BITS, PILANI --DUBAI CAMPUS
Knowledge Village, Dubai.
II year — Semester-II (2004-2005)
(Computer Science)

COMPREHENSIVE EXAMINATION
(CB)

DISCRETE STRUCTURES FOR COMPUTER SCIENCE
(MATH UC 222)

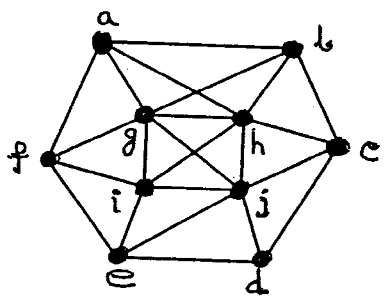
Max. Marks : 80
Weightage : 40%

Time : 03 hours
Date : 22-5-2005

Note: Answer all the questions in order.

Q1 a). Find the maximum number of edges of a complete bipartite graph on n vertices. [6]

b) Define a circuit in a graph.
State with reasons whether the following plane graph is
i) Eulerian ii) Hamiltonian : [5]

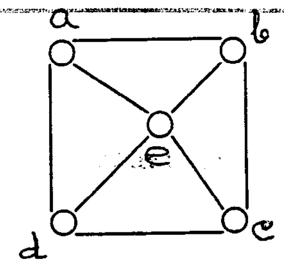


Q2 a) Use the properties of congruence to compute the last digit of 4^{100}. [5]

b) Let A= {2, 3, 4, 9, 12,18} and | is a relation defined on A by $x \mid y$ iff x divides y. Draw the Hasse diagram of the poset [A, |]. Find the maximal elements, minimal elements, if any. [6]

Q3 a) Let R be a relation defined on the set $A = \{2,3,4,5,6\}$ by xRy iff $\gcd(x, y) = 1$ and $x < y$. Draw the digraph of R^{-1}. Find in-degree and out-degree of each vertex of R^{-1}. [5]

Q3. b) Verify Grinberg's theorem for the Hamiltonian cycle $a \to b \to c \to e \to d \to a$ for the following graph : [5]



Q4. a) Let D_{63} denotes the set of all positive divisors of 63. Is the poset [D_{63} ; |] a lattice? Justify your answer. Here | represents the relation, a | b iff a divides b. [4]

b) Solve the following recurrence relation :

$$a_n - 6a_{n-1} + 8a_{n-2} = n.4^n \text{ for } n \geq 2. \text{ Here } a_0 = 8 \text{ and } a_1 = 22.$$ [6]

Q5. a) Prove that $5n^3 - 6n^2 + 4n + 2 \in O(n^3)$. [3]

b) Draw the digraph of the relation $Q = \{(f, g) \mid f : N \to R,\ g : N \to R,\ f \in O(g)\}$ on the functions $\{f_1, f_2, f_3\}$ where $f_1 = 1, f_2 = n, f_3 = \log_2 n$. [7]

Q6. a) Construct the truth table for $\sim(p \vee q) \vee ((\sim p) \wedge q) \vee p \leftrightarrow q$. Is it a contradiction? [4]

b) Use pigeonhole principle to prove that one of any n consecutive integers is divisible by n. [6]

Q7 a) Define a cyclic group. Give one example of cyclic group. [4]

b) Let $(R^+, \cdot)$ denotes the multiplicative group of positive real numbers and $(R,+)$ denotes the additive group of real numbers. Prove that the mapping $f : R^+ \to R$ defined by $f(x) = \log_{10} x$ for all $x \in R^+$, is an isomorphism. [6]

Q8. a) How many generators are there in an infinite cyclic group? [1]

b) State Chinese Remainder theorem. [2]

c) Prove that the square of an odd integer is always of the form $8n + 1$. [5]

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II Year - II Semester (2004-2005)
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TEST – II (Open-book)

Discrete Structures for Computer Science (MATH UC 222)

Marks : 20 **Weightage : 20%**
Time : 50 minutes **Date : 08-5-2005**

*Note : **Answer all questions in the given order.***

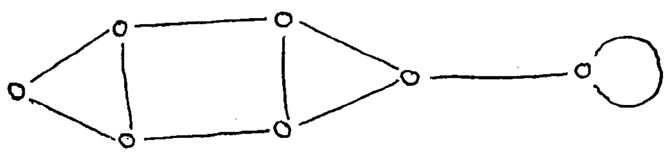
Q1. Let D_6 denotes the set of positive integers which divide 6 and R denotes a relation on D_6 such that $x\ R\ y$ iff x divides y. Find $R \cap R^{-1}$. [2]

Q2. Find the transitive closure of the relation R={(1,1),(1,2),(2,3),(1,3),(3,1),(3,2)} defined on the set $A = \{1, 2, 3\}$. [3]

Q3. Draw all possible nonisomorphic graphs of order 3. State why they are not isomorphic. [3]

Q4. State why the graph with degree sequence {2,2,2,3,5} cannot be a simple graph. [2]

Q5. Draw the dual of the following graph: [3]



Q6. Prove that every planar graph with no. of vertices $\geq$ 3 has at least two vertices of degree ≤ 5. [3]

Q7. Let a graph has the vertices a, b, c and d. Draw the graph if the adjacency matrix for the vertex ordering a, b, c, d is : [2]

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Q8. A complete bipartite graph $K_{m,n}$ has the degree sequence {3,3,3,3,4,4,4}. What are the values of m and n? [2]

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TEST – 1 (CB)

Discrete Structures for Computer Science (MATH UC 222)

Marks : 20
Time : 50 minutes

Weightage : 20%
Date : 20-3-2005

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*Note : **Answer all four questions in the given order.***

Q1. Construct the truth table of the propositional function
$\{[p \vee q \rightarrow r] \wedge (\sim p)\} \rightarrow (q \rightarrow r)$.
Is it a tautology? 5

Q2. a) Use Pigeonhole Principle to prove that if 401 letters were delivered to 50 apartments, then some apartment received at most 8 letters. 2

b) Prove that for each integer $n \geq 1$, the nth Fibonacci number F_n is less than $\left(\frac{7}{4}\right)^n$. 4

Q3. a) Use the method of elimination of cases to establish the following :
" Every odd integer is either of the form $4n+1$ or $4n+3$. 3

b) Solve the following recurrence relation using the characteristic roots :
$a_n - 5a_{n-1} + 6a_{n-2} = 0$ for $n \geq 2$ and $a_0 = 2$, $a_1 = 5$. 3

Q4. Symbolize the following argument using quantifiers and check for its validity :

Lions are dangerous animals.
There are lions.

Therefore, there are dangerous animals. 3

(All steps must be shown with reasons)