BITS PILANI, DUBAI CAMPUS
Dubai International Academic City, Dubai
First Semester 2013-14
Comprehensive Examination (Closed Book)

No. of Questions: 13
No. of Pages : 3

Course Number & Title: CS F214, Logic in Computer Science **Weightage: 40%**
Duration: 3 Hrs **Date: 30.12.2013** **Class: II year CS** **Marks: 40**

ANSWER ALL QUESTIONS SEQUENTIALLY

1. Prove the validity of the following argument using ND in propositional logic.
"John is coming to the party. If john comes then Yoko will come also. If John and Yoko both come then Paul will not come. Thus Paul will not come." 3 M

2. Identify the equivalences of propositional logic in each of the following cases as directed
   i) $p \rightarrow q \equiv$ ________________ using $\longrightarrow$.
   ii) $p \vee q \equiv$ ________________ using $\longrightarrow$.
   iii) $(p \rightarrow q) \wedge (p \longrightarrow r) \equiv$ ______________
   iv) $\neg (p \longrightarrow q) \equiv$ ______________ using $\wedge$. 2 M

3. Find the CNF of the propositional logic formula $x \vee y \longrightarrow \neg x \wedge z$, using a truth table. 3 M

4. Define tautology with an example. 2 M

5. Prove the validity of the following sequents using ND in predicate logic.
   a) $\forall x\, (P(x) \vee Q(x)),\ \exists x\, (\neg P(x)) \vdash \exists x\, Q(x)$ 5 M
   b) $\forall x\, P(x) \rightarrow S \vdash \exists x\, (P(x) \rightarrow S)$

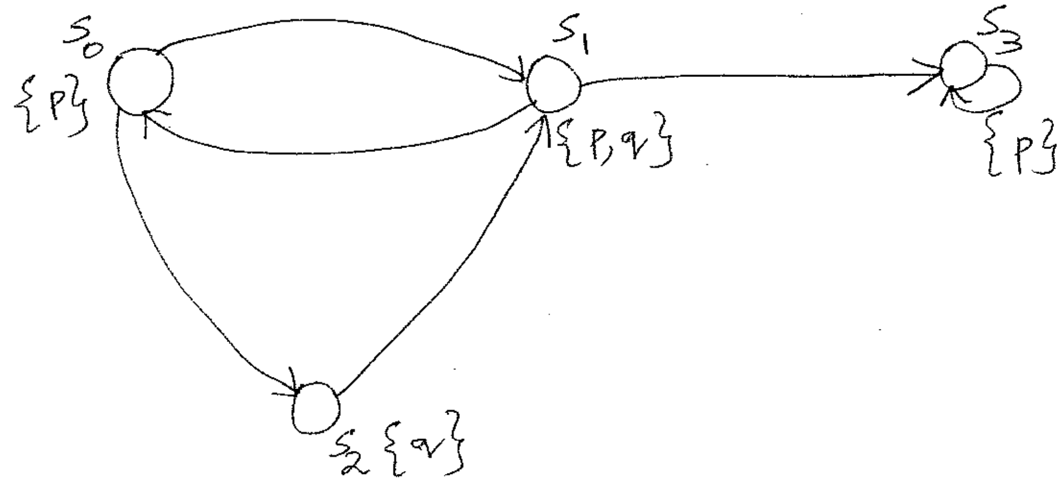
6. Find the truth values of $\propto$ under the following interpretation I. Show the detailed working. 3 M
$\propto : (\forall x)\ (\exists y)\ P(x, y)$
$D = \{1,2\}\quad I[P(1,1)] = F\quad I[P(1,2)] = T\quad I[P(2,1)] = T,\ I[P(2,2)] = F$

7. Given below is a Kripke's structure $\mathcal{M}$ and a set of valid CTL formulas. For each formula identify the state(s) in which it holds and the state(s) in which the formula doesn't hold. 4 M

   a) $EG\ p$

   b) $EF\ (EG\ p)$

   c) $A\ (p\ U\ q)$ d) $E(\ p\ U\ (\neg p\ \wedge A(\neg p\ U\ q)))$



8. Define the temporal connectives of LTL namely U and W with respect to a model M and a path π in M. Also, Express the equivalence between them. 3 M

9. Answer the following with respect to the following program P.

```
a = 0; b = x;
while b >= y do
        b = b-y;
        a = a + 1;
end
```

   a) Show the variable bindings in each iteration assuming x = 10 and y = 3.

   b) Is the program totally correct? Justify your answer.

   c) Write a specification for the program. 3 M

10. Using proof rules show the validity of

a)

$$\vdash_{par} (\!|x > 1|\!)\ a = 1; y = x; y = y - a\ (\!|y > 0 \wedge x > y|\!)$$

b) $\vdash_{tot} (\!| y \geq 0 |\!) \; P \; (\!| z = x * y |\!)$ where the program P is

```
a = 0; z = 0;
while (a != y) {
        z = z + x;
        a = a + 1;
}
```

3 M

11. Using ND prove the validity of the following sequents

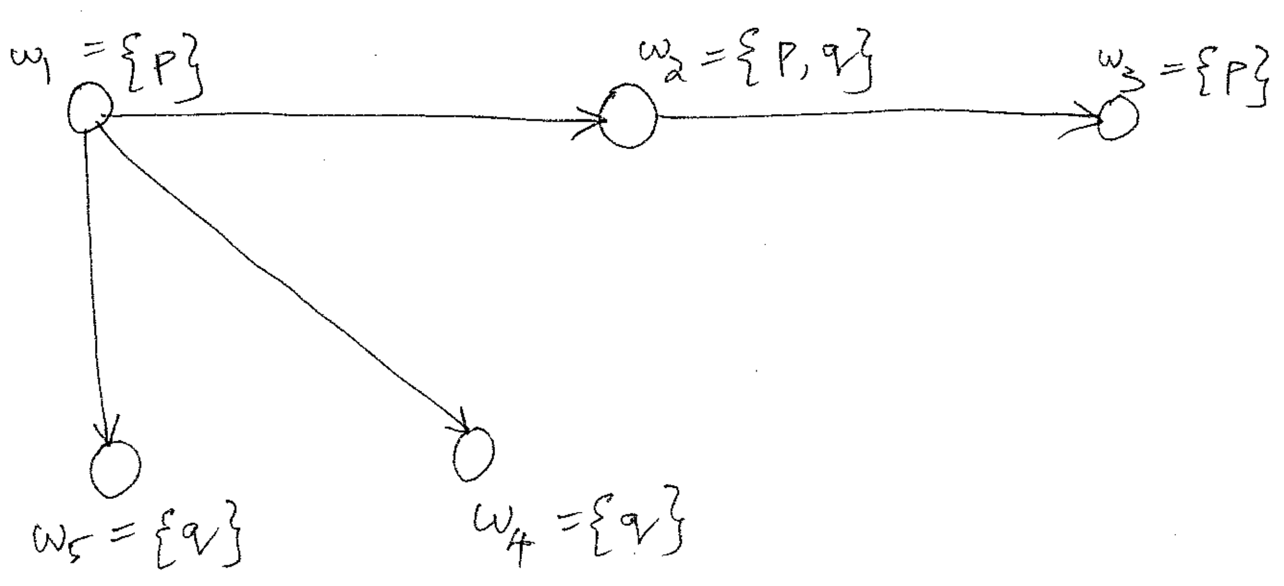
a) $\Box(p \longrightarrow q) \ \vdash \Diamond p \longrightarrow \Diamond q$ using modal logic K

b) $\Diamond(p \longrightarrow q) \ \vdash \Box p \longrightarrow \Diamond q$ using modal logic K 5 M

12. Given the following Kripke's model M, state whether the satisfaction relation hold/ does't hold in each of the following cases.

a) $\mathcal{M}, w_1 \vDash \neg\Box\, p$ b) $\mathcal{M}, w_1 \vDash \neg\Box \neg\, p$ c) $\mathcal{M}, w_1 \vDash \Diamond\, p$

c) $\mathcal{M}, w_1 \vDash \Diamond \Box\, p$ 2 M



13. Write short notes on Prolog. 2 Mark.

****************BEST OF LUCK *****************

BITS PILANI, DUBAI CAMPUS

I Semester13-14

Test 2 – Open Book

Course No. & Title: CS F214, Logic in Computer Science   Date: 21.11.13   Duration: 50 mins

Weightage: 20%   Max Marks: 20   Class: II CS

ANSWER ALL QUESTIONS SEQUENTIALLY

1. Prove that the following equivalence in predicate logic is a tautology.
   $(\neg((\forall x)P(x))) \equiv (\exists x)(\neg P(x))$ 2 M

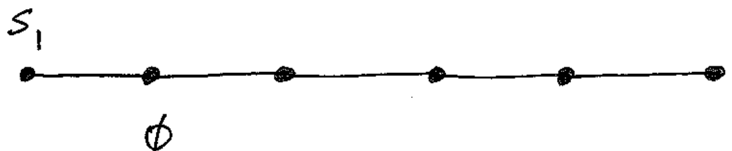
2. Consider a new quantifier ∇ meaning "for no" (Ex: for no x, can x be larger than itself). Express ∇ in terms of the quantifiers (only once) in predicate logic. 1.5 M

3. Express each one of the following using a predicate logic formula. Clearly explain the symbols used in the formula. 2.5 M
   a) All natural numbers that are not even are prime.
   b) Every student who likes a course must like CS F214.
   c) None of John's friend's friend likes him.
   d) There is someone who likes everyone.
   e) Everyone has someone who likes them.

4. Prove the following using natural deduction in predicate logic 7 M
   a) $\forall x\,(P(x) \vee Q(x)) \vdash \forall x\,P(x) \vee \exists x\,Q(x)$
   b) $\forall x\,A(x) \vdash \neg\exists x\,\neg A(x)$
   c) $\exists x\,P(x),\ \forall x \forall y\,(P(x) \rightarrow Q(y)) \vdash \forall y\,Q(y)$

5. With respect to S_1 write an equivalent temporal logic formula for each one of the following:- 1.5 M

a)



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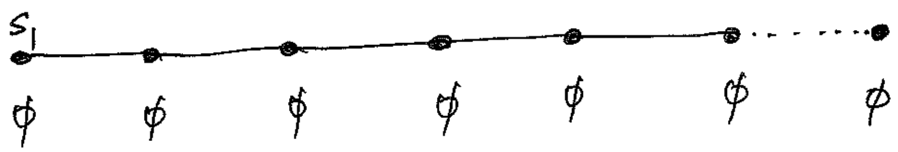

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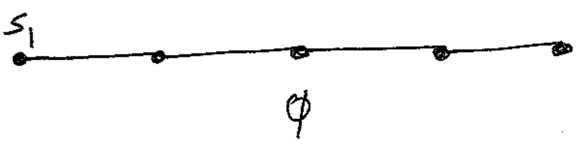

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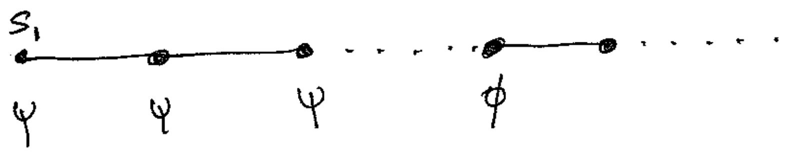

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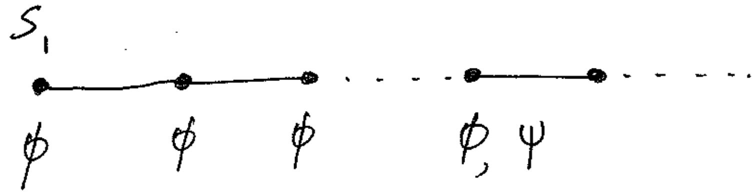

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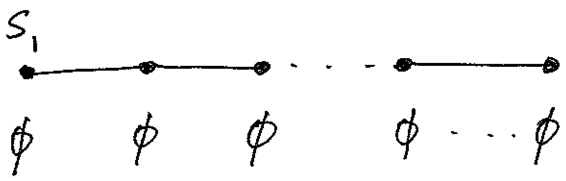

6. Assume F, G and R are not in LTL. Using U, write an LTL formula that is semantically equivalent to each one of the following.
   a) FA b) GA c) $A\ R\ B$ 1.5 M

7. Five philosophers are sitting around a table, taking turns at thinking and eating. Interpret in simple English each one of the following CTL formula assuming the following atomic propositions: 4 M
   e_i = philosopher i is currently eating
   f_i = philosopher i has just finished eating

   a) $AG\ \neg(e_1 \wedge e_4)$
   b) $AG\ (f_4 \rightarrow A(\neg e_4\ W\ e_3)$
   c) $A(\neg\ (e_1 \vee e_3 \vee e_4 \vee e_5\ )\ U\ e_2)$

******************BEST OF LUCK**********************

BITS PILANI, DUBAI CAMPUS
I Semester13-14
Test 1 – Closed Book

Course No. & Title: CS F214, Logic in Computer Science Date: 26.09.13 Duration: 50 mins
Weightage: 25% Max Marks: 25 Class: II CS

ANSWER ALL QUESTIONS SEQUENTIALLY

1. Given the following argument,

If a baby is hungry, then the baby cries. If the baby is not mad, then he does not cry. If a baby is mad, then he has a red face. Therefore, if a baby is hungry, then he has a red face.

Assuming the following propositions,

h : a baby is hungry c : a baby cries m : a baby is mad r : a baby has a red face

a) What is the propositional sequent for this argument?
b) Prove the validity of this argument using natural deduction in propositional logic.

[2 + 3 = 5 Marks]

2. Prove the validity of the following using Natural Deduction

a) $p \wedge q \longrightarrow r,\ q \longrightarrow p,\ q \ \vdash\ r$

b) $p \longrightarrow q,\ q \longrightarrow r\ \vdash p \longrightarrow q \wedge r$

c) $A \vee B,\ A \longrightarrow C,\ \neg D \longrightarrow \neg B\ \vdash C \vee D$ [1 + 2 + 3 = 6 Marks]

3. Using Natural Deduction prove the validity of the following

a) $(A \wedge B) \vee (A \wedge C) \dashv\vdash A \wedge (B \vee C)$

b) $\vdash (p \longrightarrow q) \longrightarrow ((\neg p \longrightarrow q) \longrightarrow q)$ using $p \vee \neg p$ as LEM [3 + 3 = 6 Marks]

4. What is PBC? Derive it using the basic rules. [1 + 2 = 3 Marks]

5. Prove the following using Mathematical induction

For any natural number n , $1^2 + 2^2 + ... + n^2 = n(n + 1)(2n + 1)/6$. [5 Marks]

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BITS PILANI, DUBAI CAMPUS
I Semester13-14
Quiz 1 – Closed Book

A

Course No. & Title: CS F214, Logic in Computer Science Date: 24.10.13 Duration: 20 mins
Weightage: 8% Max Marks: 8 Class: II CS
NAME: **ID NO.**

ANSWER ALL QUESTIONS

1. State whether the following are TRUE or FALSE
   a) $A \rightarrow \bot \equiv \neg A$ Ans:
   b) $\top \longrightarrow A \equiv A$ Ans:
   c) $A \wedge B \equiv \neg(\neg A \vee \neg B)$ Ans:
   d) $A \wedge B \equiv \neg A \longrightarrow B$ Ans: [1 Mark]

2. Consider a formula, $A = (p \longrightarrow q) \wedge (((p \wedge q) \longrightarrow r) \longrightarrow (p \longrightarrow r))$ and the interpretation I = { p=1, q = 0, r = 1}. Write all sub-formulas of this formula in a table, in such a way that every formula is above all of its proper sub-formulas. Then evaluate all sub-formulas and fill up the table given below. [2.5 Marks]
   Ans:

No.	Sub-formula	Value

3. Find the CNF of the formula f, whose truth table is given below. [1 Mark]

P1	P2	P3	f(P1, P2, P3)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

Ans:

4. Prove that the formulas $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$ are not equivalent. Find an interpretation(s) in which they have different values. [1 Mark]

Ans:

5. Find the satisfiability and validity of the formula $(p \wedge \neg q) \longrightarrow (q \vee \neg p)$ [1 Mark]
Ans:

6. Complete these notions between satisfiability and validity. [0.5 Mark]
   a) A formula A is valid iff ¬A is ____________________.
   b) A formula A is satisfiable iff ¬A is ____________________.

7. Draw the parse tree of the PLF in each of the following cases [1 Mark]
   a) Negation of an implication
      Ans:

   b) Conjunction of conjunctions
      Ans:

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