

**BITS PILANI, DUBAI CAMPUS**  
 DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI  
 II YEAR – I SEMESTER 2013-14  
 DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)  
 COMPREHENSIVE EXAMINATION (Closed Book)

Time: 3 Hours

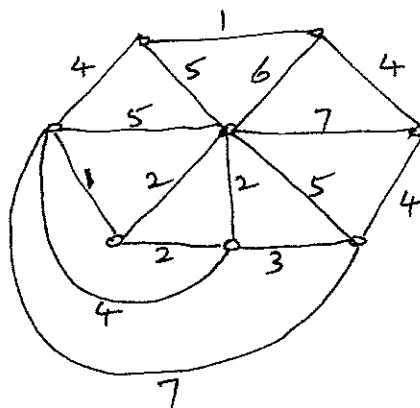
Max. Marks: 40

Date: December 29, 2013

Weightage: 40%

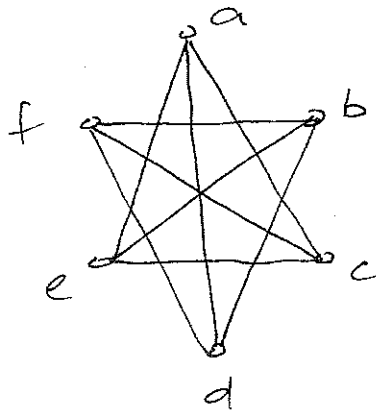
Answer all questions

1. (i) Prove that  $B - A$  is a subset of  $\bar{A}$ . [1]  
 (ii) If set  $A$  has  $k$  elements how many elements will there be in the power set  $\wp(A)$ . [1]
2. Let  $A$  be the set of nonzero rational numbers. For  $a, b \in A$  define  $aRb$  if  $\frac{a}{b}$  is an integer. Verify whether the above relation is reflexive, symmetric and transitive also check whether it is an equivalence relation or not. [3]
3. Draw the Hasse diagram of  $(D_{30}, |)$  and determine all minimal and maximal elements and greatest and least elements if they exist. Specify whether it is a Lattice with proper justification. [3]
4. Prove that for all integers  $n \geq 4$ ,  $3^n > n^3$ . by mathematical induction. [3]
5. Find a generating function for  $a_r =$  the number of ways the sum  $r$  can be obtained when 2 distinguishable dice are tossed. [2]
6. Calculate  $B(x) = \sum_{r=0}^{\infty} b_r x^r = \frac{1}{1-7x+12x^2}$ . [3]
7. Solve the inhomogeneous recurrence relation  $a_n - 2a_{n-1} + a_{n-2} = 2^n$  using the method of undetermined coefficients. [3]
8. The length of the longest simple path between two distinct vertices in a graph  $G$  is called the diameter of  $G$ . The length of the shortest cycle in  $G$  is the girth of  $G$  while the length of the longest cycle is the circumference of  $G$ . Find the diameter, girth and circumference of the graph  $K_5$ . [1]
9. Prove that a simple non directed graph  $G$  is a tree iff  $G$  is connected and contains no cycles. [3]
10. Find a minimal spanning tree of the following graph by Kruskal's algorithm. [2]

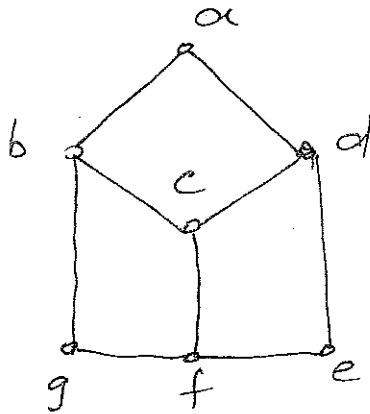


11. A regular binary tree is a binary tree where each vertex has 0 or 2 children. Draw four regular binary trees with exactly 9 vertices. [2]

12. Check whether the following graph is planar. Justify your answer. [2]



13. Check whether the following graph is Eulerian and Hamiltonian. Justify your answer. [2]



14. Is the set of all matrices of the form  $\begin{pmatrix} x & x \\ x & x \end{pmatrix}$  where  $x$  is a nonzero real number under the matrix multiplication a group? Justify. [3]

15. Let  $H$  be a subgroup of  $G$ . Show that the identity element of  $H$  is the same as that of  $G$  and for each  $a \in H$  the inverse of  $a$  in  $H$  is the same as the inverse of  $a$  in  $G$ . [3]

16. Let  $G = \{1, i, -1, -i\}$  be a group under multiplication. Is this group isomorphic to  $(Z_4, \oplus)$ ? Justify your answer. [3]

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DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)  
TEST – II (Open Book)

Time: 50 Minutes

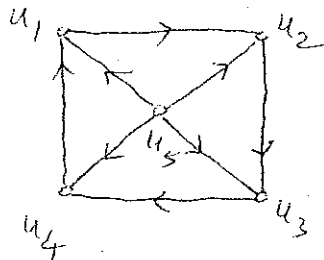
Max. Marks: 20

Date: November 06, 2013

Weightage: 20%

Answer all questions

1. Let  $H$  be a subgroup of  $G$ . The number of distinct left cosets of  $H$  in  $G$  is called the index of  $H$  in  $G$ . Find the index of  $\{1, 6\}$  in  $(Z_7 - \{0\}, \odot)$ . [3]
2. Let  $A$  and  $B$  be two finite subgroups of group  $G$  such that  $|A|$  and  $|B|$  have no common divisors. Then show that  $A \cap B = \{e\}$ . [3]
3. Let  $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}, \text{ where } a \in R^* \right\}$  be a group under matrix multiplication. Is  $G$  isomorphic to  $(R^*, \cdot)$ ? Justify your answer. [4]
4. Verify whether the set of all matrices of the form  $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$  where  $a, b \in R$ , a ring under matrix addition and matrix multiplication. [4]
5. Is the sequence  $(1, 1, 3, 3, 3, 3, 4, 5, 8, 9)$  graphic? Justify your answer. [3]
6. Specify the in-degree and out-degree of each vertex in the following digraph: [3]



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TEST – I (Closed Book)

Time: 50 Minutes

Max. Marks: 25

Date: October 02, 2013

Weightage: 25%

Answer all questions

1. Show that if  $A$ ,  $B$  and  $C$  are three sets then  $A - (B \cup C) = (A - B) \cap (A - C)$ . [3]
2. Compute  $A - B$ ,  $B - A$  and  $(A - B) \cup (B - A)$  if  $A = \{1, 2, 3\}$  and  $B = \{1, 2, 3, 4\}$ . [3]
3. In the set  $S = \mathbb{Z}$ ,  $a R b$  means  $ab$  is odd. Is this relation an equivalence relation? Justify your answer. [3]
4. Let  $R$  be the relation in the natural numbers  $N = \{1, 2, 3, \dots\}$  defined by  $x + 2y = 10$ . Find the domain and range of  $R$ . [3]
5. Consider the function  $f: \mathbb{R} \rightarrow \mathbb{R}^+$  given by  $f(x) = e^x$ . Is this function a bijection? Justify your answer. [4]
6. Let  $G = \{a + b\sqrt{2} / a, b \in \mathbb{Z}\}$ . Is  $G$  a group under usual addition? Justify your answer. [4]
7. Use mathematical induction to prove that  $1^3 + 2^3 + \dots + n^3 = \left[ \frac{n(n+1)}{2} \right]^2$ . [5]

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DISCRETE STRUCTURES FOR COMPUTER SCIENCE (CS F222)  
QUIZ – II (Closed Book)

Time: 20 Minutes

Max. Marks: 7

Date: December 08, 2013

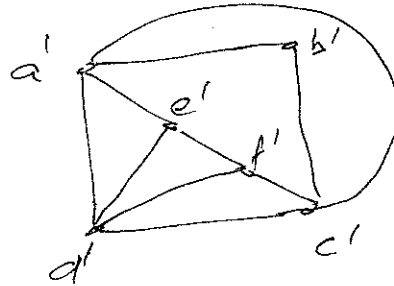
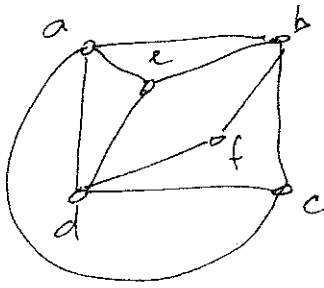
Weightage: 7%

ID.No:

Name:

Answer all questions

1. Check whether the following graphs are isomorphic. If they are isomorphic, construct an isomorphism between them. If they are not isomorphic state a graph-theoretic property satisfied by one and not the other. [2]

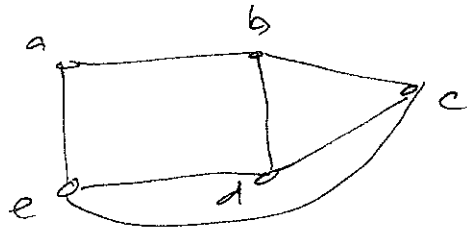


2. Determine the following statements are true or false: [1]
- (a) In a tree every vertex is a cut vertex.
- (b) Any connected  $(p, q)$  graph with  $p+1=q$  is a tree.

3. What is the maximum number of pendant vertices a tree on  $n$  vertices can have? [1]

4. Find the number of vertices and number of edges in the graph  $K_{m,n}$ . [2]

5. Draw two different spanning trees of the following graph. [1]



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QUIZ – I (Closed Book)

Time: 20 Minutes

Max. Marks: 8

Date: October 21, 2013

Weightage: 8%

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ID.No:

Name:

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Answer all questions

1. Prove that in a group identity element is unique. [1]
2. In an abelian group prove that  $(ab)^2 = a^2b^2$ . [2]
3. Is  $\{1, -1\}$  is a subgroup of  $(R^*, \cdot)$ . Justify your answer. [1]
4. Is  $\{2Z, +\}$  a cyclic group. If yes list all the generators. [1]
5. Find the order of  $-1$  and  $3$  in  $(R^*, \cdot)$ . [1]
6. Prove that in a finite group every element is of finite order [2]