BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH C222)

COMPREHENSIVE EXAM

Time: 3 Hours | Max.Marks: 40

Date: June 01, 2009 | Weighage: 40 %

Note: 1. Solve all the four questions.

1. Explain the rules of inference used to obtain the conclusion from the given premises. Given the hypotheses: *If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on If the sailing race is held, then the trophy will be awarded. The trophy was not awarded*
   Can you conclude: It rained?

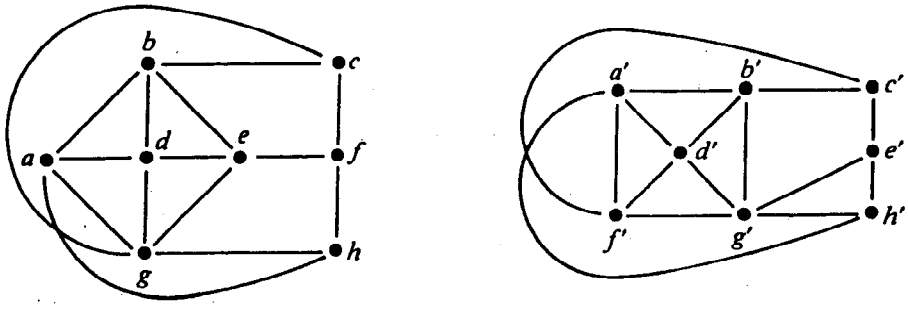
   (b) Prove that if any six numbers from 1 to 9 are chosen, then two of them will must add to 10.

   (c) Prove that $g(n) = 5\ 2^n + 1$ is the unique function defined by
   1. $g(0) = 6\ g(1) = 11$
   2. $g(n) = 3g(n-1) - 2g(n-2)$ for $n \geq 2$. (4+3+3)

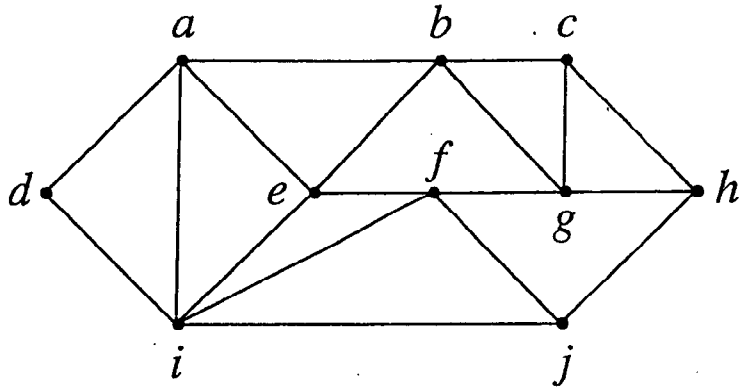
2. (a) Find the general solution of the the *recurrence relation*:
   $a_n + 6a_{n-1} + 12a_{n-2} + 8a_{n-3} = 3^n$ for $n \geq 3$.

   (b) Solve the *recurrence relation*:
   $a_n - 10a_{n-1} + 21a_{n-2} = 3^{n-2}$; $n \geq 2$ with initial conditions $a_0 = 1$ and $a_1 = 10$ by method of *generating function*. (4+4)

3. (a) Determine whether the given pair of graphs is Isomorphic? Justify.



(b) Determine whether the following graph has an Hamiltonian circuit, an Hamiltonian path but no Hamiltonian circuit, or neither. Give reasons.



(c) Prove the *Euler's Formula*: If G is connected plane graph, then $v - e + r = 2$, where v, e and r denotes respectively number of vertices, edges and regions.

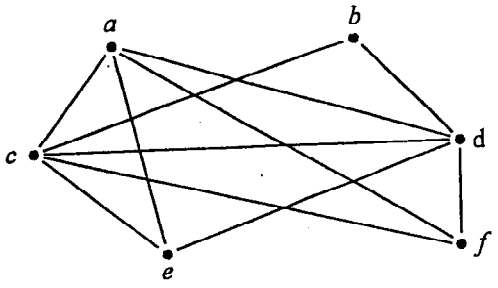
(d) Draw a Hasse diagram for the divisibility relation on I_{12}. Find (i) Find all maximal elements of the poset $(I_{12}; |)$. Consider the subset $B = \{2, 3\ 6\}$. (ii) Find $glb(B)$ and $lub(B)$. (2.5+2.5+4+3)

4. (a) Define a cyclic group. Prove that the group $(G, +_6)$ is a cyclic group where $G = \{0, 1, 2, 3, 4, 5\}$.

(b) Sate the Lagrange's Theorem. Prove that every group of the prime order is cyclic.

(c) Let $G = \{1, -1, i, -i\}$ be multiplicative group. Find the order of every elements.

(d) Is the following graph planar? Justify. If so, redraw the graph without crossing edges. (3+3+2+2)



BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR COMPUTER SCIENCE

(MATH C222) TEST - II (Open-Book)

Time: 50 Minutes | Max.Marks: 20

Date: April 12, 2009 | Weighage: 20 %

Note: 1. Text-book and class notes are allowed. 2. Solve all the four questions.

1. (a) Find the solution of the recurrence relation: $a_n = ka_{n/b} + c$ with $a_1 = d$.

   (b) Solve the following recurrence relation by *method of generating functions*

   $h_n + h_{n-1} - 16h_{n-2} + 20h_{n-3} = 0$ $(n \geq 3)$.

   where $h_0 = 0,\ h_1 = 1,\ h_2 = -1$. (2+3)

2. (a) Find the *general solution* of the recurrence relation

   $a_n - 3a_{n-1} + 3a_{n-2} - a_{n-3} = 3 + 5n,\ n \geq 0$.

   (b) Find the coefficient in x^{15} in each of the following

   (a) $x^3(1-2x)^{10}$ (b) $(1+x)^4/(1-x)^4$ (3+2)

3. let $A = \{1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 9,\ 12,\ 18,\ 24\}$ be ordered set with relation "x divides y" $(x \mid y)$. Draw a Hasse diagram for $(A;\ |)$. Determine all *maximal and minimal elements and greatest and least* elements if they exists. Let $B = \{2,\ 4,\ 6\}$ be a subset of A. Also find, if they exists, *all lower bounds, all upper bounds,* glb *and* lub of B. (5)

4. (a) Is the following relations R on $\{1, 2, 3, 4, 5, 6\}$ an *equivalence relations*? Justify

   $R = \{(1,1), (1,3)(1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5), (2,2)(2,6)(6,2)(6,6)(4,4)\}$.

   Also draw the diagraphs for R.

   (b) Prove or disprove: *the relation "|" is antisymmetric on set of integers Z.*

   (3.5+1.5)

Good Luck . . .

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SECOND SEMESTER 2008-09

DISCRETE STRUCTURE FOR COMPUTER SCIENCE (MATH C222)

TEST - I (CLOSED-BOOK)

Time: 50 Minutes | Max. Marks: 25

March 1, 2009 | Weightage: 25 %

Note: 1. Answer all the questions sequentially. 2. Calculators are not allowed

1. (a) Is the compound statement $[(p \rightarrow r) \wedge (\sim q \rightarrow p) \wedge (\sim r)] \rightarrow q$ a *tautology*? Justify. (3+3)

   (b) Prove that: *If p is an odd prime, then p has the form $6n+1$ or $6n+5$ or $p = 3$.*

2. (a) Prove that the following *inference pattern is valid or invalid.* Give the reasons for each steps to validate the argument.

   If the band could not play rock music or the refreshment were not delivered on time, then the New Years's party would have been cancelled and Alicia would have been angry. If the party were cancelled, then refunds would have had to be made. No refunds were made. *Therefore the band could play rock music.* (5)

3. (a) Let $S = [3, 5, 11, \ldots, 95, 103]$. How many elements must we select from S to ensure that there will be at least two whose sum is 110?

   (b) Prove that: *there are infinitely many prime numbers.* (3+3)

4. (a) Prove that $a_n = 5(2^n) + 1$ is the unique function defined by
   (1) $a_0 = 6$, $a_1 = 11$. (2) $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$. (3+2)

   (b) How many times must we roll a single die to get the same score at least three?

5. (a) For the universe of all students, translate each of the following *statements into symbols using quantifiers.*

   (a) There is a mathematics major in the class who is junior. (b) No graduate in the class is a physics major. (c) Every student in the is majoring in mathematics or physics. (3)

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR

COMPUTER SCIENCE (MATH C222)

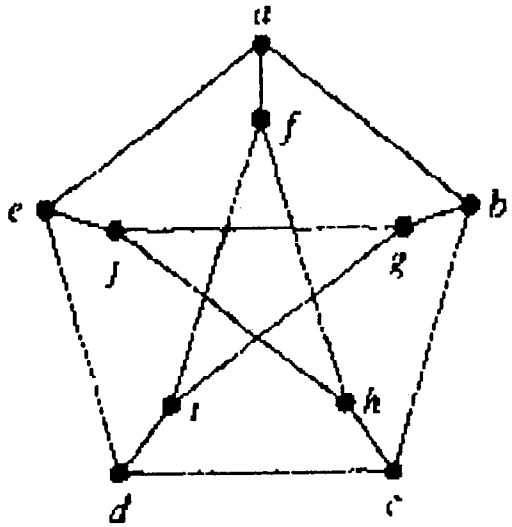
SURPRISE QUIZ III (Open-Book)

Time: 15 Minutes Max. Marks: 5

ID No: Section No: Name:

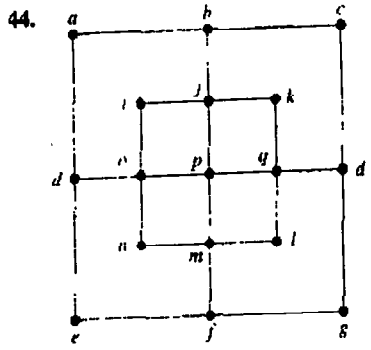
Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Is the given graph *planar*?.



2. Suppose that an N-node planar graph is drawn in the plane (with no edges crossing) with R regions. How many does the graph have?

3. Is the given graph *Eulerian*? Justify.



4. Define the term *Circuit* in graph theory. Draw a circuit of length 1, if exists.

5. Draw two 3-regular graph with 6 vertices.

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR

COMPUTER SCIENCE (MATH C222)

QUIZ II (Closed-Book)

Time: 15 Minutes Max. Marks: 5

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Solve the following recurrence relation using generating function: $a_n - 10a_{n-1} + 21a_{n-2} = 3^{n-2}$ for $n \geq 2$, where $a_0 = 1$ and $a_1 = 10$.

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR

COMPUTER SCIENCE (MATH C222)

QUIZ I (Closed-Book)

Time: 15 Minutes Max. Marks: 5

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. The generating function for the sequence $0,\ 0,\ a,\ a^2,\ a^3,\ a^4, \ldots$ is given by:

2. Write an expression for a_r, where a_r is the coefficient of X^r in the generating function

$$\frac{X+21}{(X-5)(2X+3)}$$

3. The coefficient of X^{11} in $(1 + X + X^2 \ldots)(X + X^2 + X^3 \ldots)^3(1 + X + X^2 \ldots)$ is:

4. If $C(t) = (t-2)^2(t-3)^3 = 0$, then the corresponding recurrence relation has the form

5. The general solution of the recurrence relation $a_n - 6a_{n-1} + 12a_{n-2} - 8a_{n-3} = 0 is$:

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR

COMPUTER SCIENCE (MATH C222)

SURPRISE QUIZ III (Open-Book)

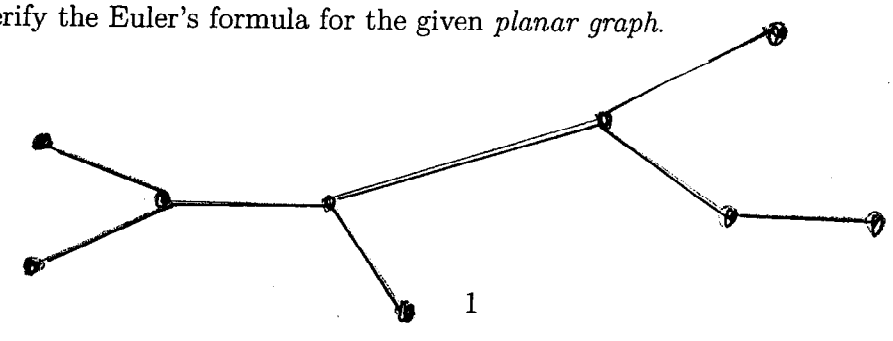
Time: 15 Minutes Max. Marks: 5

ID No: Section No: Name:

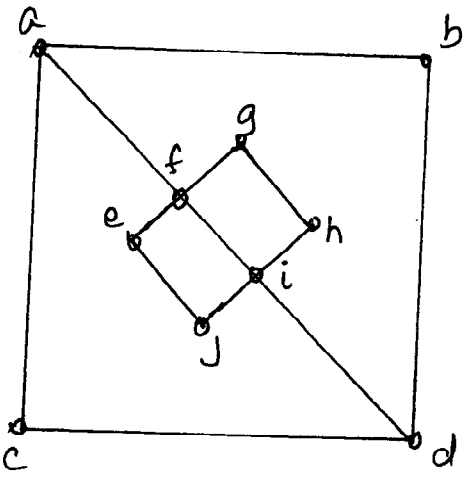
Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Prove that the graph $K_{2,4}$ is not *planar.*

2. Verify the Euler's formula for the given *planar graph.*



3. Is the given graph *traversable*? Justify.



4. Define the term *Hamilton's Circuit.*

5. Draw a graph with 6 vertices which is niether *Eulerian nor Hamiltonian*?

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR

COMPUTER SCIENCE (MATH C222)

QUIZ II (Closed-Book)

Time: 15 Minutes Max. Marks: 5

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Solve the following recurrence relation using generating function: $a_n - 10a_{n-1} + 21a_{n-2} = 3^{n-2}$ for $n \geq 2$, where $a_0 = 1$ and $a_1 = 10$.

BITS, PILANI - DUBAI

DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI

II - Year - SEMESTER - II (2008-09)

DISCRETE STRUCTURES FOR

COMPUTER SCIENCE (MATH C222)

QUIZ I (Closed-Book)

Time: 15 Minutes Max. Marks: 5

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. The generating function for the sequence $(r+3)(r+2)(r+1)(3/2)^r$ is given by:

2. Write an expression for a_r, where a_r is the coefficient of X^r in the generating function

$$A(X) = \frac{1}{X^2 - 5X + 6}$$

3. The coefficient of X^{15} in $(X^2 + X^3 + X^4 + X^5)(X + X^2 + \ldots + X^7)$ is: