BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - II

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH UC222)

COMPREHENSIVE EXAMINATION - (Closed-Book)

Time: 03 Hours | Max. Marks: 40

Date: May 31, 2007 | Weightage: 40 %

Note:- All questions are compulsory and should be answered sequentially.

1. (a) Verify that the following arguments are valid:

   Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised.

   Therefore, babies cannot manage crocodile.

   (b) Is the statement $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ a tautology? Justify.

   (c) Prove that if $\text{GCD}(a, b) = 1$, then $\text{GCD}(a^n, b^n) = 1$ for all $n \geq 1$.

   (d) Ten people volunteer for a three-person committee. Every possible committee of three can be formed from the ten names is written on a slip of paper, one slip for each possible committee, and slips are put in ten hats. Show that one hat contains 12 or more slips of paper. (2×4)

2. (a) Find an expression for a_r, where a_r is the coefficient of X^r, in the following:

$$\frac{1 - 7X + 3X^2}{(1 - 3X)(1 - 2X)(1 + X)}$$

   (b) Solve the following recurrence relations using generating functions $a_n + a_{n-1} - 16a_{n-2} + 20a_{n-3} = 0$ for $n \geq 3$ and with initial conditions $a_0 = 0$, $a_1 = 1$ and $a_2 = -1$.

   (c) Solve the following recurrence relation $a_n - 6a_{n-1} + 8a_{n-2} = n4^n$; $n \geq 2$, where $a_0 = 8$ and $a_1 = 22$

   (d) Solve the following recurrence relation $t_n - 5t_{n-1} + 8t_{n-2} - 4t_{n-3} = 0$; $n \geq 3$ subject to initial conditions $t_n = n$ if $n = 0,\ 1,\ 2$. (2+3+2.5+2.5)

3. (a) If n is a positive integer, let I_n denote the set of integer $\{x \mid 1 \leq x \leq n\}$. Then $(I_n; \mid)$ is a poset. Draw a Hasse Diagram for the poset $(I_{12}; \mid)$. Determine all maximal and minimal elements and greatest and least elements if they exists.

(b) Let G = (V, E) be a connected simple planar graph with $|V| = v$, $|E| = e > 2$, and r regions. Then prove that $3r \leq 2e$ and $e \leq 3v - 6$.

(c) Draw digraph for the relation $\subseteq$ on all the non-empty set {0, 1, 2}. Is the relation an equivalence relation?

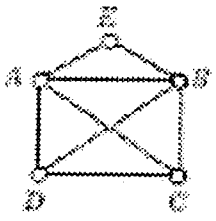
(d) (b) Suppose that G is a connected plane graph with less than 12 regions and such that each vertex of G has degree ≥ 3. Then prove that G has region of degree ≤ 4. (2.5×4)

4. (a) Draw a graph with six vertices which is (i) Hamiltonian but not Eulerian (ii) Eulerian but not Hamiltonian.

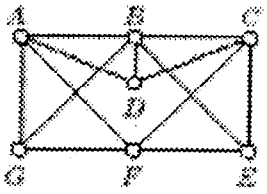
(b) Prove that a necessary and sufficient condition for a non-empty subset H of group $(G, *)$ to be a subgroup is: (1) if $a,\ b \in H \implies a * b^{-1} \in H$.

(c) Let G and H be two groups with identities e_G and e_H respectively. If $f : G \to H$ be an isomorphism then prove that: (i) $f(e_G) = e_H$ (ii) $f(a)^{-1} = f(a^{-1})$.

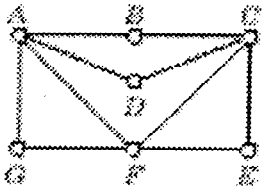
(d) Determine whether the following graphs has an Euler circuit, an Euler path but no Euler circuit, or neither. (3×4)



Graph 1



Graph 2



Graph 3

Good Luck . . .

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - II (2006-07)

DISCRETE STRUCTURES FOR COMPUTER SCIENCE

(MATH UC222) TEST - II (Open-Book)

Time: 50 Minutes | Max.Marks: 20

Date: May 13, 2007 | Weighage: 20 %

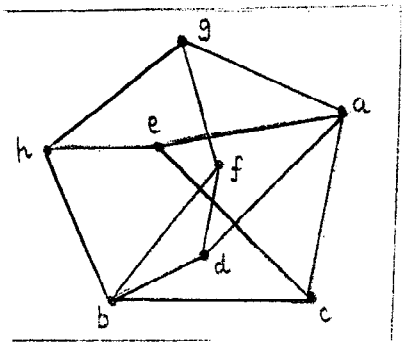
Note: 1. Text-book and class notes are allowed. 2.Solve all the four questions.

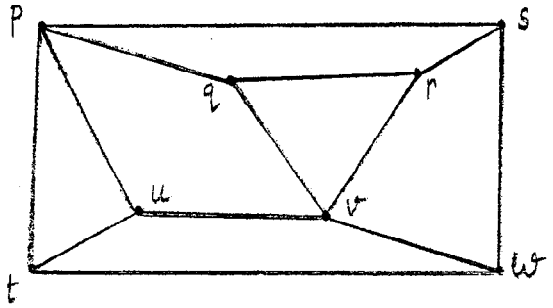
1. Let $G = \{e,\ a,\ b,\ c,\ d,\ f\}$ with the following operation table

*	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	e	c	b	f	d
b	b	d	e	f	a	c
c	c	f	a	d	e	b
d	d	b	f	e	c	a
f	f	c	d	a	b	e

Show that $(G, *)$ is a group. Is $(G, *)$ an abelian group? Justify. Let $H = \{e,\ c,\ d\}$ be a subgroup of G. Find the left cosets of H in G. (5)

2. (a) Determine whether the given pair of graphs is Isomorphic? Justify.

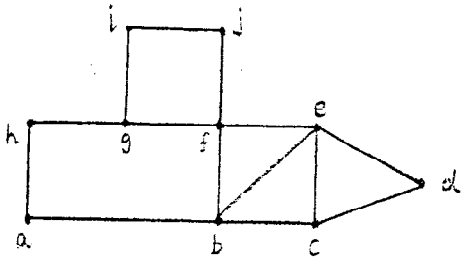




(b) Does there exist a simple planar graph with exactly 8 vertices, having 6 vertices of degree 4 and 2 vertices of degree 7? Either give an example of such a graph or explain why no such graph exists. (3+2)

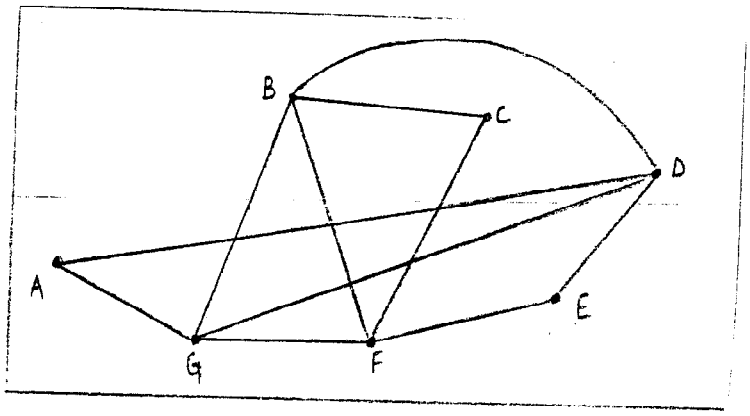
3. (a) Determine whether the following graph has an Euler circuit, an Euler path but

no Euler circuit, or neither. Also draw the path or circuit, if exist.

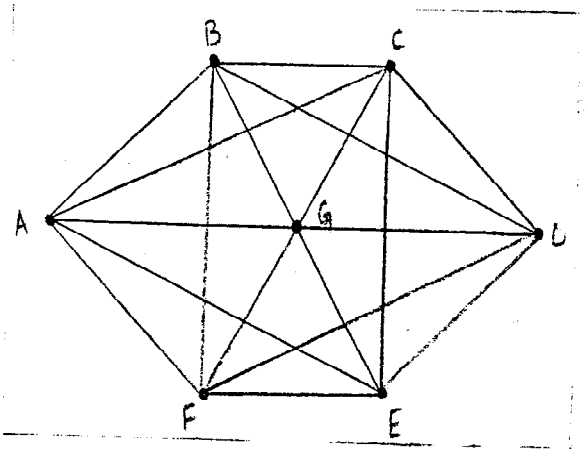


(b) Does there exist a simple graph with five vertices of the degree spectrum 0, 1, 2, 2 3 ? If so, draw such a graph. (3+2)

4. (a) Determine whether the following graph has an Hamiltonian circuit, an Hamiltonian path but no Hamiltonian circuit, or neither. Also draw the path or circuit, if exist.



(b) Is the given graph planar? Justify (3+2)



A

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - II

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH UC222)

QUIZ II - (Closed-Book)

Time: 30 Minutes | Dated: April 11, 2007 | Max. Marks: 10

ID No: | Section No: | Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. Draw a digraph with three vertices of the relation which is transitive and reflexive, but not antisymmetric:

2. Is antisymmetry implies asymmetry? Justify.

3. If in a inhomogeneous recurrence relation the function on $rhs$is $f(n) = Dn^s$ and 1 is root of recurrence relation of multiplicity m, the form of particular solution a_n^p is .....

4. Define a totally ordered set.

5. Answer the following for the poset $(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,3,4\},\{2,3,4\}\},\subseteq)$.

   a. Write the greatest element and maximal elements, if they exist.

   b. Write the upper bounds and least upper bound of the set { { 2 }, { 4 } }, if they exist.

   c. Write the lower bounds and greatest lower bound of the set { {1,3,4 }, {2,3,4} }, if they exist.

6. A circuit in a graph is defined as .....

7. The particular solution for the inhomogeneous recurrence relation is $a_n - 3a_{n-1} + 2a_{n-2} = 5n + 3$ is .....

8. How many vertices will the graph have if it contains 21 edges, 3 vertices of degree 4, and the other vertices of degree 3 .....

B

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - II

DISCRETE STRUCTURES FOR COMPUTER SCIENCE (MATH UC222)

QUIZ I - (Closed-Book)

Time: 30 Minutes | Dated: February 27, 2007 | Max. Marks: 10

ID No: Section No: Name:

Note: (1) Write ID No., Name, Sec. No. and Answer in the space provided. (2) Overwriting will be treated as wrong answer.

1. State the contrapositive of the implications: *We play the game if it is sunny.*

2. Let p and q be the propositions: *p: Your car is out of gas. q: You can't drive your car.* Write the following propositions using p and q and logical connectives: *a) You can't drive your car if it is out of gas. b) Your car is not out of gas if you can drive it.*

3. What rule of inference is used in the following arguments? *If roses are red and violets are blue, then sugar is sweet and so are you. Roses are red and violets are blue. Therefore, sugar is sweet and so are you.*

4. Is the implications $\sim (p \rightarrow q) \rightarrow \sim q$ a tautology? (Yes/No) .....

5. What rule of inference is used in the following arguments? *To go to Tahiti, one must fly or take a boat; there is no seat on any flight to Tahiti this year. Therefore, one must take a boat to go to Tahiti this year.*

6. Let $P(x)$ be the statement "x is happy," where the universe of discourse for x is the set of students. Express each of the following quantifications in English: 1. $\forall\, x \sim P(x)$ 2. $\exists\, x \sim P(x)$

7. If 30 dictionaries in a library contains 61327 pages, then one of them must have at-least ..... pages. (Fill in the blank).

8. Given any set of 7 distinct integers, then there must exists ..... integers in the set whose sum or difference is divisible by 10. (Fill in the blank).

9. Let B(x), E(x) and G(x) be the statements *"x is a book, " "x is expensive, "and "x is good, "* respectively. Express each of the following statements using quantifiers; logical connectives; and *B(x), E(x)* and *G(x)*,where the universe of discourse is the set of all objects. *a. No books are expensive. b. No books are good.*

10. If the product of two integers a and b is even, the either a is even or b is even.

BITS, PILANI - DUBAI CAMPUS

Knowledge Village, DUBAI

II - Year - SEMESTER - II (2007)

DISCRETE STRUCTURE FOR COMPUTER SCIENCE

(MATH UC222)

Test - I (Closed-Book)

Time: 50 Minutes
March 18, 2007

Max. Marks: 20
Weightage: 20 %

Note: 1. Answer all four questions. 2. All questions carries equal marks.

1. (a) For the following arguments, explain which rules of inference are used for each step. The universe is the set of people.
"Someboday in this class enjoys hiking. Every person who enjoys hiking also likes biking. Therefore, there is a person in this class who likes biking."

   (b) The nth term of the Fibonacci series would be $a_n = a_{n-1} + a_{n-2}$ for all n ≥ 3. Prove that $a_n < 2^n$ for all n ≥ 3, using strong mathematical induction.

2. (a) Prove that if n pigeons are assigned m pigeonholes, then one of the pigeonholes must contain at least $[(n-1)/m] + 1$ pigeons.

   (b) Prove the following inference pattern:

$$\begin{aligned} &\sim r \rightarrow (s \rightarrow \sim t) \\ &\sim r \vee w \\ &\sim p \rightarrow s \\ &\sim w \\ \text{Therefore, } &t \rightarrow p \end{aligned}$$

3. (a) Find the coefficient of x^{14} in $(X + X^2 + X^3 + \ldots + X^5)(X^2 + X^3 + X^4 + \ldots)^5$

   (b) Solve the recurrence relation $a_n - 7a_{n-1} + 16a_{n-2} - 12a_{n-3} = 0$ for $n \geq 3$ with initial conditions $a_0 = 1$, $a_1 = 4$, and $a_2 = 8$.

4. (a) Find a general expression for a_n using generating functions $a_n - 9a_{n-1} + 27a_{n-2} - 27a_{n-3} = 0$ for $n \geq 3$.

   (b) Solve the following recurrence relation by method of generating function $a_n + 2a_{n-1} - 4a_{n-2} = 0$; $n \geq 2$ with initial conditions $a_0 = 3$ and $a_1 = -2$

Good Luck . . .