BITS Pilani, Dubai Campus
Dubai International Academic City, Dubai
Second Year – Second Semester 2010 – 2011
MATH C231 – Number Theory
Comprehensive Examination

Date: 9.06.2011 **Max. Marks: 40**
Time: 3 hours **Weightage: 40%**

Q1 a). Find the g.c.d of (12378, 3054). [2]

b). Show that for an arbitrary integer a, [2]

$2 \mid a(a+1)$

Q2 a). Determine whether 701 is a prime or not? [2]

b). A customer bought a dozen pieces of, apples and oranges for $1.32. If an apple costs 3 cents more than an orange and more apples than oranges were purchased .How many pieces of each kind were bought? [4]

Q3.a)Find the remainder when 2^{50} is divide by 7 [2]

b) Use Fermats theorem to prove that 17 divides $11^{104}+1$. [2]

Q4. Find a complete set of mutually incongruent solution for the following congruence 9x $\equiv 21 (\text{mod } 30)$. [4]

Q5. Solve the simultaneous congruences [4]

$x \equiv 2 \ (\text{mod } 3)$, $x \equiv 3 \ (\text{mod } 5)$, $x \equiv 2 \ (\text{mod } 7)$.

Q6.a) For n=6 show

$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3)=0$ [2]

Q7 a).Find the value of the following Legendre symbol [2]

$(\frac{461}{773})$

b). If p $\equiv 1 (\text{mod } 20)$, then $(\frac{5}{p}) = ?$ [2]

Q8 . Solve the quadratic congruence [4]

$x^2 \equiv 196 (\text{mod } 1357)$

Q9.Solve 364x + 227y = 1 using continued fractions. Give the general Solution. [4]

Q10.Show that if a is an odd integer, then g.c.d (3a, 3a+2) =1. Without taking any particular value(give a general proof). [4]

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MATH C231 – Number Theory
Test 2 (Open Book)

Date: 5.05.2011 **Max. Marks: 20**
Time: 50 Minutes **Weightage: 20%**

Q1.a) Find the value of the following Legendre symbol

$(\frac{-72}{131})$ [2]

b) Is it true that $(\frac{6}{37}) = (\frac{6}{13})$ [2]

Q2. a) Find $\sigma(180)$ and d (180) [2]

b) Show that If m=2 and n=10, $\sigma(mn) \neq \sigma(m)\sigma(n)$
explain why? [2]

Q3. Show

$\sum_{d|n} \mu(n) = 0$ for n=10 [2]

Q4.Write

$\frac{111}{345}$ as a continued fraction [2]

Q5.Solve 18x +5y = 24 using continued fractions. Give the general Solution. [4]

Q6. Find out whether the quadratic congruence
$x^2 \equiv 7(\mathrm{mod}\, 3^3)$ has a solution or not? If yes find it. [4]

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MATH C231 – Number Theory
Test 1 (Closed Book)

Date: 3.04.2011 **Max. Marks: 25**
Time: 50 Minutes **Weightage: 25%**

Q1. Show if a is an odd integer, then $a^2 + (a+2)^2 + (a+4)^2 + 1$ is divisible by 12. [2]

Q2. Find the g.c.d of (910,780,286,195). [2]

Q3. Find the l.c.m and g.c.d of (p^2q, pqr) where p, q, r are all primes. [2]

Q4. Find the solution of $10x - 8y = 42$. [2]

Q5. Find $\tilde{a}$ the inverse of a (mod c) when a=12 and c=17. [2]

Q6. Prove that $1 + a + a^2 + \ldots + a^{\Phi(m)-1} \equiv 0 \pmod m$
if g.c.d (a, m) = 1 and g.c.d (a-1,m) = 1. [3]

Q7. Find a complete set of mutually incongruent solution for the following congruence $9x \equiv 12 \pmod{15}$. [3]

Q8. What is the remainder when 2^{68} is divided by 19. [3]

Q9. Solve the simultaneous congruences
$x \equiv 2 \pmod 7$, $x \equiv 7 \pmod 9$, $x \equiv 3 \pmod 4$. [3]

Q10. What is the perpendicular distance to the origin (0, 0) from the line defined by the equation $a x - b y = 1$. [3]

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MATH C231 – Number Theory
Quiz- 2 (Closed Book)

Date: 18.04.2011 Max. Marks: 7
Time: 20 Minutes Weightage:7%

Q1. Find σ (999) and d (150)?

Q2. Prove If 2n is an even number, there exist integers q and r such that
$\Phi(q) + \Phi(r) = 2n$.

Q3.State the Chinese Remainder theorem

Q4. Find μ (7). Show that μ (n) is a multiplicative function

Q5. For which integer n is d(n) odd

Q6. What is a Fermats Number?

Q7. If number of prime's not exceeding x is given by $\pi(x)$ then the order of magnitude of $\pi(x)$ is

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MATH C231 – Number Theory
Quiz- 1 (Closed Book)

Date: 1.03.2011 Max. Marks: 8
Time: 20 Minutes Weightage:8%

Q1. Show if a, b, c, d are integers and an integer e divides both a and c then it also divides ab + cd.

Q2. Fill in the blanks for the following problem to find the gcd 'd' of 299 and 481

481 = 299× 1 + ...
299 = ...× 1 + 117
182 = ... × 1 +....
.... = ... ×1 + ...
65 = 52 ×1 +....
52 = ...×... + ...

So the gcd is ....

Q3.Find the solution of 17x +19y = 23 if it exists.

Q4.Find the gcd of 51 and 187 using the prime factorization of 51 and 187.

Q5. The perfect faro shuffle is used for a pack of 62 cards .In how many shuffles will the cards return to original order.

Q6.If a and b are integers, p is a prime, p divides ab, and p does not divide a then p divides b.Prove by taking an example.

Q7. Given { 0, 1, 2, 3, 4........n-1} forms a complete residue system mod n
Then will {1,2,3} {2,3,4}, {-1,0,1} ,{3,1,0} form a complete residue system mod 3?

Q8. Find an integer n such that it satisfies

$$140n \equiv 28 \pmod 3$$

by replacing the integers in the congruence by elements from {0,1,2}