BITS, PILANI – DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

Second Year-CSE (2011 -2012)

Comprehensive Examination (Closed Book)

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH C222)

Time: 3 Hours | Max. Marks: 80

Date: June 12, 2012 | Weightage: 40%

Note: All questions are compulsory and should be answered sequentially.

1) a) Establish the validity of the following argument using the rules of inferences and the laws of logic

$$\begin{array}{l} \sim p \vee s \\ \sim t \vee (s \wedge r) \\ \sim q \vee r \\ p \vee q \vee t \\ \hline \therefore\ r \vee s \end{array}$$

[4]

b) Give a contradiction proof that the square root of 2 is not a rational number. [4]

2) a) Let S={3,7,11,15,19,⋯ 95, 99,103}. How many elements must we select from S to insure that there will be atleast two whose sum is 110? (Use Pigeon-Hole principle) [4]

b) Write the following argument in symbolic form. Then either verify the validity of the argument or explain why it is invalid (Assume here that the universe comprises all adults in a particular city. Two of those individuals are Rose and John) [4]

All Credit Union employees must know COBOL. All Credit Union employees who write loan applications must know EXCEL. Rose works for the Credit Union, but she does not know EXCEL. John knows EXCEL but does not know COBOL. Therefore Rose does not write loan applications and John does not work for the Credit Union.

3) Use strong mathematical induction to prove the following:

$a_n = 5(2^n) + 1$ is the unique function defined by

i) $a_0 = 6, \quad a_1 = 11$

ii) $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$ [8]

4) a) Find the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$ [4]

b) Solve the recurrence equation using method of generating functions

$a_n - 7a_{n-1} + 10a_{n-2} = 0$ for $n \geq 2$, where $a_0 = 10,\ a_1 = 41$ [4]

5) a) Find the solution of the recurrence equation

$a_n - 7a_{n-1} + 12a_{n-2} = 3^n$ for $n \geq 2, \quad a_0 = 3,\ a_1 = 6$ [4]

b) Solve the following recurrence equation by making an appropriate substitution to transfer the equation into linear recurrence equation with constant coefficients.

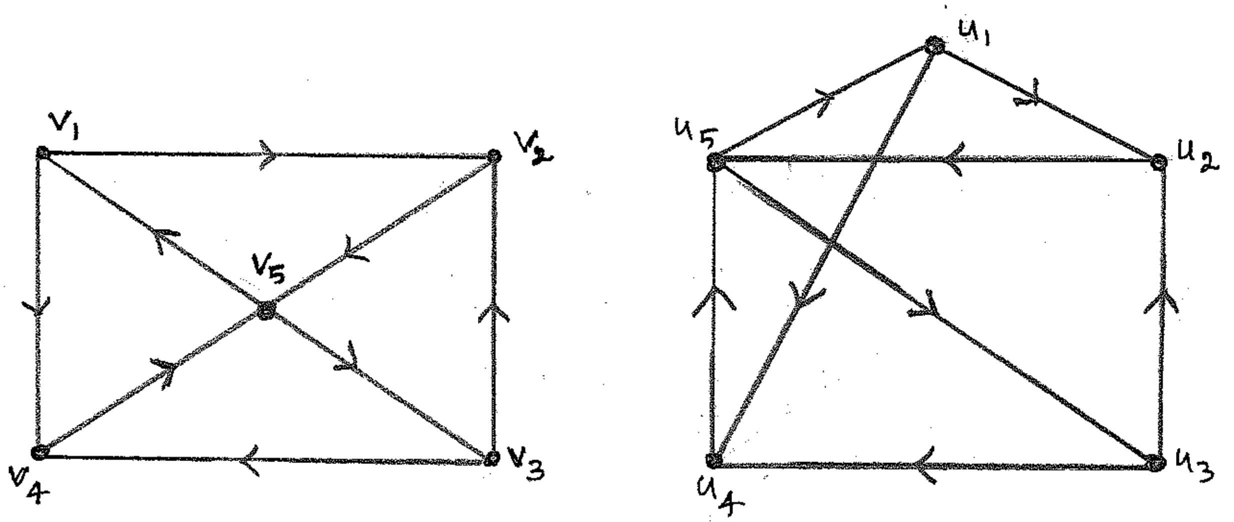
$a_n = (a_{n-1})^2.(a_{n-2})^3$ where $a_0 = 4$ *and* $a_1 = 4$ [4]

6) a) Determine whether the relation is reflexive, symmetric, antisymmetric or transitive.

i) For a given universe *U* and a fixed subset C of *U*, define $\mathcal{R}$ on *P(U)* as follows:
A, B ⊆ *U* we have A $\mathcal{R}$ B if *A*∩*C* = *B*∩*C*

ii) Let *T* be the set of all triangles in R^2. Define $\mathcal{R}$ on *T* by $t_1 \mathcal{R}\, t_2$ if $t_1$and t_2 have an angle of the same measure. [4]

b) Check whether the two directed graphs given below are Isomorphic or not. Give reasons for your answer. [4]

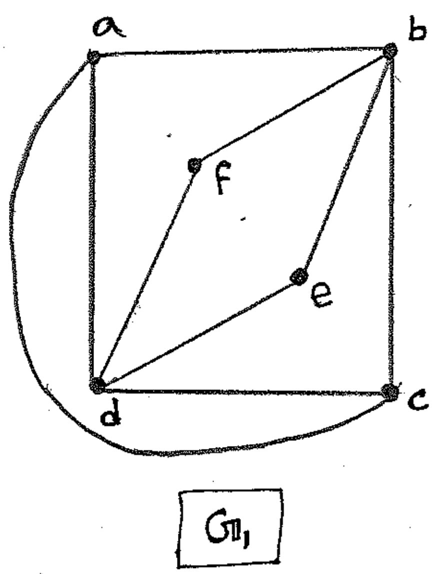


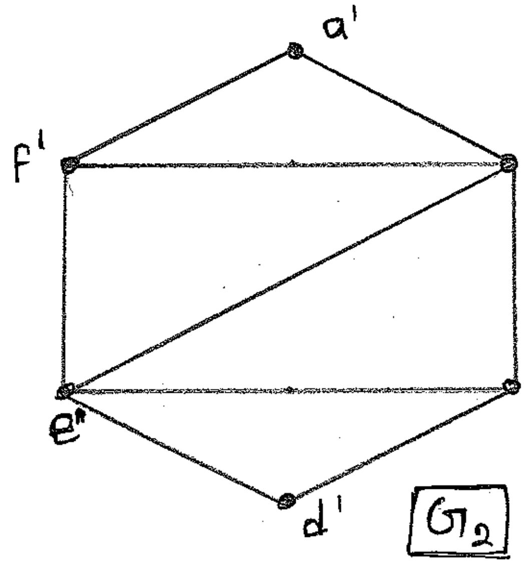
7) a) i) For the following Poset draw a Poset diagram and determine all maximal and minimal elements.
[A: /] where A={2,3,5,6,8,10,12,15,24}
ii) For subset B={8,6,12} of A, find the g.l.b and l.u.b [4]

b) Let A be a Poset with respect to relation $\mathcal{R}$. Then prove or disprove the following:
If [A, $\mathcal{R}$] is a totally ordered, then [A, $\mathcal{R}$] is a Lattice. [4]

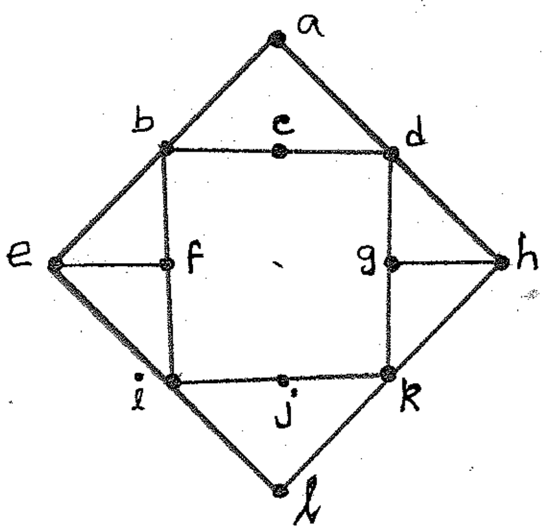
8) a) Prove that if V={v_1, v_2, $\cdots$ v_n} is the vertex set of a undirected k-regular graph then prove that [4]
$k|V| = \sum_{v \in V(G)} \deg(v) = 2|E|$ where $|V|$=Number of vertices and $|E|$= Number of edges in graph G.

b) Determine whether the given pair of undirected graphs is isomorphic or not? Justify your answer. [4]





9) a) Determine whether or not the following graph contains a Hamiltonian cycle. If there is a Hamiltonian cycle exhibit it. Otherwise give an argument that shows there is no Hamiltonian cycle. [4]



b) Using Euler Planar graph theorem, prove the following:
For a connected simple planar graph with $|V|$=v, $|E|$= e $>$ 2 and r regions, $e \leq 3v-6$ [4]

10) a) i) If H and K are subgroups of a group G, prove that H∩K is also a subgroup of G
ii) Give an example of a group G with subgroup H and K such that H∪K is not a subgroup of G. [4]

b) Given U = {1, 2, 4, 5, 7, 8} is a Group under the binary operation of multiplication modulo 9. Check whether U is a cyclic Group or not? If it is cyclic then give all possible generators of U, otherwise explain why it is not cyclic. [4]

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

(II year – II semester - Section 4 2011-2012) TEST – II (Open Book)

Course Title: Discrete Structures for Computer Science **Course Code : MATH C 222**

Max. Marks: 40 **Weightage: 20%** **Date: 13-05-2012** **Time: 50 min.**

Answer all the questions.

1. Solve the recursive relation $a_n^2 - 5a_{n-1}^2 + 6a_{n-2}^2 = 7n - 14, \quad n \geq 2, \; a_0 = a_1 = 1$ {8M}

2. For the non-homogeneous recurrence relation $a_n - 10a_{n-1} + 21a_{n-2} = f(n), \quad n \geq 2$

a) Find the solution of the corresponding homogeneous recurrence relation. {2M}

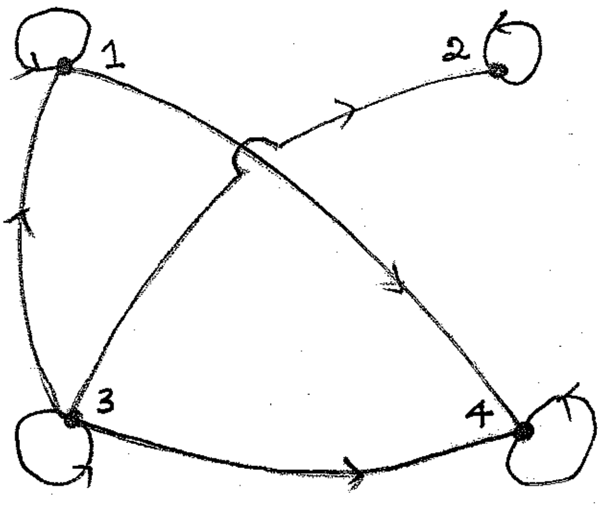
b) Also write the trial particular solution for the following choices of $f(n)$

i) $f(n) = 5$ ii) $f(n) = 3n^2 - 2$ iii) $f(n) = 7(11)^n$ iv) $f(n) = 3(r^n),\; r \neq 3,7$

v) $f(n) = 4(3^n) + 3(7^n)$ vi) $f(n) = 2(3^n) - 8(9^n)$ {6M}

3.(a) Define the relation $\mathcal{R}$ on the set A of positive integer by $(a,b) \in \mathcal{R}$ if and only if $\frac{a}{b}$ can be expressed in the form 2^m, where m is arbitrary integer(negative, zero or positive). Is $\mathcal{R}$ is an equivalence relation. {4M}

(b) The directed graph G for a relation $\mathcal{R}$ on set A={1,2,3,4} is shown in the following figure. Check whether {A, $\mathcal{R}$} is a poset and find its Hasse diagram. {4M}



4. For a given set A={a,b,c,d,e,v,w,x,y,z}, consider the poset {A, $\mathcal{R}$} whose Hasse diagram is given below. Is {A, $\mathcal{R}$} a lattice? {2M}

BITS PILANI – DUBAI CAMPUS

International Academic City, Dubai

(II year – II semester - Section 4 2011-2012) TEST – I (CB)

Course Title: Discrete Structures for Computer Science **Course No. : MATH C 222**

Max. Marks: 50 **Weightage: 25%** **Date: 22-03-2012** **Time: 50 min.**

Answer all the questions.

1) Use division algorithm and proof by cases to prove the following:
Any integer n>0 is either of the form 6k, 6k+1, 6k+2, 6k+3, 6k+4, 6k+5. Conclude that n(n+1)(2n+1) is divisible by 6. [8]

2) Give an argument using rules of inferences to show the conclusion follows from the given premises.

a) Premises: Everyone in the class has a graphing calculator.
Everyone who has a graphic calculator understands the trigonometric functions.
Conclusion: Sam, who is in the class, understands the trigonometric functions. [4]

b) Check the following argument is valid or invalid. Provide an explanation for your answer.
Premise (1): All law-abiding citizens pay taxes.
Premise (2): Mr. Anil pays his taxes.
Conclusion: Therefore Anil is a law-abiding citizen. [4]

3) Show that the conclusion is logically valid in the following argument.

$\forall x[p(x) \to (q(x) \wedge r(x))]$

$\forall x[p(x) \wedge s(x)]$

$\therefore\ \forall x[r(x) \wedge s(x)]$ [8]

4) Using Pigeonhole principle show that in a given set of 7 distinct integers there must exist 2 integers in this set whose sum or difference is divisible by 10. [9]

5) Use Mathematical Induction to prove the following:

$$1^3 + 2^3 + 3^3 + \cdots n^3 = \left[\frac{n(n+1)}{2}\right]^2$$ [8]

6) Use partial fraction to compute the sequence represented by

$$A(x) = \frac{X+21}{(X-5)(2X+3)}$$ [9]

BITS PILANI, DUBAI CAMPUS

DUBAI INTERNATIONAL ACADEMIC CITY

CSE- SECOND YEAR – II SEMESTER 2011 – 2012 **B**

Discrete Structures for Computer Science (MATH C222) QUIZ-II

Date: 23.04.2012

NAME: ______________________________ ID: ______________________

DURATION: 20 MINUTES MAXIMUM: 14 MARKS

Answer all the questions:

1) Find a particular solution to the Inhomogeneous recurrence equation $a_n - 2a_{(n-1)} + a_{(n-2)} = 4$ (2M)

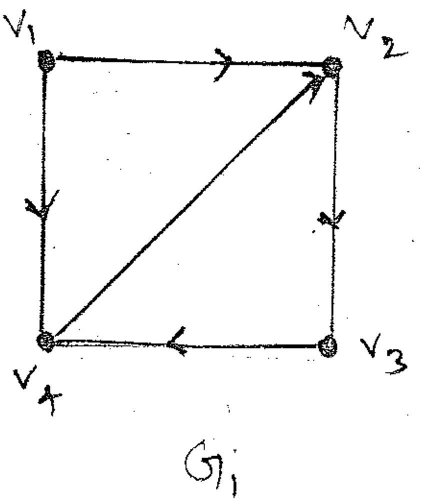
2) Solve $a_n - n\, a_{(n-1)} = n!$ for $n \geq 1$ and $a_0 = 2$ (3M)

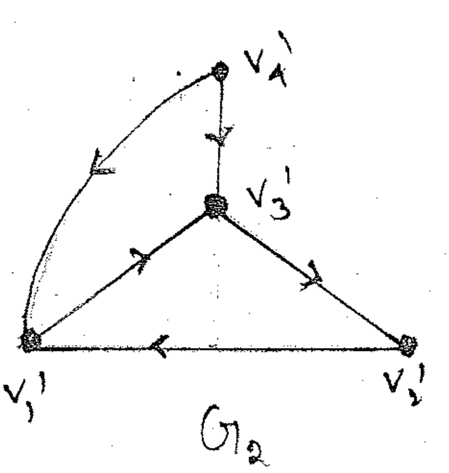
3) Find the homogeneous recurrence equation whose characteristic equation is

$C(t) = (t-2)^2(t-4)(t-5)$ (2M)

4) Find the solution of the recurrence equation $a_n - 6\,a_{(n-1)} = 0$ for $n \geq 1$ and $a_0 = 1$ (2M)

5) Check whether the following graphs are isomorphic. (3M)





6) If A be the set of non-zero rational numbers, then for $a, b \in A$ define $a\ \mathcal{R}$ b if b divides a. Give any five relational properties valid for the above relation $\mathcal{R}$. (2M)

BITS PILANI, DUBAI CAMPUS

DUBAI INTERNATIONAL ACADEMIC CITY

CSE- SECOND YEAR – II SEMESTER 2011 – 2012 **A**

Discrete Structures for Computer Science **(MATH C222) QUIZ-I**

Date: 05.03.2012

NAME: ______________________________ ID: ________________________

DURATION: 20 MINUTES MAXIMUM: 16 MARKS

Answer all the questions:

1) Show that the following expression is a tautology

$\{[p \to (q \to r)] \wedge (\sim q)\} \to (p \to r)$ (3M)

2) Determine whether each of the following inferences is valid. Justify your answer by giving the name of the inference rule or the name of the fallacy. (1+1 M)

a) If the patient has a virus, he must have a temperature above 99^o

The patient's temperature is not above 99^o

Hence, the patient does not have a virus.

b) AB is parallel to EF or CD is parallel to EF

AB is parallel to EF

Hence, CD is not parallel to EF.

3) Test the validity of the following argument using rules of inference. (2M)

If I study then I will not fail Mathematics

If I do not play basketball then I will study

But I failed in Mathematics

Therefore I must have played basketball.

4) Prove by contrapositive method (ie) indirect proof method. (3M)

If a is an odd integer then there are no integral roots for the polynomial $f(x) = x^2 - x - a$

5) Use division algorithm to prove the following (3M)

The square of any integer is either of the form $3n$ or $3n + 1$.

6) Prove by contradiction. (3M)

If the sum of 11 real numbers is greater than 100 prove that one of the numbers is greater than 9.