

BITS, PILANI – DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II – Year – Semester – II (2009 -2010)

Discrete Structures for Computer Science (MATH C222)

Comprehensive Examination (Closed Book)

Time: 3 Hours

Max. Marks: 120

Date: May 25, 2010

Weightage: 40%

Note: All questions are compulsory and should be answered sequentially.

1. a) Prove the following is a tautology

$$\{[(p \rightarrow (q \vee r)) \wedge (\sim q)] \rightarrow (p \rightarrow r)\}$$

(5-Marks)

- b) Verify that the following argument is valid by using the rules of inference.

If Clifton does not live in France, then he does not speak French.

Clifton does not drive a Datsun. ?

If Clifton lives in France, then he rides a bicycle.

Either Clifton speaks French, or he drives a Datsun.

Hence Clifton rides a bicycle.

(8-Marks)

- c) Prove that the square of any ^{odd} positive integer is of the form $8k + 1$ (7-Marks)

2. a) verify the following is an valid inference

$$\sim p \rightarrow (q \rightarrow \sim w)$$

$$\sim s \rightarrow q$$

$$\sim t$$

$$\sim p \vee t$$

$$\therefore w \rightarrow s$$

(7-Marks)

- b) Prove by contrapositive method: If a is an odd integer then there are no integral roots for

the polynomial $f(x) = x^2 - x - a$

(5-Marks)

- c) Suppose that the circumference of a circular wheel is divided into 50 sectors and that the numbers 1 and 50 are randomly assigned to these sectors then show that there are three consecutive sectors whose sum of assigned numbers is at least 77 (8-Marks)

3. a) Write the negations of the following sentences.

$$1) \exists x, [(x^2 \geq 10) \wedge (x \text{ is even})]$$

$$2) \exists x, \{ \forall y, [x^2 = y] \}$$

(5-Marks)

b) Prove that by using strong Mathematical Induction, If g_n is given as follows

then $g_n = 3(5)^n + 7(2)^n$ is the unique function defined by

$$(1) g(0) = 10$$

$$g(1) = 29$$

$$(2) g(n+1) = 7g(n) - 10g(n-1)$$

For n greater than equal to 1.

(7-Marks)

c) $X^4(1+X+X^2+X^3)(1+X+X^2+X^3+X^4)(1+X+X^2+\dots+X^{12})$ Find the

coefficients X^{10} in the above expression

(8-Marks)

4. a) Solve the following Recurrence equation using generating function

$$a_n - a_{n-1} - 9a_{n-2} + 9a_{n-3} = 0; n \geq 3$$

$$\text{and } a_0 = 0, a_1 = 1, a_2 = 2$$

(7-marks)

b) Find the solution of the Recurrence equation using method of undetermined coefficients

$$a_n - 6a_{n-1} + 8a_{n-2} = n4^n \text{ for } n \geq 2$$

(8-Marks)

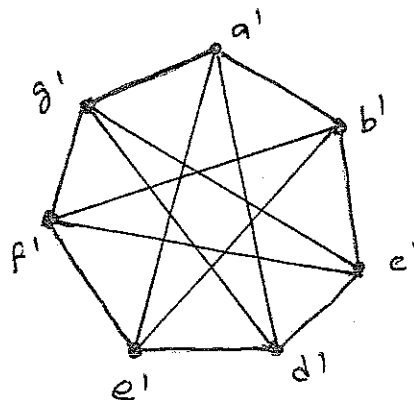
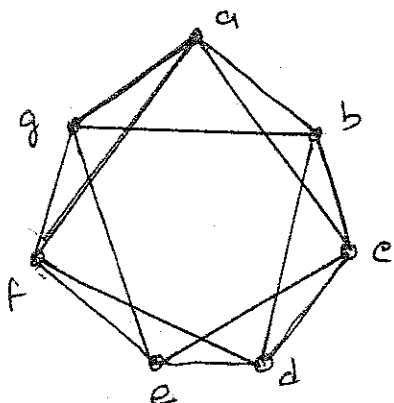
c) For the following Poset, draw a Poset diagram and determine all maximal, minimal elements, greatest and least elements if they exists.

$$[A; /] \text{ where } A = \{2, 3, 4, 6, 12, 18, 24, 36, 72\}$$

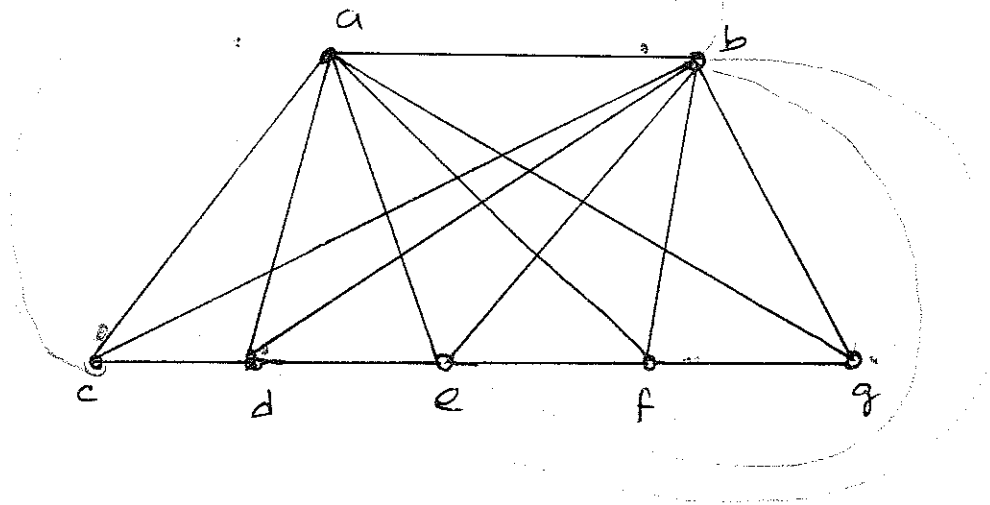
(5-Marks)

5. a) Check whether the given pair of graph is Isomorphic or not. Justify your answer with reasons

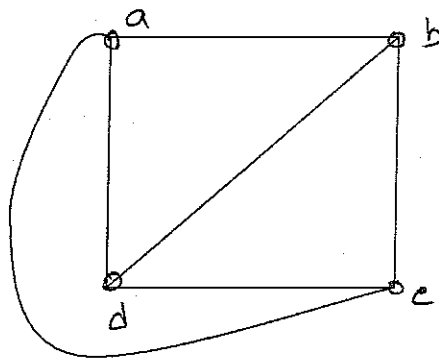
(8-Marks)



- b) Check whether the graph given below is Planar or not. Justify your answer. If Planar redraw the graph without crossover.



- c) Show that the graph G given below has a Hamiltonian cycle but no Euler circuit: (5-Marks)



- 6 a) If G is a connected plane graph then prove that $v - e + r = 2$ where v represents number of vertices, e represents the number of edges and r represents the number of regions in the graph G . (8-Marks)
- b) If $G = \{0, 1, 2, 3, 4, 5, 6\}$ then show that G is a abelian group under addition modulo 7. (7- Marks)
- c) Prove that the intersection of the two subgroups of a group is again a subgroup. (5-marks)

...Good Luck...

BITS, PILANI – DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II – Year – Semester – II (2009 -2010)

Discrete Structures for Computer Science (MATH C222) TEST (Open Book)

Time: 50 Minutes

TEST – 2

Max. Marks: 60

Date: May 2, 2010

Weightage: 20%

Note: (1) All questions are compulsory and should be answered sequentially.

(2) Only Prescribed Text Book and hand written class notes are allowed.

1.(a) Solve $a_n - 4a_{(n-1)} + 4a_{(n-2)} = n2^n$; $n \geq 2, a_0 = 0, a_1 = 1$ (8 Marks)

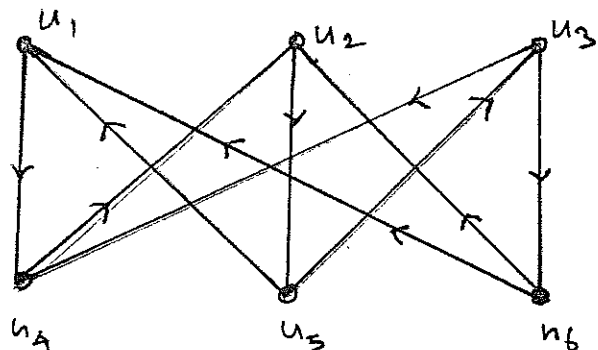
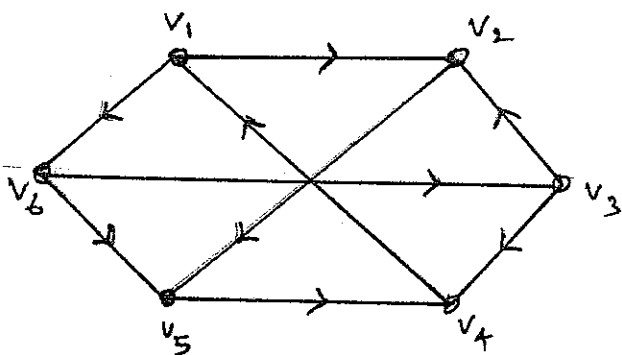
(b) Solve by divide and conquer relation

$$a_n - 2a_{(n/4)} = n \text{ for } n = 4^k \geq 4 \text{ and } a_1 = 1 \quad (7 \text{ Marks})$$

2. a) Solve the recurrence relation by making an appropriate substitution to transform it into linear recurrences with constant coefficients.

$$\sqrt{a_n} - \sqrt{a_{(n-1)}} - 2\sqrt{a_{(n-2)}} = 0 \text{ where } a_0 = a_1 = 1 \quad (8 \text{ Marks})$$

b) Determine if the given pair of directed graphs are Isomorphic. Justify
(7 Marks)



(a) Prove that $[D_{30}; /]$ is a Poset. Draw a Hasse diagram and determine all maximal and minimal elements and greatest and least elements if they exist. Also find the greatest lower bound (glb) and least upper bound (lub) for the entire set. Specify whether this Poset is a Lattice or not. (10 Marks)

(b) Draw Digraph for the relation \subseteq on all the non-empty set $\{0,1,2\}$. Is the relation an equivalence relation. Explain (5 Marks)

4. a) Let $[A; \leq]$ be a Poset and let B be a subset of A. Prove

(i) If $b \in B$ is an Upper bound of B then b is the greatest element of B

(ii) If b and b' are greatest elements of B then $b = b'$ (4+4=8 Marks)

b) Recall that if R is a relation on A then R^{-1} is also a relation on A defined by

$$R^{-1} = \{(a,b) \text{ such that } (b,a) \in R\}$$

(i) Prove that if $[A; R]$ is totally ordered then $[A; R^{-1}]$ is also a Poset

(ii) Prove that if $[A; R]$ is Lattice ordered then $[A; R^{-1}]$ is also a Lattice ordered (3+4=7 Marks)

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DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II – Year – Semester – II (2009 -2010)

Discrete Structures for Computer Science (MATH C222) TEST-I (Closed Book)

Time: 50 Minutes

Max. Marks: 75

Date: March 21, 2010

Weightage: 25%

Note: All questions are compulsory and should be answered sequentially.

1. (a) Verify that the following argument is valid by translating into symbols and using the rules of inference

If Joe is a Mathematician, then he is ambitious

If Joe is an early riser, then he does not like oatmeal

If Joe is ambitious, then he is an early riser

Hence, if Joe is a Mathematician, then he does not like oatmeal (7 Marks)

- (b) Prove or disprove the validity of the following arguments using quantifiers & inference rules.

Every living thing is a plant or an animal.

David's dog is alive and it is not a plant.

All animals have hearts.

Hence, David's dog has a heart. (7 Marks)

- c) Determine whether the following inference pattern is valid or invalid, and give reasons for answers.

$\sim r \rightarrow (s \rightarrow \sim t)$

$\sim r \vee w$

$\sim p \rightarrow s$

$\sim w$

Hence $t \rightarrow p$

(6 Marks)

2. (a) Suppose that the circumference of a circular wheel is divided into 40 sectors and that the integers 1 through 40 are randomly assigned to these sectors. Prove that there is at least one group of four consecutive sectors whose sum of assigned numbers is 82 or more. (10 Marks)

(b) Use division algorithm and proof by cases or elimination of cases to prove any integer $n > 0$ is either of the form $6k, 6k+1, 6k+2, 6k+3, 6k+4, 6k+5$. Hence show that $\{n(n+1)(n+2)\}/6$ is an integer. (10 Marks)

3. (a) Given 20 French, 30 Spanish, 25 German, 20 Italian, 50 Russian and 17 English books, how many books must be chosen to guarantee that at least
- 50 books of one language were chosen.
 - 6 French, 11 Spanish, 7 German, 4 Italian, 20 Russian or 8 English were chosen. (10 Marks)

(b) Use Mathematical Induction to prove the following statement is true for all integers.

$$1.3+3.5+5.7+\dots+(2n-1)(2n+1) = \{n[4n^2n+6n-1]/3\} \quad (10 \text{ Marks})$$

4. (a) Use partial fraction to compute the coefficients of

$$\sum_{r=0}^{\infty} a_r x^r = \frac{x^2-5x+3}{x^4-5x^2+4} \quad (10 \text{ Marks})$$

(b) Find the coefficient of x^{30} in

$$(x^{10} + x^{11} + \dots + x^{25})(x + x^2 + \dots + x^{15})(x^{20} + x^{21} + \dots + x^{45}) \quad (5 \text{ Marks})$$

DISCRETE STRUCTURE FOR COMPUTER SCIENCE (MATH C222)

Quiz - 2

Time: 20 Minutes	Max Marks: 21	Weightage: 7%	13.4.2010
Name:	ID:	Section:	

Answer all the questions:

1. Solve the recurrence relation using the characteristic roots: (5)
 $a_n = a_{n-1} + 6 a_{n-2}, \quad n \geq 2$ subject to the initial condition $a_0 = 12$ and $a_1 = -1$
2. Find the coefficient of X^{12} in $\frac{1 - X^4 - X^7 + X^{11}}{(1 - X)^5}$ (5)
3. Find the generating function for the sequence $\{a_r\}_{r=0}^{\infty}$ defined by: (5)
 $a_r = (r + 2)(r + 1)3^r$
4. Solve the recurrence relation using generating function: (6)
 $a_n = 5a_{n-1} - 6 a_{n-2}, \quad n \geq 2 \quad \text{and} \quad a_0 = 1, a_1 = -2$

BITS, Pilani-Dubai
Dubai International Academic City, Dubai
Second year – Second semester 2009 – 2010

B

DISCRETE STRUCTURE FOR COMPUTER SCIENCE (MATH C222)
Quiz - 2

Time: 20 Minutes

Max Marks: 21

Weightage: 7%

13.4.2010

Name:

ID:

Section:

Answer all the questions:

1. Solve the recurrence relation using the characteristic roots: (5)

$$a_n = 6a_{n-1} - 9a_{n-2}, \quad n \geq 2 \text{ subject to the initial condition } a_0 = 1 \text{ and } a_1 = 1$$

2. Find the coefficient of X^{12} in $\frac{1 - X^4 - X^7 + X^{11}}{(1 + X)^5}$ (5)

3. Find the generating function for the sequence $\{a_r\}_{r=0}^{\infty}$ defined by: (5)

$$a_r = (r+2)(r+1)3^r$$

4. Solve the recurrence relation using generating function: (6)

$$a_n = 9a_{n-1} - 20a_{n-2}, \quad n \geq 2 \quad \text{and} \quad a_0 = -3, a_1 = -10$$

BITS PILANI, DUBAI A
Dubai International Academic City, DUBAI
II-Year, Semester-II, 2009-2010

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH C222) QUIZ-I
(Closed Book)

Time: 20 Minutes Date: 02-03-2010 Max. Marks: 24 Weightage:8%

ID No: Section: Name:

Note:(i) Write ID No., Name and Section. (ii)Write all the answers in the space provided.(iii) Each question carry 3 marks

1)Find the contrapositive and converse of the implication. "If today is Thursday then I have a test today"

2)State whether the following arguments are valid or invalid.

a) $p \rightarrow q, p \Rightarrow q$

b) $p \rightarrow q, q \Rightarrow p$

3)Check the validity of each of the argument

If it rains, Eric will be sick.

Eric was not sick.

.....

It did not rain.

4)By using division algorithm or otherwise show that the square of any Odd integer is of the form $8k + 1$

5) What is the minimum number of students required in a Discrete Mathematics class to be sure that at least six will receive the same grade. If there are five possible grades A, B, C, D and F. (Use Pigeon hole principle)

6) Using Pigeon hole principle show that in a given 37 positive integers then there must be at least 4 of them that have the same remainder when divided by 12.

7) Give direct proof. for integers a, b and c show that if a divides b and a divides c then a^2 divides bc

8) Prove by contradiction. If the sum of five real numbers is greater than 100 prove that one of the numbers is greater than 16.

BITS PILANI, DUBAI B
Dubai International Academic City, DUBAI
II-Year, Semester-II, 2009-2010

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH C222) QUIZ-I
(Closed Book)

Time: 20 Minutes Date: 02-03-2010 Max. Marks: 24 Weightage:8%

ID No: Section: Name:

Note:(i) Write ID No., Name and Section (ii)Write all the answers in the space provided. (iii) Each question carry 3 marks

1)Check the validity of each of the argument

If it rains, Eric will be sick.

Eric was not sick.

.....

It did not rain.

2)Find the contrapositive and converse of the implication. "If today is Thursday then I have a test today"

3)State whether the following arguments are valid or invalid.

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4)What is the minimum number of students required in a Discrete Mathematics class to be sure that atleast six will receive the same grade.If there are five possible grades A, B, C, D and F. (Use Pigeon hole principle)

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6) Prove by contradiction. If the sum of five real numbers is greater than 100 prove that one of the numbers is greater than 16.

7) Using Pigeon hole principle show that in a given 37 positive integers then there must be atleast 4 of them that have the same remainder when divided by 12.

8) Give direct proof. for integers a, b and c show that if a divides b and a divides c then a^2 divides bc