BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, Pilani

Dubai Campus, Dubai International Academic City, Dubai, U.A.E.

Second Semester, Academic Year 2012-13

ECE / EEE / INSTR C272 Circuits and Signals

COMPREHENSIVE EXAMINATION (Closed Book)

Date: 4^{TH} June, 2013 — Max. Marks: 40

Time: 3 Hrs. — Weightage: 40%

NOTE: Answer All Questions.

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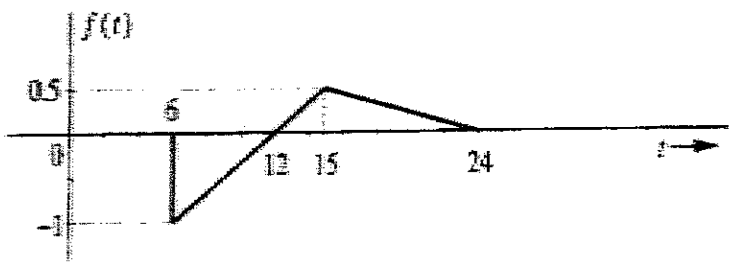
1. Consider the signal f(t) shown in Fig. below:



Sketch its: [1+1+1.5+1.5=5M]

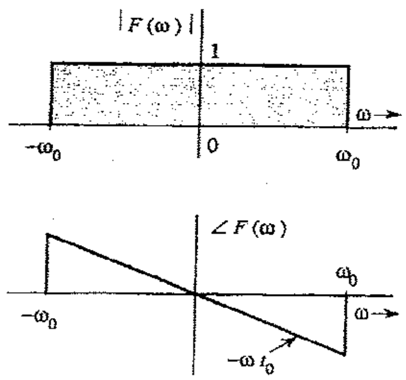
A. f(-t); B. f(t+6); C. f(3t); D. $f(\frac{t}{2})$

2. Let the signal c(t) is the convolution of two functions f(t) and g(t). What is the time-scaling property of this convolution? Show also that the convolution of
   A. an odd and an even function is an odd function
   B. two odd or two even functions is an even function.

[2+1.5+1.5=5 M]

3. Consider a signal f(t) which is equal to t for all t.
   A. Find the function that represents trigonometric Fourier series of f(t) over the interval $(-\pi, \pi)$, [3M]
   B. Sketch both f(t) and its function that represents its Fourier series. [2M]

4. Find the inverse Fourier transform of $F(\omega)$ for the spectra illustrated in figure below. Note that $F(\omega) = |F(\omega)|e^{j\angle F(\omega)}$ [5M]



5. A Compact Disc (CD) records audio signals digitally by using Pulse Code Modulation (PCM). Assume the audio signal bandwidth to be 20 kHz.

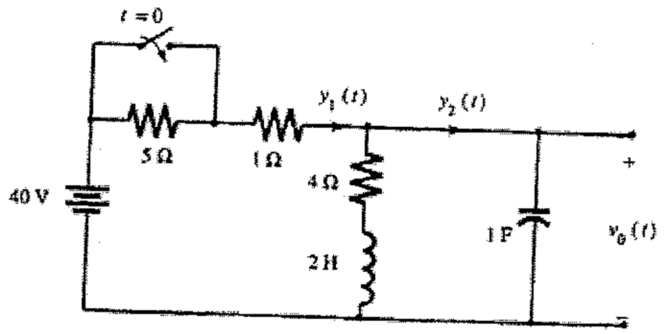
[1+1.5+1.5+1=5M]

   A. What is the Nyquist rate?
   B. If the Nyquist samples are quantized into 32768 levels and then binary-coded, what number of binary digits is required to encode a sample?
   C. Find the number of binary digits (bits) required to encode the audio signal
   D. What is the sampling rate employed in practical CDs and comment on how it compares with the Nyquist rate?

6. What is zero-padding employed in the computation of DFT. Compute 4-point DFT of a signal x[n]={1, 0, 1, 0} using both the FFT algorithms as below
   A. Decimation in Time (DIT) FFT Algorithm
   B. Decimation in Frequency (DIF) FFT Algorithm

[1+2+2=5M]

7. For the circuit shown in Figure below, the switch is in open position for a long time before t=0, when it is closed instantaneously.



   A. Write loop equations (in time domain) for $t \geq 0$ [2 M]
   B. Solve for $y_1(t)$ and y2(t) by taking the Laplace transform of the Loop equations found in part (a) [3 M]

8. Write Short notes on: [(1.5+1.5)+2=5M]
   A. Representation of circuits and systems employing
      i. Z-parameters
      ii. h-parameters
   B. Estimation of Complete & Total response of discrete time systems.

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ALL THE BEST
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ECE / EEE / INSTR C272 Circuits and Signals

TEST-2 (Open Book)

Date: 2^{nd} May, 2013 Max. Marks: 20

Time: 50mts. Weightage:20%

NOTE: 1. Answer ALL Questions; 2.Only the prescribed Text Books & handwritten notes is permitted.

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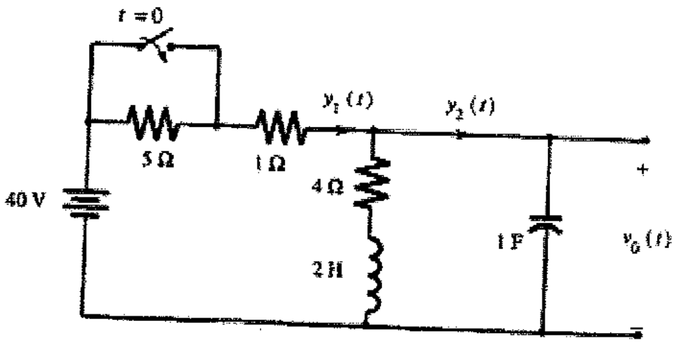
1. A Compact Disc (CD) records audio signals digitally by using Pulse Code Modulation (PCM). Assume the audio signal bandwidth to be 20 kHz. (1Mx3=3Marks)
   a. What is the Nyquist rate?
   b. If the Nyquist samples are quantized into 32768 levels and then binary-coded, what number of binary digits is required to encode a sample?
   c. Find the number of binary digits (bits) required to encode the audio signal.

2. Compute 4-point DFT of a signal x[n]={1, 2, 2, 1} using both the FFT algorithms as below: (3M+3M=6Marks)
   a. Decimation in Time (DIT) FFT Algorithm and
   b. Decimation in Frequency (DIF) FFT Algorithm.

3. A first-order all pass filter impulse response is given by $h(t) = -\delta(t)+2e^{-t}u(t)$. (2Mx2=4 Marks)
   a. Find the zero-state response of this filter for theinput $e^{t}u(-t)$.
   b. Sketch the input and the corresponding zero-state response.

4. Find (2Mx2=4Marks)
   a. The Laplace Transform of the function f(t)= u(t)-u(t-1)
   b. The Inverse Laplace Transform of $F(s)=\frac{5}{s^2(s+2)}$

5. For the circuit shown in Figure below, the switch is in open position for a long time before t=0, when it is closed instantaneously.



   a. Write loop equations (in time domain) for $t \geq 0$ (1 Mark)
   b. Solve for $y_1(t)$ and y2(t) by taking the Laplace transform of the Loop equations found in part (a) (3 Marks)

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TEST-2 (Closed Book)

Date: 2^{ND} May, 2013 | Max. Marks: 20

Time: 50mts. | Weightage:20%

Key / Answering Scheme

1.
   a. the Bandwidth is 20 kHz. The Nyquist rate is 40 kHz.
   b. $32768 = 2^{15}$, so 15 bits are needed to encode each sample.
   c. 40000x15=600000 bits/s

2. By DIT and DIF methods student will need to arrive at the same result, as below:

$\{F_0=6;\ F_1=-1-j;\ F_2=0;\ F_3=-1+j\}$

This is a 4-point signal starting at $k = 0$. The four points are 1, 2, 2, 1. Also $\Omega_0 = \pi/2$. Hence, the 4-point DFT is

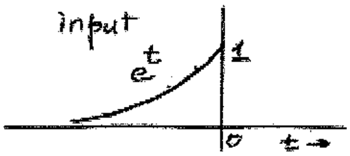
$$F_r = \sum_{k=0}^{3} f[k]e^{-jr\frac{\pi}{2}k} = 1 + 2e^{-jr\frac{\pi}{2}} + 2e^{-jr\pi} + e^{-jr\frac{3\pi}{2}}$$

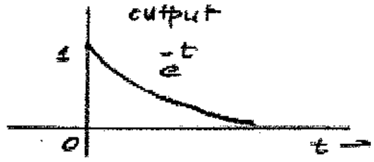
This yields

$$F_0 = 6,\ F_1 = -1-j,\ F_2 = 0,\ F_3 = -1+j$$

3.

$$\begin{aligned} y(t) &= [-\delta(t) + 2e^{-t}u(t)] * e^{t}u(-t) \\ &= -\delta(t) * e^{t}u(-t) + 2e^{-t}u(t) * e^{t}u(-t) \\ &= -e^{t}u(-t) + [e^{-t}u(t) + e^{t}u(-t)] \\ &= e^{-t}u(t) \end{aligned}$$





4.
   a.

$$F(s) = \frac{5}{s^2(s+2)} = \frac{k}{s} + \frac{2.5}{s^2} + \frac{1.25}{s+2}$$

To find k set $s = 1$ on both sides to obtain

$$\tfrac{5}{3} = k + 2.5 + \tfrac{5}{12} \quad \Longrightarrow \quad k = -1.25$$

and

$$F(s) = -\frac{1.25}{s} + \frac{2.5}{s^2} + \frac{1.25}{s+2}$$

$$f(t) = 1.25(-1 + 2t + e^{-2t})u(t)$$

b.

2/2

and

$$f(t) = u(t) - u(t-1)$$

$$\begin{aligned} F(s) &= \mathcal{L}[u(t)] - \mathcal{L}[u(t-1)] \\ &= \frac{1}{s} - e^{-s}\frac{1}{s} \\ &= \frac{1}{s}(1 - e^{-s}) \end{aligned}$$

5.

a. At $t=0^-$, the inductor current $y_1(0)=4$ and the capacitor voltage is 16 volts. After $t=0$, the loop equations are

$$2\frac{dy_1}{dt} - 2\frac{dy_2}{dt} + 5y_1(t) - 4y_2(t) = 40$$

$$-2\frac{dy_1}{dt} - 4y_1(t) + 2\frac{dy_2}{dt} + 4y_2(t) + \int_{-\infty}^{t} y_2(\tau)\,d\tau = 0$$

b. If,

$$y_1(t) \Longleftrightarrow Y_1(s), \quad \frac{dy_1}{dt} = sY_1(s) - 4$$

$$y_2(t) \Longleftrightarrow Y_2(s), \quad \frac{dy_2}{dt} = sY_2(s)$$

$$\int_{-\infty}^{t} y_2(\tau)\,d\tau \Longleftrightarrow \frac{1}{s}Y_2(s) + \frac{16}{s}$$

Laplace transform of the loop equations are

$$2(sY_1(s) - 4) - 2sY_2(s) + 5Y_1(s) - 4Y_2(s) = \frac{40}{s}$$

$$-2(sY_1(s) - 4) - 4Y_1(s) + 2sY_2(s) + 4Y_2(s) + \frac{1}{s}Y_2(s) + \frac{16}{s} = 0$$

Or

$$(2s+5)Y_1(s) - (2s+4)Y_2(s) = 8 + \frac{40}{s}$$

$$-(2s+4)Y_1(s) + (2s + 4 + \frac{1}{s})Y_2(s) = -8 - \frac{16}{s}$$

Cramer's rule yields

$$Y_1(s) = \frac{4(6s^2 + 13s + 5)}{s(s^2 + 3s + 2.5)} = \frac{8}{s} + \frac{16s + 28}{s^2 + 3s + 2.5}$$

$$y_1(t) = [8 + 17.89e^{-1.5t}\cos(\tfrac{t}{2} - 26.56^\circ)]u(t)$$

$$Y_2(s) = \frac{20(s+2)}{(s^2 + 3s + 2.5)}$$

$$y_2(t) = 20\sqrt{2}e^{-1.5t}\cos(\tfrac{t}{2} - \tfrac{\pi}{4})u(t)$$

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ECE / EEE / INSTR C272 Circuits and Signals

TEST-1 (Closed Book)

Date: 21st March, 2013 Max. Marks: 20

Time: 50mts. Weightage:20%

NOTE: Answer All Questions.

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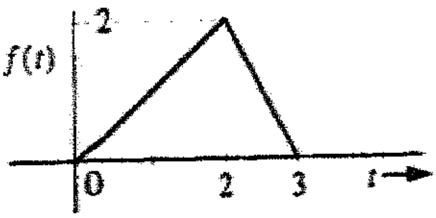
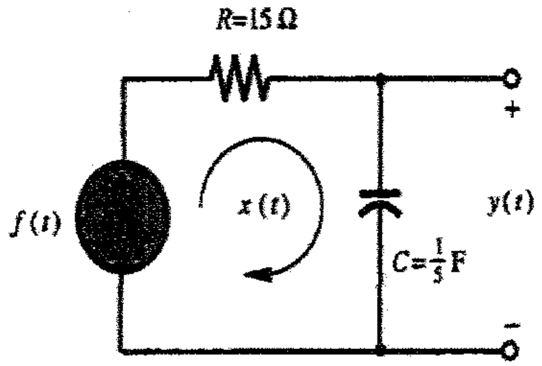
1. Determine whether the system described by its input f(t) and output y(t) using the equations below is causal or non-causal. 2 Marks
   a. $y(t)=f(t-2)+f(t+2)$
   b. $y(t)=f(t-4)+f(t)$
2. Describe the signal in Fig. below employing the standard signal models & the operations, if any, needed on them. 3 Marks



3. The unit impulse response of an LTIC system is $h(t)=e^{-t}u(t)$. Find the system's (zero-state) response y(t) if the input $f(t) = e^{-t}u(t)$. 2 Marks
4. Find the equation relating the input to output for the series RC Circuit of figure below, if the input is the voltage f(t) and the output is: 2 + 2 = 4 Marks



   a. The loop current x(t)
   b. The capacitor voltage y(t).

5. Determine the discrete time convolution of f[k]={0,1,2,3} with g[k]={0,1,0,1}. 3 Marks
6. Define Fourier Transform of a signal and Briefly explain any two of its properties. 1+2=3 Marks
7. A Signal $f(t)=\text{Sinc}(200\pi t)$ is sampled (using uniformly spaced impulses) at a rate of 200 Hz.
   a. sketch the spectrum of the sampled signal 1.5 Marks
   b. explain if you can recover the signal f(t) from the sampled signal 1.5 Marks

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TEST-1 (Closed Book)

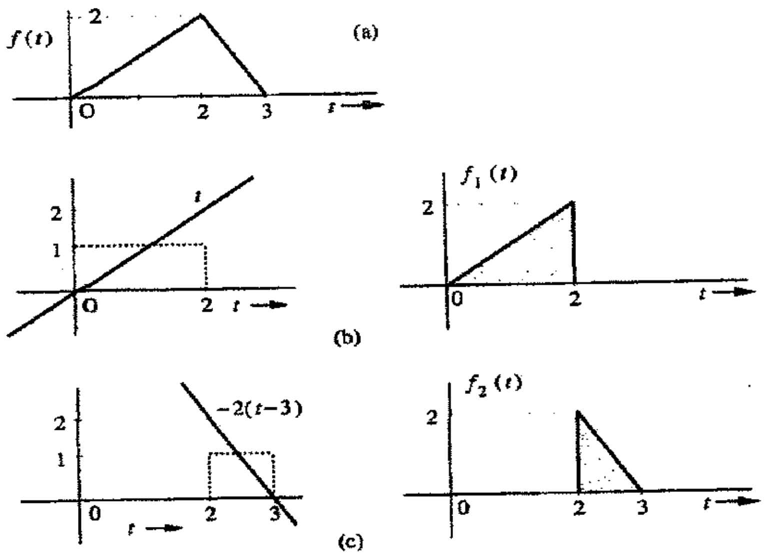
Date: 21st March, 2013 | Max. Marks: 20

Time: 50mts. | Weightage:20%

- - -

Key / Answering Scheme

1. While system (b)is causal because its output is a function of only the current f(t) and the past input f(t-4) the system (a) non-causal because the output is a function the future input "f(t+2)"
2.



The signal illustrated in Fig. 1.16a can be conveniently handled by breaking it up into the two components $f_1(t)$ and $f_2(t)$, depicted in Figs. 1.16b and 1.16c respectively. Here, $f_1(t)$ can be obtained by multiplying the ramp t by the gate pulse $u(t) - u(t-2)$, as shown in Fig. 1.16b. Therefore

$$f_1(t) = t\,[u(t) - u(t-2)]$$

The signal $f_2(t)$ can be obtained by multiplying another ramp by the gate pulse illustrated in Fig. 1.16c. This ramp has a slope -2; hence it can be described by $-2t+c$. Now, because the ramp has a zero value at $t=3$, the constant $c=6$, and the ramp can be described by $-2(t-3)$. Also, the gate pulse in Fig. 1.16c is $u(t-2) - u(t-3)$. Therefore

$$f_2(t) = -2(t-3)\,[u(t-2) - u(t-3)]$$

and

$$\begin{aligned} f(t) &= f_1(t) + f_2(t) \\ &= t\,[u(t) - u(t-2)] - 2(t-3)\,[u(t-2) - u(t-3)] \\ &= tu(t) - 3(t-2)u(t-2) + 2(t-3)u(t-3) \quad \blacksquare \end{aligned}$$

3. Since (a) $e^{at}u(t)*u(t)=\{(1-e^{at})/-a\}\,u(t)$ and that (b) $u(t)*u(t)=tu(t)$ we can solve $y(t)=h(t)*f(t)$ as:

$$y(t) = h(t) * f(t) = e^{-t}u(t) * e^{-t}u(t) = te^{-t}u(t)$$

4.

The loop equation for the circuit is

$$Rx(t) + \frac{1}{C}\int_{-\infty}^{t} x(\tau)\,d\tau = f(t) \tag{1.56}$$

or

$$15x(t) + 5\int_{-\infty}^{t} x(\tau)\,d\tau = f(t) \tag{1.57}$$

With operational notation, this equation can be expressed as

$$15x(t) + \frac{5}{D}x(t) = f(t) \tag{1.58}$$

Multiplying both sides of the above equation by D (that is, differentiating the above equation), we obtain

$$(15D + 5)x(t) = Df(t) \tag{1.59a}$$

or

$$15\frac{dx}{dt} + 5x(t) = \frac{df}{dt} \tag{1.59b}$$

Moreover,

$$x(t) = C\frac{dy}{dt} = \frac{1}{5}Dy(t)$$

Substitution of this result in Eq. (1.59a) yields

$$(3D + 1)y(t) = f(t) \tag{1.60}$$

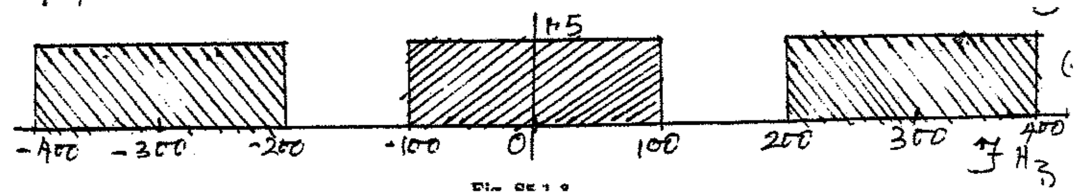
or

$$3\frac{dy}{dt} + y(t) = f(t) \tag{1.61}$$

5. Student may choose either a mathematical or graphical method of convolving the two discrete time sequences : f[k]={0,1,2,3} with g[k]={0,1,0,1} and get the result.
6. Student will need to define Fourier transform and briefly explain any two of its properties.
7.

a.

The spectrum of $f(t) = \text{sinc}(200\pi t)$ is $F(\omega) = 0.005\,\text{rect}(\frac{\omega}{400\pi})$. The bandwidth of this signal is 100 Hz (200π rad/s). Consequently, the Nyquist rate is 200 Hz, that is, we must sample the signal at a rate no less than 200 samples/second.



b. Since repeating spectra do not overlap, it is possible to recover f(t).