t}$$

$$I = \frac{1}{10}\left[\sin 3t - 3\cos 3t + 3e^{-t}\right]$$

$\longrightarrow$ [5 MARKS]

4]



To find impulse response we differentiate the step response.

Step — $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$

$$\frac{V - V_c}{10k} = \frac{V_c}{10k} + C \cdot \frac{dV_c}{dt}$$

→ [2 MARKS]

$$\frac{V - V_c}{10k} = \frac{V_c}{10k} + 10^{-5} \frac{dV_c}{dt}$$

$$\frac{V}{10k} = \frac{2V_c}{10k} + 10^{-5} \frac{dV_c}{dt}$$

$$\Rightarrow V = 2V_c + 10^{-1} \frac{dV_c}{dt}$$

$$\Rightarrow 10u(t) = 20V_c + \frac{dV_c}{dt}$$

As $u(t) = 1$

$$\frac{dV_c}{dt} = 10 - 20V_c$$

$$\Rightarrow \frac{dV_c}{10 - 20V_c} = dt$$

After integrating

$$\Rightarrow \frac{\log(10 - 20V_c)}{20} = -t + C$$

$10 - 20V_c = C \cdot e^{-20t}$ ⊛ —— (2 MARKS)

At $t = 0$.

$V_c(0) = 0$.

$10 = C \cdot e^{-20 \times 0}$

$\Rightarrow C = 10$

$10 - 20V_c(t) = 10e^{-20t}$

$\Rightarrow V_c(t) = \frac{10 - 10e^{-20t}}{20} = V_c(t)$.

$\Rightarrow V_c(t) = \frac{1 - e^{-20t}}{2}$ ⟶ 2 MARKS

The derivative of the step response give impulse response:

$V_{s(t)} = \frac{dV_c(t)}{dt} = \frac{e^{-20t}}{2} \cdot 20 = \frac{10e^{-20t}}{}$.

⟶ [2 MARKS]

5]

$$U(\omega) = \int_{-\infty}^{\infty} u(t) e^{-j\omega t} dt.$$

$$= \int_{0}^{\infty} e^{-j\omega t} dt = \frac{-1}{j\omega} e^{-j\omega t} \Big|_{0}^{\infty}.$$

$$u(t) = \lim_{a \to 0} e^{-at} u(t)$$

$$U(\omega) = \lim_{a \to 0} F\{ e^{-at} u(t) \} = \lim_{a \to 0} \frac{1}{a + j\omega}.$$

$$U(\omega) = \lim_{a \to 0} \left[\frac{a}{a^2 + \omega^2} - j \cdot \frac{\omega}{a^2 + \omega^2} \right]$$

$$= \lim_{a \to 0} \left[\frac{a}{a^2 + \omega^2} \right] + \frac{1}{j\omega}.$$

$$\int_{-\infty}^{\infty} \frac{a}{a^2 + \omega^2} d\omega = \tan^{-1} \frac{\omega}{a} \Big|_{-\infty}^{\infty} = \pi$$

Thus $U(\omega) = \pi \delta(\omega) + \frac{1}{j\omega}$ ⟶ [6 MARKS]

6] $[D^2 + 4D + 4] = 0.$

$\lambda_1 = \lambda_2 = \underline{2}$

$y(t) = (c_1 + c_2 t) e^{-2t}$ for $t > 0$

To find c_1 and c_2.

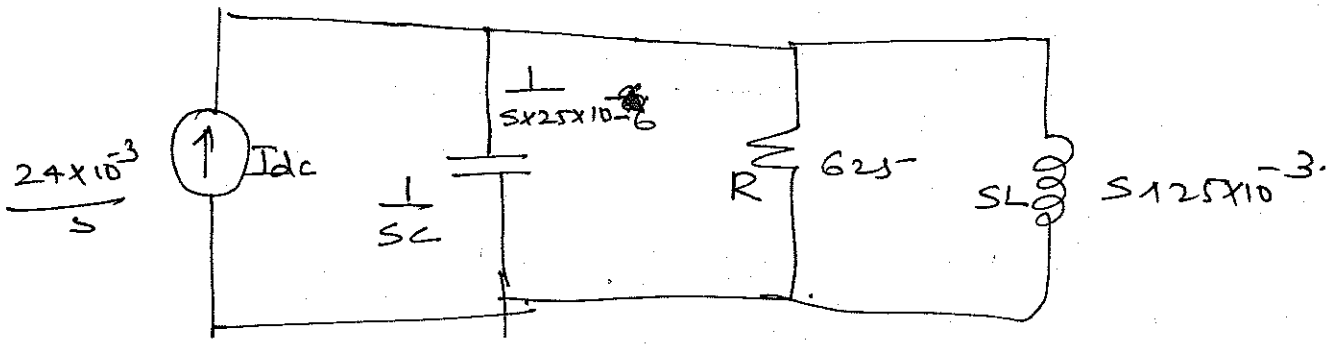
$y(0) = 3 \qquad \dot{y}(0) = -4$

$c_1 = 3 \qquad c_2 = 2.$ ⟶ [3 MARKS]

$y(t) = \underline{(3+2t)e^{-2t}}$ for $t \geq 0$ see

↳ 3 [MARKS]

7]



By applying kirchoffs current law we get

$$sCV + \frac{V}{R} + \frac{V}{sL} = \frac{Idc}{s}.$$

$$V = \frac{Idc/C}{s^2 + \left(\frac{1}{RC}\right)s + \frac{1}{LC}}.$$

and $I_L = \frac{V}{sL}.$ ⟶ [2 MARKS]

Substituting.

$$I_L = \frac{I_{dc}/LC}{s\left[s^2 + \left(\frac{1}{RC}\right)s + \left(\frac{1}{LC}\right)\right]}$$

$$I_L = \frac{384 \times 10^5}{s(s^2 + 64000s + 16 \times 10^8)} \longrightarrow$$

$$I_L = \frac{384 \times 10^5}{s(s + 32000 - j24000)(s + 32000 + j24000)} \longrightarrow (2 \text{ MARKS})$$

$$I_L = \frac{K_1}{s} + \frac{K_2}{(s + 32000 - j24000)} + \frac{K_2^*}{(s + 32000 + j24000)}$$

$K_1 = 24 \times 10^{-3}$

$K_2 = 20 \times 10^{-3} \angle 126.87^\circ \longrightarrow [3 \text{ MARKS}]$

$$i_L = \left[24 + 40e^{-32,000t} \cos(24000t + 126.87^\circ)\right] \text{mA}$$

$\longrightarrow$ 3 MARKS]

8) $$\frac{F[z]}{z} = \frac{2z^2 - 11z + 12}{(z-1)(z-2)^3}$$

$$= \frac{K_1}{z-1} + \frac{a_0}{(z-2)^3} + \frac{a_1}{(z-2)^2} + \frac{a_2}{(z-2)}$$

$k = -3$

$q_0 = -2$

$q_2 = 3$

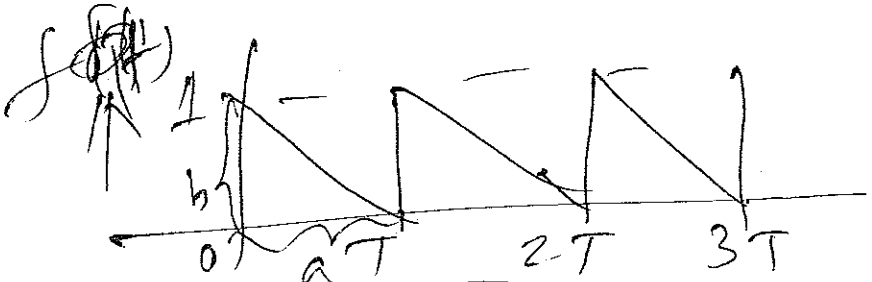
$q_1 = -1.$

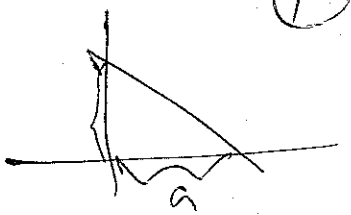
} ⟶ [4 MARKS] using partial fractions

$$f[k] = -\left[3 + \frac{1}{4}\left(k^2 - k - 12\right) 2^k\right] u[k]$$

⟶ [4 MARKS]

9]





$T = 1$

$f(t)$

$\frac{x}{a} + \frac{y}{b} = 1$

$x = t$

$y = f(t)$

$a = T, \quad b = 1$

$\frac{t}{T} + \frac{f(t)}{1} = 1$

$f(t) = 1 - \frac{t}{T} = 1 - \frac{t}{1} = 1 - t.$

$$a_n = \frac{2}{1}\int_0^1 (1-t)\cos n\omega t\, dt$$

$$= 2\left[\frac{\sin n\omega t}{n\omega}\Big|_0^1 - \int_0^1 (t)\cos n\omega t\, dt \right]$$

$$= 2\left[\frac{1}{n\omega}\sin n\omega - \left\{ t\,\frac{\sin n\omega t}{n\omega}\Big|_0^1 - \int_0^1 \left\{ (1)\int \cos n\omega t\, dt \right\} dt \right\} \right]$$

$$= 2\left[\frac{\sin n\omega}{n\omega} - \left\{ \frac{1}{n\omega}\sin n\omega - \frac{1}{n\omega}\int_0^1 \sin n\omega t\, dt \right\} \right]$$

$$= 2\left[\frac{\sin n\omega}{n\omega} - \left\{ \frac{\sin n\omega}{n\omega} + \frac{1}{(n\omega)^2}(\cos n\omega - 1) \right\} \right]$$

$$= 2\left[\frac{\sin n\omega}{n\omega} - \frac{\sin n\omega}{n\omega} + \frac{1}{(n\omega)^2}(1-\cos n\omega)\right]$$

$$= \frac{2}{(n\omega)^2}\left[1-\cos n\omega\right] \longrightarrow (3\text{MARKS})$$

$$= \frac{2}{(n\omega)^2}[1-1] \quad \omega = \frac{2\pi}{T} = 0 \longrightarrow$$

$$b_n = \frac{2}{1}\int_0^1 (1-t)\sin n\omega t\, dt$$

$$= 2\left[-\frac{\cos n\omega t}{n\omega}\Big|_0^1 - \int_0^1 +\sin n\omega t\, dt\right]$$

$$= 2\left[\left(-\frac{1}{n\omega}\right)(\cos n\omega - 1) - \left\{\frac{t\,(-\cos n\omega t)}{n\omega}\Big|_0^1 - \int_0^1 \{(1)\int \sin n\omega t\, dt\}\, dt\right\}\right]$$

$$= 2\left[-\frac{\cos n\omega}{n\omega} + \frac{1}{n\omega} + \left(\frac{1}{n\omega}\right)\cos n\omega - \frac{1}{(n\omega)^2}\sin n\omega t\Big|_0^1\right]$$

$$= 2\left[\frac{1}{n\omega} - \frac{1}{(n\omega)^2}\sin n\omega t\right] = \frac{2}{2n\pi} = \frac{1}{n\pi} = b_n \longrightarrow (3\text{MARKS})$$

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\int_0^T t \Rightarrow (1-t)\,dt$$

$$= 1\int_0^1 (1-t)\,dt$$

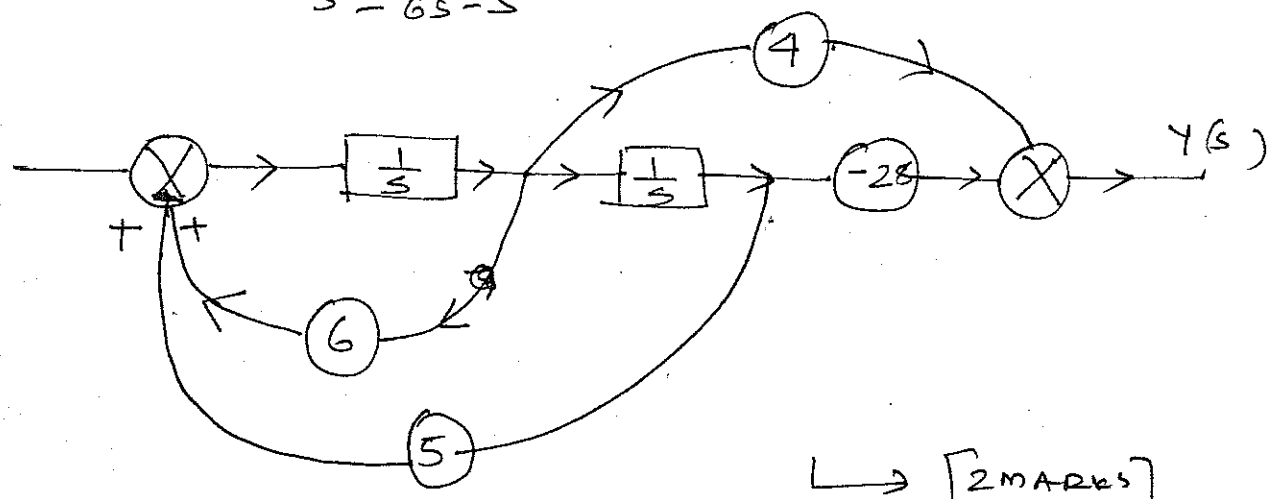
$$= \left[t - \frac{t^2}{2}\right]\Big|_0^1$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$ —— [2 MARK]

10]

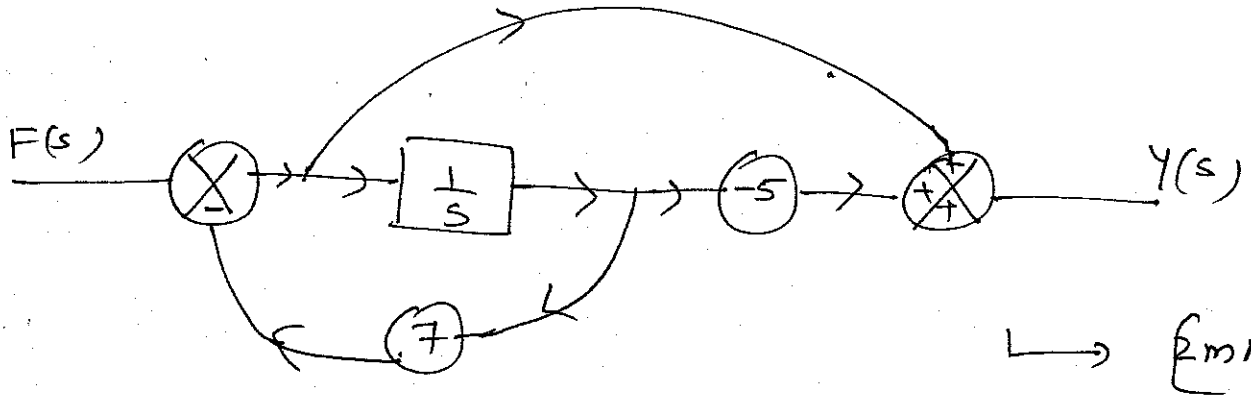
$$H(s) = \frac{4s - 28}{s^2 - 6s - 5}$$

a)



↳ [2 MARKS]

b) $H(s) = \frac{s-5}{s+7}$



↳ [2 MARKS]

11]

$$F(s) = \underbrace{\frac{s+3}{(s+1)(s+2)}}_{F_1(s)} + \underbrace{\frac{5e^{-2s}}{(s+1)\cdot(s+2)}}_{F_2(s)\cdot e^{-2s}}$$

$$F_1(s) = \frac{2}{s+1} - \frac{1}{s+2}$$

$$F_2(s) = \frac{5}{s+1} - \frac{5}{s+2}$$

$$\left.\begin{aligned} f_1(t) &= (2e^{-t} - e^{-2t})\,u(t) \\ f_2(t) &= 5(e^{-t} - e^{-2t})\,u(t) \end{aligned}\right\} \rightarrow (3 \text{ marks})$$

because $F(s) = F_1(s) + F_2(s)e^{-2s}$.

$$f(t) = f_1(t) + f_2(t-2)$$

$$f(t) = (2e^{-t} - e^{-2t})\,u(t) + 5\left[e^{-(t-2)} - e^{-2(t-2)}\right] u(t-2)$$

$\rightarrow$ (3 marks)

12) (a) Standard definition & mathematical expressions given in text book

(b) "

(c) "

$\rightarrow$ (2+2+2 = 6 MARKS)

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SECOND SEMESTER 2012 – 2013
EEE / INSTR / ECE F243 Signals & Systems
TEST 2(OPEN BOOK)

MAXIMUM MARKS: 40 **WEIGHTAGE: 20%**
DATE: 06/05/13 **DURATION: 50 MINUTES**

1. Find the convolution integral of $f_1(t)$ and $f_2(t)$ given that $f_1(t) = (1-e^{-at})$ and $f_2(t)=1$; for $a > 0$. **[10 marks]**

2. Find the Trigonometric Fourier series for the function shown in Figure.1. **[10 marks]**

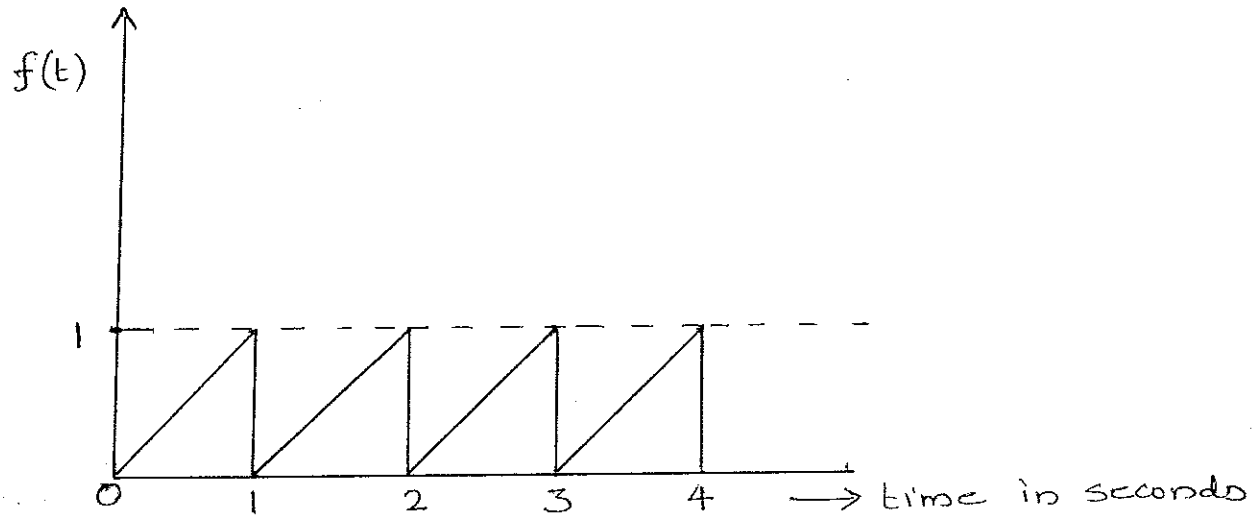

Figure 1

3. Find the Fourier transform for the function $f(t)= e^{-at} \cos\omega_o t.u(t)$
(Show appropriate mathematical steps to arrive at the answer.)
[10 marks]

4. Find the minimum sampling frequency to digitize the signal given by the expression $S = A_c.\text{Cos}(\omega_o.t)$

Where; $\omega_o = \omega_c + A_m.m(t)$

$A_c = 100$

$A_m = 50$ and $\omega_c = 10\text{KHz}$ **[10 marks]**

The signal m(t) is shown in Figure. 2.

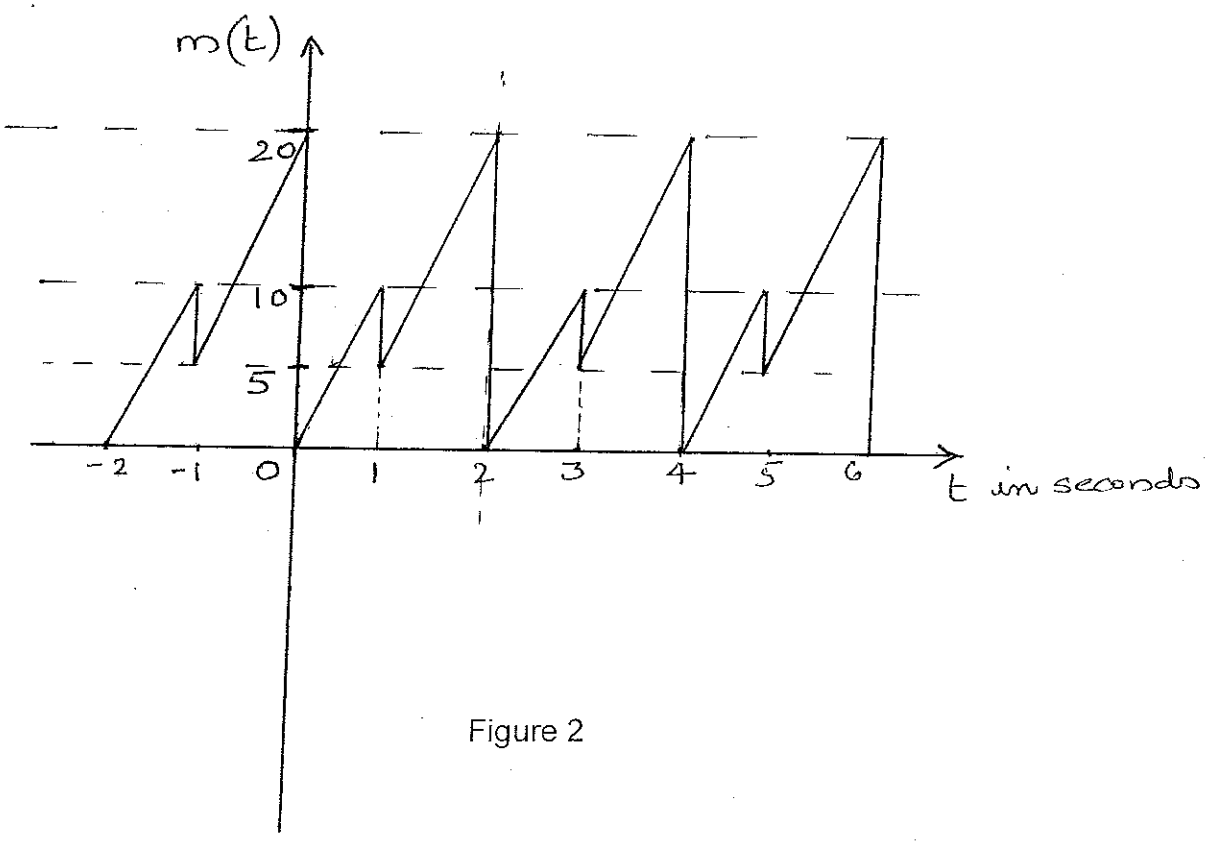

Figure 2

Test - 2 (open book) 2 hrs 06/05/13

EEE/INSTR/ECE F243 SIGNALS AND SYSTEMS

1) $f_1(t) * f_2(t)$.

$$f_1(t) = 1 - e^{-at}$$

$$f_2(t) = 1$$

$$f_1(t) * f_2(t) = \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$$

→ [2 MARKS]

$$= \int_0^t (1 - e^{-a\tau}) 1\, d\tau$$

$$= [t-0] - \int_0^t e^{-a\tau}\, d\tau$$ → [2 MARKS]

$$= t + \frac{1}{a}\left(e^{-at} - e^0\right)$$

$$= t + \frac{1}{a}\left(e^{-at} - 1\right)$$ → [2 MARKS]

2]

$$a_0 = \frac{1}{T}\int_0^T (t)\, dt$$

$$= \frac{1}{T}\int_0^1 t\, dt$$

$$= \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}(1-0)$$

$$= \frac{1}{2}$$ → [2 MARKS]

$$\omega = \frac{2\pi}{T} = 2\pi$$

$$T = 1 \text{ sec}$$

$$a_n = \frac{2}{1} \int_0^1 (t) \cos n\omega t \, dt$$

$$= 2\left[\frac{1}{n\omega} (t) \left\{ \sin n\omega 1 - \sin n\omega_0 \right\} \right]$$

$$- 2 \int_0^1 \left\{ (1) \int \cos n\omega t \, dt \right\} dt$$

$$= 2 \left(\frac{t}{n\omega}\right) \sin 2n\pi + \frac{2}{(n\omega)^2} \left[\cos n\omega(1) - \cos(n\omega)0 \right]$$

$$= 0 + \frac{2}{(2n\pi)^2} \left[\cos 2n\pi - 1 \right]$$

$$= \frac{2}{(2n\pi)^2} \left[1 - 1 \right] = \underline{0} \longrightarrow [4 \text{ MARKS}]$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin n\omega t \, dt$$

$$= \frac{2}{T} \int_0^1 t \sin n\omega t \, dt$$

$$= \frac{2}{T} \left[(t) \left\{ -\frac{\cos n\omega t}{n\omega} \right\} \right]_0^1 - \int_0^1 \left[(1) \int \sin n\omega t \, dt \right] dt$$

$$= \frac{2}{T} \left[\frac{1}{n\omega} \left[-\cos n\omega \right] + \frac{1}{(n\omega)^2} \left[\sin n\omega - 0 \right] \right]$$

$$= \left[\frac{\sin n\omega}{(n\omega)^2} - \frac{\cos n\omega}{n\omega} \right] \frac{2}{T}$$

$$= \frac{2}{T} \left[\frac{\sin 2n\pi}{4n^2\pi^2} - \frac{\cos 2n\pi}{2n\pi} \right]$$

$$= \frac{2}{T} \left[0 - \frac{1}{2n\pi} \right]$$

$$= \frac{2}{1} \left(-\frac{1}{2n\pi} \right)$$

$$= \frac{-1}{n\pi}$$

$\longrightarrow$ [4 MARKS]

3] $f(t) = e^{-at} \cos \omega_0 t \cdot u(t)$

$$F(\omega) = \int_0^\infty e^{-\alpha t} \left[\frac{e^{+j\omega_0 t} + e^{-j\omega_0 t}}{2} \right] e^{-j\omega t} \, dt \longrightarrow$$

$$= \frac{1}{2} \int_0^\infty e^{-\{\alpha - j(\omega_0 - \omega)\}t} \, dt + \frac{1}{2} \int_0^\infty e^{-\{\alpha + j(\omega_0 + \omega)\}t} \, dt$$

$$= \frac{1}{2} \cdot \frac{1}{-\alpha + j(\omega_0 - \omega)} [0 - 1] + \frac{1}{-2} \frac{1}{\alpha + j(\omega_0 + \omega)} [0 - 1].$$

$\longrightarrow$ [5 MARKS]

$$= \frac{1}{2}\left[\frac{1}{\alpha - j(\omega_0 - \omega)} + \frac{1}{\alpha + j(\omega_0 + \omega)}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(\alpha + j\omega) - j\omega_0} + \frac{1}{(\alpha + j\omega) + j\omega_0}\right]$$

→ [4 MARK]

Let $\alpha + j\omega = c_1$

$j\omega_0 = c_2$.

$$\therefore F(\omega) = \frac{1}{2}\left[\frac{1}{c_1 - c_2} + \frac{1}{c_1 + c_2}\right]$$

$$= \frac{1}{2}\left[\frac{2c_1}{c_1^2 - c_2^2}\right]$$

$$= \frac{\alpha + j\omega}{(\alpha + j\omega)^2 - (j\omega_0)^2}$$

$$\boxed{F(\omega) = \frac{\alpha + j\omega}{(\alpha + j\omega)^2 + \omega_0^2}}$$

→ [1 MARK]

* Note :- Direct writing of final result without sufficient mathematical step would carry no weightage.

4) $F_s \geq 2B$. ⟶ [2 MARKS]

or $\omega_s \geq 2\omega_0$.

here $\omega_0 = [\omega_c + A_m \text{ Max } m(t)]$ ⟶ [2 MARKS]

$= 0$

$\omega_s \geq 2[10k + 50 \cdot (20)]$ ⟶ [2 MARKS]

$\omega_s \geq 2(10k + 1k)$

$\omega_s \geq 22k\text{Hz}$ ⟶ [4 Marks]

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EEE / INSTR / ECE F243 Signals & Systems
TEST 1(CLOSED BOOK)

MAXIMUM MARKS: 50 WEIGHTAGE: 25%
DATE: 18/03/13 DURATION: 50 MINUTES

1. Describe the signal shown in Figure 1, by a single mathematical expression valid for all time't'. **[8 marks]**

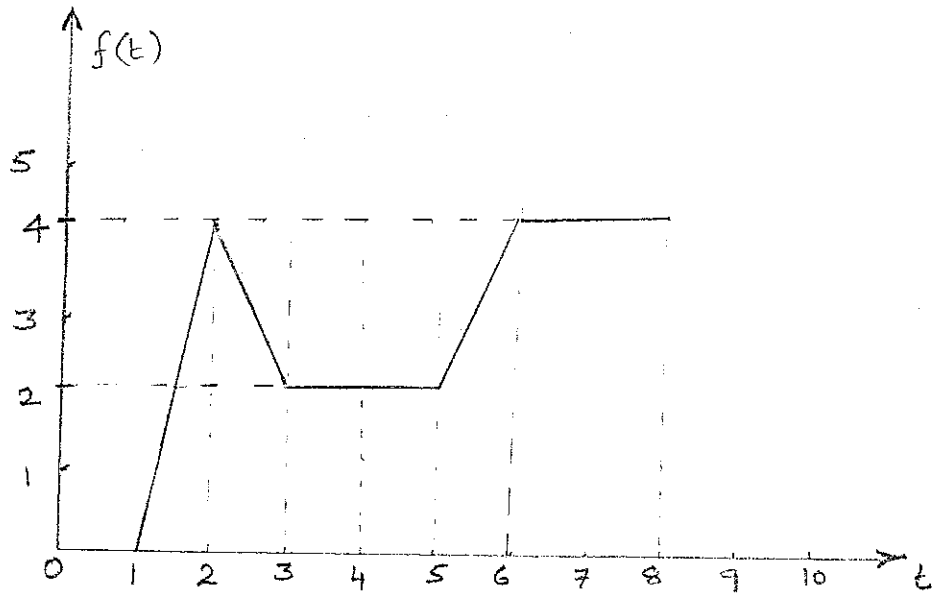

Figure 1

2. Determine the power of the signal given by the expression $f(t) = V_a \sin\omega t + V_a \sin(\omega t - 120) + V_a \sin(\omega t - 240)$, where the symbols have their usual meanings. **[13 marks]**

3. Find the unit impulse response of a system specified by the equation $(D^2+5D+6)\, y(t) = (D^2+7D+11)\, f(t)$; where y(t) is the output and f(t) is the input to the system. **[14 marks]**

4. For the parallel RLC circuit shown in Figure 2,switch S_1 opens at time t=0 second and switch S_2 closes at time t=0 second, find
   (a) The characteristic equation
   (b) The characteristic roots
   (c) Inductor current , $i_L(t)$ for $t \geq 0$ seconds **[15 marks]**

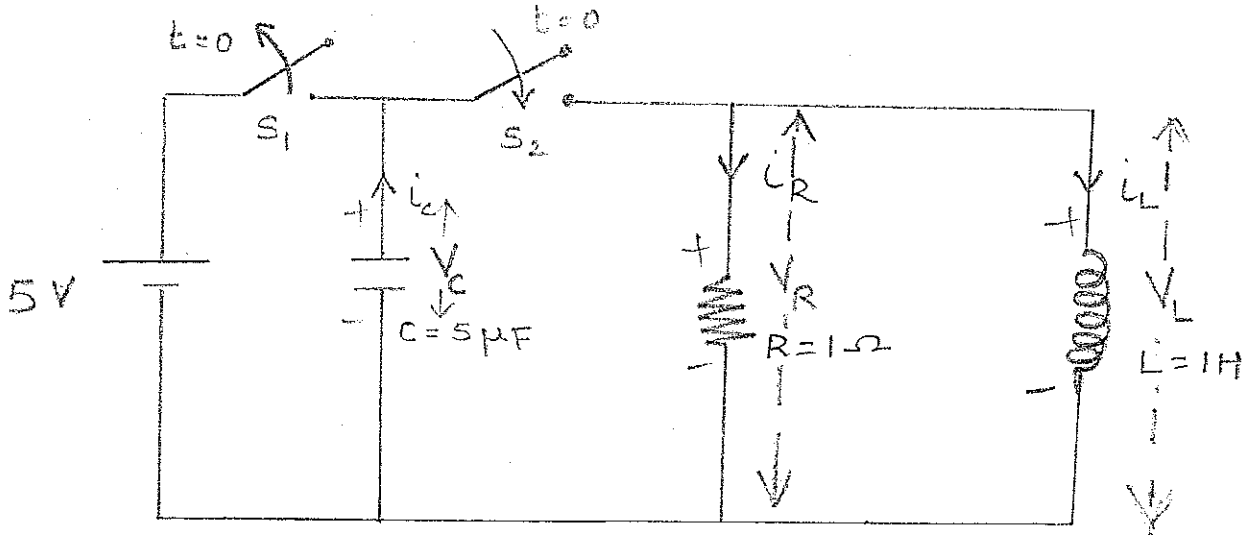

Figure 2

18/3/13

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Test1 (CLOSED BOOK)
Answering Scheme

1) $f_1(t) = 4(t-1)$

$f_2(t) = 8-2t$

$f_3(t) =$

$f_4(t) = 2t+8$

$f_5(t) = 4$ ⟶ [5 MARKS]

$f(t) = f_1(t) + f_2(t) + f_3(t) + f_4(t) + f_5(t)$

$= 4(t-1)[u(t-1) - u(t-2)] + (8-2t)[u(t-2) - u(t-3$

$+ 2[u(t-3) - u(t-5)] + (2t+8)[u(t-5) - u(t-6)]$

$+ 4[u(t-6) - u(t-8)]$ ⟶ [3 MARKS]

2) $f(t) = V_a \sin\omega t + V_a \sin(\omega t - 120) + V_a \sin(\omega t - 240)$

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Answering Scheme

$$P = \frac{1}{T}\int_0^T [f_1(t)]^2\,dt + \frac{1}{T}\int_0^T f_2^2(t)\,dt + \frac{1}{T}\int_0^T f_3^2(t)\,dt \rightarrow [1M]$$

$$= \frac{1}{T}\int_0^T V_a^2 \sin^2 \omega t + \frac{1}{T}\int_0^T V_a^2 \sin^2(\omega t - 120) + \frac{1}{T}\int_0^T V_a^2 \sin^2(\omega t + 120)\,dt$$

$$= \frac{V_a^2}{2T}\int_0^T 1 - \cos 2\omega t\,dt + \frac{V_a^2}{2T}\int_0^T 1 - \cos(2\omega t - 240)\,dt + \frac{V_a^2}{2T}\int_0^T 1 - \cos(2\omega t - 480)\,dt$$

$$= \frac{V_a^2}{2T}\cdot T - 0 + \frac{V_a^2}{2T}\cdot T - 0 + \frac{V_a^2}{2T}\cdot T - 0$$

↳ [4 M] ↳ [4M] ↳ [4M]

$$= \frac{3V_a^2}{2}$$

3] characteristic equation: $\lambda^2 + 5\lambda + 6 = 0$.

$\lambda_1 = -2 \quad \lambda_2 = -3 \rightarrow [2M]$

$y(t) = c_1 e^{-2t} + c_2 e^{-3t}$

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Answering Scheme

Applying initial conditions:

$y(0) = 0$.

$\dot{y}(0) = 1$.

$0 = c_1 + c_2 \qquad c_1 = -c_2$

$\dot{y}(0) = -2c_1 e^{-2t} - 3c_2 e^{-3t}$.

$c_1 = 1, \quad c_2 = -1 \longrightarrow [5\ M]$

$h(t) = b_n \delta(t) + \left[P(D)\, y_n(t)\right] u(t)$

↳ [2M]

$= \delta(t) + \left[(D^2 + 7D + 11)(e^{-2t} - e^{-3t})\right] u(t)$

$= \delta(t) + [D^2 e^{-2t} - D^2 e^{-3t} + 7De^{-2t} - 7De^{-3t} + 11e^{-2t} - 11e^{-3t}]\, u(t)$

$h(t) = \delta(t) + [e^{-2t} + e^{-3t}]\, u(t)$

↳ [5M]

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Test1 (CLOSED BOOK)
Answering Scheme

4] $i_c = i_R + i_L$.

$V_c = V_R = V_L$.

$$i_R = \frac{V_R}{R} = \frac{V_L}{R} = \frac{L\,di_L}{R\,dt}.$$

$$i_c = -C\frac{dV_c}{dt} = -C\frac{dV_L}{dt} = -LC\frac{d^2 i_L}{dt^2}$$

$$-LC\frac{d^2 i_L}{dt^2} = \frac{L}{R}\frac{di_L}{dt} + i_L.$$

$$\left(LCD^2 + \frac{L}{R}D + 1\right) i_L = 0.$$

$$\left(5\times10^{-6}D^2 + D + 1\right) i_L = 0.$$

root 1 = 0,

root 2 = -2×10^5

$$i_L(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}.$$

$$i_L(t) = C_1 + C_2 e^{-2\times10^5 t}$$

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Test1 (CLOSED BOOK)
Answering Scheme

$i_L(0) = 0$
$V_c(0) = 5V$
} (Applying initial conditions from inspection from figure)

$0 = C_1 + C_2 \qquad C_1 = -C_2.$

$$V_L(0) = V_c(0) = L \frac{di_L}{dt}\Big|_{t=0}$$

$$-L C_2 2\times10^5 e^{-2\times10^5 t}\Big|_{t=0} = 5.$$

$$-2\times10^5 C_2 = 5$$

$$C_2 = \frac{5}{-2}\times10^{-5} = -2.5\times10^{-5}$$

$$i_L(t) = C_1 - C_1 e^{-2\times10^5 t}.$$

$$= C_1\left(1 - e^{-2\times10^5 t}\right)$$

$$i_L(t) = 2.5\times10^{-5}\left(1 - e^{-2\times10^5 t}\right) \quad \text{for } t \geq 0 \text{ sec}$$

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QUIZ 2 (CLOSED BOOK)

MAXIMUM MARKS: 15 SET 1 **WEIGHTAGE: 7.5 %**
DATE: 15.04.13 **DURATION: 20 MINUTES**

NAME:	Id. No.:

1. Using direct integration , find $e^{at}\,u(t) * e^{-bt}\,u(t)$ [4 M]

2. Find the Fourier transform for the signal shown below [5 M]

3. Find the Fourier series for the signal described by the function f (t) = cos (1000t) [4M]

4. If f (t)$\leftrightarrow$F (ω), then k f (t) $\leftrightarrow$ k F (ω) _____ (TRUE / FALSE) [2 M]

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QUIZ 2 (CLOSED BOOK)

MAXIMUM MARKS: 15 SET 1 **WEIGHTAGE: 7.5 %**
DATE: ~~14.0.13~~ 15·04·13 **DURATION: 20 MINUTES**

NAME:	**Id. No.:**

1. Using direct integration , find $e^{+at}u(t) * e^{-bt}u(t)$ [4 M]

$$= \int_0^t e^{a(t-\tau)} \cdot e^{-b\tau} \, d\tau$$

$$= \int_0^t e^{(at - a\tau - b\tau} \, d\tau$$

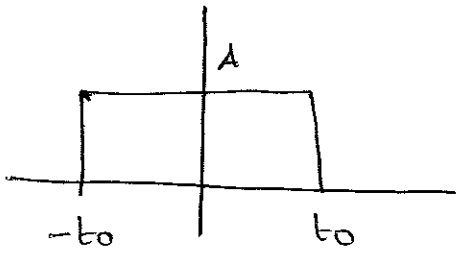
$$= e^{at} \int_0^t e^{-(a+b)\tau} \, d\tau$$

$$= \frac{e^{at}}{(a+b)} e^{-(a+b)\tau} \Big|_t^0$$

$$= \frac{e^{at}}{a+b} \left[1 - e^{-(a+b)t} \right]$$

$$= \frac{e^{at} - e^{-bt}}{(a+b)}$$

2. Find the Fourier transform for the signal shown below [5 M]



$$F(\omega) = \int_{-t_0}^{t_0} A \cdot e^{-j\omega t}\, dt$$

$$= A \cdot \frac{e^{-j\omega t}}{-j\omega} \Bigg|_{-t_0}^{t_0}$$

$$= A \cdot \frac{e^{-j\omega t_0} - e^{j\omega t_0}}{-j\omega}$$

$$= A \cdot \left[\frac{e^{j\omega t_0} - e^{-j\omega t_0}}{j\omega} \right]$$

$$= A \cdot t_0 \left[2 \frac{\sin \omega t_0}{\omega t_0} \right]$$

$$= A\, t_0\, 2 \operatorname{sinc}(\omega t_0)$$

3. Find the Fourier series for the signal described by the function f (t) = cos (1000t) [4M]

$$f(t) = a_0 + \sum a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$a_0 = \frac{1}{T}\int_0^T \cos(1000t)\,dt$$

$$= \frac{1}{1000T}\left[\sin 1000t\right]_0^T = 0$$

$$a_n = \frac{2}{T}\int_0^T \cos(1000t)\cos n\omega_0 t\,dt$$

$$= \frac{2}{2T}\int_0^T \left[\cos(1000t + n\omega_0 t) + \cos(1000t - n\omega_0 t)\right]dt$$

$$= \begin{cases} 1 & \text{if } n\omega_0 = 1000 \\ 0 & \text{if } n\omega_0 \neq 1000 \end{cases}$$

then $a_n = 1$ (For proof, see backside)

then $a_n = 0$.

By inspection (or logically)

$b_n = 0$

"OR"

Alternative Answer :- As the given signal is originally a sinusoidal function, theoretically its Fourier series should not exist

Hence, $a_0 = 0$, $a_n = 1$ $b_n = 0$

4. If f (t) ↔ F (ω), then k f (t) ↔ k F (ω) TRUE (TRUE / FALSE) [2 M]

*

$$a_n = \frac{1}{T}\left[\frac{\sin(1000+n\omega_0)T - 0}{(1000+n\omega_0)}\right] + \frac{1}{T}\left[\frac{\sin(1000-n\omega_0)T - 0}{(1000-n\omega_0)}\right]$$

At $n\omega_0 = 1000$, the value of a_n becomes:—

$$a_n = \frac{1}{T}\left[\frac{\sin 2n\omega_0 T}{2n\omega_0}\right] + \frac{1}{T}\left\{\left[\frac{\sin 0}{0}\right] \text{form}\right\}$$

The second part of a_n is indeterminate form, ∴ Using L' Hospitals Theorem, we get

$$\lim_{(1000-n\omega_0)\to 0}\left\{\frac{1}{T(1000-n\omega_0)}\left[\sin(1000-n\omega_0)T\right]\right\} = a_{n_2}$$

$$\therefore a_{n_2} = \lim_{(1000-n\omega_0)\to 0}\frac{1}{T}\left\{\frac{\frac{d}{d\omega_1}(\sin\omega_1 T)}{\omega_1}\right\}$$

where, $\omega_1 = 1000 - n\omega_0$

$$\therefore a_{n_2} = \lim_{\omega_1\to 0}\frac{1}{T}\left[\frac{(\cos\omega_1 T)T}{1}\right] = \frac{1}{T}\left(\frac{\cos 0}{1}\right)(T)$$

$$= \frac{T}{T}(1) = 1$$

$$\therefore a_n = \frac{1}{2n\omega_0 T}\sin 2n\omega_0 T + 1 = 0 + 1 = 1$$

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QUIZ 1 (CLOSED BOOK)

MAXIMUM MARKS: 15 | SET 2 | WEIGHTAGE: 7.5 %
DATE: 28.02.13 | DURATION: 20 MINUTES

NAME:	Id. No.:

1. Sketch the signal f(t)=(t-4)[u(t-5)-u(t-7)] [3 M]

2. Find the power for the signal described by the function f(t)=10cos(100t+60) [3M]

3. All ramp signals are geometrically straight lines but all geometric straight lines do not indicate a ramp signal_____ (TRUE / FALSE) [2 M]

4. For the system described by the equation

$$\frac{dy(t)}{dt} + y^2(t) = f(t)$$

where f(t) and y(t) are the input and output to the system respectively. Determine whether the system is linear or not? [4 M]

5. Find and sketch the odd and even components of the signal described by the function f (t) = tu(t) [3 M]

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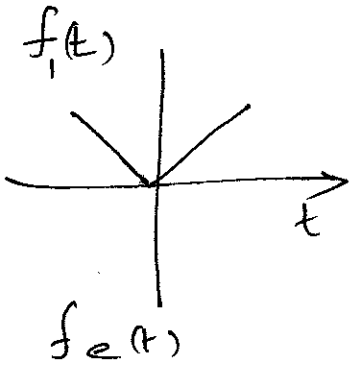
QUIZ 1 (CLOSED BOOK)

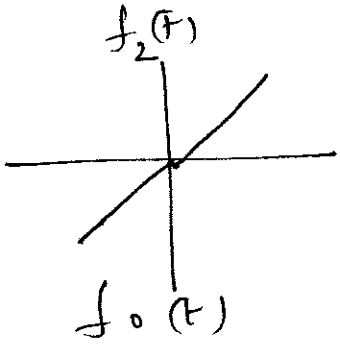
MAXIMUM MARKS: 15 | SET 1 | WEIGHTAGE: 7.5 %
DATE: 28.02.13 | DURATION: 20 MINUTES

NAME: Id. No.:

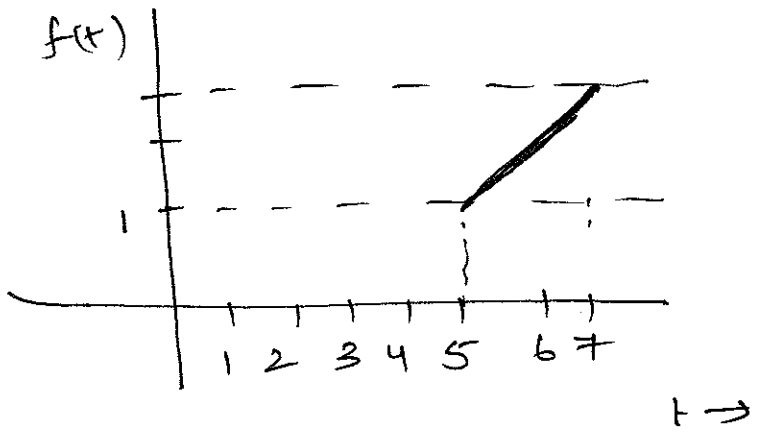
1. Find and sketch the odd and even components of the signal described by the function f (t) = tu(t) [3 M]

$$f(t) = \frac{1}{2}[t\,u(t) - t\,u(-t)] + \frac{1}{2}[t\,u(t) + t\,u(-t)]$$





2. Sketch the signal f(t)=(t-4)[u(t-5)-u(t-7)] [3 M]



3. Find the power for the signal described by the function f(t)=10cos(100t+60) [3M]

$f(t) = 10\cos(100t+60) \Rightarrow f^2(t) = 100\cos^2(100t+60)$

$f^2(t) = \frac{100}{2}\left[1+\cos(200t+120)\right]$

$= \left[50 + 50\cos(200t+120)\right]$

$\text{Power} = P_f = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 50\,dt + \lim_{T\to\infty}\int_{-T/2}^{T/2}\cos(200t+120)\,dt$

$= 50$

4. All ramp signals are geometrically straight lines but all geometric straight lines do not indicate a ramp signal TRUE. (TRUE / FALSE) [2 M]

5. For the system described by the equation

$$\frac{dy(t)}{dt} + y^2(t) = f(t)$$

where f(t) and y(t) are the input and output to the system respectively. Prove that the system is non-linear. [4 M]

$y_1(t) = \frac{d}{dt}y_1(t) + y_1^2(t) = f_1(t)$ ——— (1)

$y_2(t) = \frac{d}{dt}y_2(t) + y_2^2(t) = f_2(t)$ ——— (2)

$1\times k_1 + 2\times k_2$

$\frac{d}{dt}\left(k_1 y_1(t) + k_2 y_2(t)\right) + \left[k_1 y_1^2(t) + k_2 y_2^2(t)\right] =$

$k_1 f_1(t) + k_2 f_2(t)$

because of square term the system is non-linear.