

BITS PILANI, DUBAI CAMPUS
Dubai International Academic City, Dubai
I Semester, 2013-2014
MATHEMATICS - III (MATH F211)
Comprehensive Examination

Time: 3 Hours
Date: 02-01- 2014

Max. Marks: 80
Weightage: 40%

NOTE: 1) Answer Part-A and Part-B in two separate answer sheets and write (A) or (B) on the top of the respective answer sheets in CAPITAL BOLD LETTERS.
2) All questions are compulsory

PART-A (40 Marks)

1) Check the exactness and solve

$$\left(2x(1 + \sqrt{x^2 - y})\right) dx - (\sqrt{x^2 - y}) dy = 0 \quad (5 \text{ Marks})$$

2) Solve the following linear differential equation

$$y' + y = \frac{1}{1+e^{2x}} \quad (5 \text{ Marks})$$

3) Solve by the method of undetermined coefficients and find the general solution for

$$y'' + 10y' + 25y = 14e^{-5x} \quad (5 \text{ Marks})$$

4) Solve $y'' + 2y' + y = e^{-x}\log(x)$ and find the general solution by Variation of Parameter method (5 Marks)

5) Find only the particular solution for $y'' + 4y' + 4y = 10x^3e^{-2x}$ (4 Marks)

6) Find the general solution of $(1 - x^2)y'' - xy' + 9y = 0$ using series solution method near the ordinary point $x = 0$ (8 Marks)

7) (i) Find $L^{-1} \left\{ \frac{8}{3p^2+12} + \frac{3}{p^2-49} \right\}$ (3 Marks)

(ii) Solve using Laplace transform $y'' + 2y' + 5y = 3e^{-t}\sin(t)$; $y(0) = 0, y'(0) = 3$ (5 Marks)

PART-B (40 Marks)

1) Find the general solution of the differential equation $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$ near the singular point $x = -1$ using Hypergeometric series. (8 Marks)

2) Find the Legendre series, up to the first three terms, for the following function

$$f(x) = \begin{cases} 0, & -1 \leq x < 0 \\ x, & 0 \leq x \leq 1 \end{cases} \quad (4 \text{ Marks})$$

3) Solve the vibrating string problem (One dimensional wave equation) $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ with the

initial shape of the string given by $f(x) = \begin{cases} x, & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$ (6 Marks)

4) Find the Fourier cosine series of the function $f(x) = \begin{cases} \frac{\pi}{4} - x, & 0 \leq x \leq \frac{\pi}{2} \\ x - \frac{3\pi}{4}, & \frac{\pi}{2} \leq x \leq \pi \end{cases}$ (6 Marks)

5) Find the eigenvalues λ_n and eigenfunctions $y_n(x)$ for the equation $y'' + \lambda y = 0$ when

$$y(0) = 0, y(L) = 0; L > 0 \quad (5 \text{ Marks})$$

6) Prove that $J_{-\frac{1}{2}}(x) = \left(\sqrt{\frac{2}{\pi x}}\right) \cos(x)$ (5 Marks)

7) Find the general solution of the following system:

$$\frac{dx}{dt} = 7x + 6y$$

$$\frac{dy}{dt} = 2x + 6y$$

(6 Marks)

-----BEST OF LUCK-----

BITS PILANI, DUBAI CAMPUS

II Semester (2013-2014)

MATHEMATICS - III (MATH F 211)

Test II (Open book)

Time: 50 Minutes
Date: 11-11- 2013

Max. Marks: 40
Weightage: 20%

Answer all questions

1. Find the general solution for $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$ (8M)

2. Find only the particular solution for $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = x^3$ (8M)

3. Find the general solution for $\frac{d^2y}{dx^2} + y = e^{-2x}\sin(x)$ (8M)

4. Find a power series solution for the following equation at the ordinary point $x=0$ (find solution up to x^6 term)

$$y'' + x^2y' - 2y = 0 \quad (10M)$$

5. a) For the following differential equation locate and classify its singular points on the x -axis:

$$x^2(x+1)^2y'' + (x^2-1)y' + 2y = 0 \quad (3M)$$

b) Find the Indicial equation and Indicial values for the differential equation

$$x^3y'' + (\cos(x) - 1)y' - \sin(x)y = 0 \quad (3M)$$

BITS PILANI, DUBAI CAMPUS
I Semester (2013-2014)
MATHEMATICS - III (MATH F 211)
Test I (Closed Book)

Time: 50 Minutes
Date: 30-09- 2013

Max. Marks: 50
Weightage: 25%

Answer all the questions

1. Solve the following non-exact differential equation by finding a suitable Integrating factor

$$(y \log(y) - 2xy) dx + (x + y) dy = 0 \quad (9 \text{ Marks})$$

2. Solve the following linear differential equation

$$(y - 2xy - x^2) dx + x^2 dy = 0 \quad (7 \text{ Marks})$$

3. Solve $xy' + 2 = x^3(y - 1)y'$ (9 Marks)

4. Use reduction of order technique and solve $x^2y'' + (y')^2 = 0$ (9 Marks)

5. Find the general solution of the differential equation $x^2y'' - x(x + 2)y' + (x + 2)y = 0$, given $y_1(x) = x$ is one of its known solution (9 Marks)

6. Solve $y'' + 4y' + 2y = 0$; $y(0) = -1$, $y'(0) = 2 + 3\sqrt{2}$ (7 Marks)

-----**ALL THE BEST**-----

Date: 12.12.2013
MAXIMUM: 14 MARKS

MATHEMATICS III (MATH F211) QUIZ-II

DURATION: 20 MINUTES

| | | |
|-------------|-----------|--------------------------|
| NAME: _____ | ID: _____ | Instructor's Name: _____ |
|-------------|-----------|--------------------------|

Answer all the questions

1. Find the Eigenvalues of the differential equation $y'' + \lambda y = 0$, given $y(0) = 0, y\left(\frac{\pi}{3}\right) = 0$ (3M)

2. Evaluate $\int_{-1}^1 P_2(x)[1 + P_1(x) + P_2(x) + P_3(x)]dx$ where $P_n(x)$ is n^{th} Legendre Polynomial (3M)

3. Evaluate $[J_4(x) - J_{(-4)}(x)]$ where $J_p(x)$ is Bessel function of order p

(2M)

4. Find the Laplace transform of $f(x) = e^{-x} \sin(2x)$

(3M)

5 Find the inverse Laplace transform of $F(p) = \frac{8}{(p+4)^2+64}$

(3M)

BITS PILANI, DUBAI CAMP
DUBAI INTERNATIONAL ACADEMIC CITY
I SEMESTER 2013 – 2014

B

Date: 12.12.2013
MAXIMUM: 14 MARKS

MATHEMATICS III (MATH F211) QUIZ-II

DURATION: 20 MINUTES

NAME: _____ ID: _____ Instructor's Name: _____

Answer all the questions:

1. Find the Eigenvalues of the differential equation $y'' + \lambda y = 0$, given $y(0) = 0, y\left(\frac{\pi}{4}\right) = 0$ (3M)

2. Evaluate $\int_{-1}^1 P_3(x)[1 + P_1(x) + P_2(x) + P_3(x)]dx$ where $P_n(x)$ is n^{th} Legendre Polynomial (3M)

3. Evaluate $[J_2(x) - J_{(-2)}(x)]$ where $J_p(x)$ is Bessel function of order p

(2M)

4. Find the Laplace transform of $f(x) = e^{2x} \sin(x)$

(3M)

5 Find the inverse Laplace transform of $F(p) = \frac{6}{(p-3)^2+36}$

(3M)