BITS PILANI, DUBAI CAMPUS

Dubai International Academic City, Dubai

I Semester (2012-2013)

MATHEMATICS - III (MATH C211/MATH C241)

Comprehensive Examination

Time: 3 Hours | Max. Marks: 80

Date: 10-01-2013 | Weightage: 40%

***NOTE:** 1) Answer Part-A and Part-B in two separate answer sheets and write (A or B) on the top of the respective answer sheets in CAPITAL BOLD LETTERS.*

2) All questions are compulsory and should be answered sequentially.

PART-A (*42 Marks*)

1) Solve the following differential equation by finding an Integrating factor
$(xy - 1)dx + (x^2 - xy)dy = 0$ (5 Marks)

2) Solve the following linear differential equation
$(x\log(x))y' + y = 3x^3$ (5 Marks)

3) If $y_1 = x^2$ is one solution of $x^2y'' + xy' - 4y = 0$ then find y_2 and its general solution (5 Marks)

4) Solve $y'' - y = x^2e^{2x}$ (5 Marks)

5) Solve $y'' + y = \sec(x)\,\mathrm{cosec}(x)$ by Variation of Parameter method (5 Marks)

6) Verify that the origin is a regular singular point and calculate two independent Frobenius series solution for the differential equation $2xy'' + (3 - x)y' - y = 0$ (9 Marks)

7) Find the general solution of the differential equation $x(1 - x)y'' + \left(\frac{1}{2} - 2x\right)y' + 2y = 0$ in hypergeometric form, near the singular point x=0 (4 Marks)

8) Using the Generating function of Legendre polynomials $P_n(x)$ obtain the recursion formula
$(n + 1)P_{n+1}(x) = (2n + 1)x\,P_n(x) - n\,P_{n-1}(x)$ (4 Marks)

PART-B (*38 Marks*)

1) Prove that $J_{\left(\frac{1}{2}\right)}(x) = \left(\sqrt{\frac{2}{\pi x}}\right)\sin(x)$ (4 Marks)

2) Find the eigenvalues λ_n and eigenfunctions $y_n(x)$ for the equation $y'' + \lambda y = 0$ with

$y(0) = 0, y(2\pi) = 0$ (4 Marks)

3)Find the general solution of the following system:

$$\frac{dx}{dt} = 4x - 2y$$

$$\frac{dy}{dt} = 5x + 2y$$

(5 Marks)

4) Solve the following differential equation by the method of Laplace transforms:

$y'' + 5y' + 6y = 5e^{3t} \qquad y(0) = y'(0) = 0.$ (5 Marks)

5) Find the Fourier series of the function $f(x) = \begin{cases} 1 + x, & -1 \le x \le 0 \\ 1 - x, & 0 \le x \le 1 \end{cases}$ (4 Marks)

6) i) Find $L\{e^{3x}\cos(2x) + x^2 e^{-3x} + 5\}$ (4 Marks)

ii) Find $L^{-1}\left\{\frac{s+3}{s^2+2s+5}\right\}$ (4 Marks)

7) Solve the one dimensional heat equation $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ subject to the conditions

(i) $u(0,t) = 0$, (ii) $u(\pi,t) = 0$, (iii) $u(x,0) = \pi x(\pi - x)$; $0 < x < \pi$ (8 Marks)

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I Semester (2012-2013)

MATHEMATICS - III (MATH F 211/MATH C241)

Test II (Open Book)

Time: 50 Minutes | Max. Marks: 40

Date: 20-12- 2012 | Weightage: 20%

a) All questions are compulsory and should be answered sequentially.
b) Only prescribed text book and hand written class notes are allowed.

1. For the following equation, verify that origin is a regular singular point and calculate two independent Frobenius series solutions.

$$x^2y'' + (x+x^2)y' + (x-9)y = 0$$ **(12 Marks)**

2) Solve the following system of linear first order differential equations

$$\frac{dx}{dt} = 7x - y$$

$$\frac{dy}{dt} = 2x + 5y$$ **(10 Marks)**

3) a) Find the Laplace transform of $f(x) = 10x^{-\left(\frac{1}{2}\right)}$ **(3 Marks)**

b) Using Convolution theorem, find the inverse Laplace transform of

$$\frac{s}{(s+3)(s^2+4)}$$ **(5 Marks)**

4) Using Laplace transform, Solve

$$y''+4\,y' + 4\,y = e^{-2t}\ ;\ y(0) = 0,\ y'(0) = 0$$ **(10 Marks)**

ALL THE BEST!

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I Semester (2012-2013)

MATHEMATICS - III (MATH F 211)

Test I (Closed Book)

Time: 50 Minutes | Max. Marks: 50

Date: 04-11- 2012 | Weightage: 25%

All questions are compulsory and should be answered sequentially.

1. Find a particular solution to the equation $\frac{d^2y}{dx^2} + y = cot^2(x)$ by the method of variation of parameter (8 Marks)

2. a) Find the general solution for the differential equation $y^{(4)} + 2y''' + 2y'' + 2y' + y = 0$ (4 Marks)

b) Find the general solution for the differential equation $\frac{d^2y}{dx^2} + 10\frac{dy}{dx} + 25y = 14e^{-5x}$ (4 Marks)

3. a) Find a particular solution for the differential equation $y''' + y'' = 9x^2 - 2x + 1$ (5 Marks)

b) Find a particular solution for the differential equation $y'' - 7y' + 12y = e^{2x}(x^3 - 5x^2)$ (5 Marks)

4. Find the power series solution for the following differential equation by finding an appropriate recurrence equation.

$$y'' + xy' + y = 0$$ (12 Marks)

5. Find the general solution of the following differential equation, valid for $|x| < 1$, by finding an appropriate recurrence equation by power series method,

$$(1 - x^2)y'' - xy' + p^2y = 0$$ (Where p is a constant) (12 Marks)

B

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I SEMESTER 2012 – 2013

MATHEMATICS III (MATH F211/MATH C241) - QUIZ-II

MAXIMUM: 14 MARKS
Weightage : 7%

Date: 06.12.2012
DURATION: 20 MINUTES

NAME: ______________________	ID: ______________________
SEC: ________	Instructor's Name: ______________________

Answer all the questions

1. Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$ where $J_p(x)$ represents Bessel function. (4M)

2. Prove that $P_n(1) = 1$ where $P_n(x)$ represents n-th Legendre polynomials (3M)

3. Prove that $T_3(x) = 4x^3 - 3x$ where $T_n(x) = \cos(n\theta)$ and $x = \cos(\theta)$ (3M)

4. Verify $x\,F(1,1,2,-x) = \log(1+x)$ where $F(a,b,c,x)$ represent hypergeometric series. (4M)

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I SEMESTER 2012 – 2013

A

MATHEMATICS III (MATH F211/MATH C241) - QUIZ-II

MAXIMUM: 14 MARKS
Weightage : 7%

Date: 06.12.2012
DURATION: 20 MINUTES

NAME: ______________________ ID: ______________________

SEC: __________ Instructor's Name: ______________________

Answer all the questions

1. Verify $\mathbf{x\,F(1,1,2,-x) = \log(1+x)}$ where $\mathbf{F(a,b,c,x)}$ represent hypergeometric series. (4M)

2. Prove that $\mathbf{T_3(x) = 4x^3 - 3x}$ where $\mathbf{T_n(x) = \cos(n\theta)}$ **and** $\mathbf{x = \cos(\theta)}$ (3M)

3. Prove that $\mathbf{P_n(1) = 1}$ where $\mathbf{P_n(x)}$ represents n-th Legendre polynomials (3M)

4. Prove that $\frac{\mathbf{d}}{\mathbf{dx}}\left[\mathbf{x^p J_p(x)}\right] = \mathbf{x^p J_{p-1}(x)}$ where $\mathbf{J_p(x)}$ represents Bessel function. (4M)

A

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FIRST SEMESTER 2012 – 2013

MATHEMATICS III (MATH F211)
QUIZ-I

MAXIMUM: 16 Marks DURATION: 20 Minutes Date: 10.10.2012

NAME: ______________________ ID: ______________________

Answer all the questions:

1. Solve the differential equation by converting it in to an exact differential $\frac{x\,dx}{(x^2+y^2)^{\frac{3}{2}}} + \frac{y\,dy}{(x^2+y^2)^{\frac{3}{2}}} = 0$ (3M)

2. Find an Integrating factor which converts the following non-exact differential equation into an exact equation
$(x^3 + xy^3)dx + 3y^2dy = 0$ (3M)

3. Reduce the given Bernoulli's equation into a linear differential equation by using suitable substitution (do not solve the equation) $xy^2 \frac{dy}{dx} + y^3 = x \cos(x)$ (3M)

4. Find the particular solution of $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ with $y_1 = e^x$, $y_2 = e^{-2x}$ and with the initial conditions $y(0) = 8, y'(0) = 2$ and check whether they are linearly independent. (3M)

5. Find general solution of the differential equation $x\frac{d^2y}{dx^2} - (2x+1)\frac{dy}{dx} + (x+1)y = 0$, if one solution is $y_1 = e^x$ (4M)

B

BITS PILANI, DUBAI CAMPUS
DUBAI INTERNATIONAL ACADEMIC CITY, DUBAI
FIRST SEMESTER 2012 – 2013

MATHEMATICS III (MATH F211)
QUIZ-I

MAXIMUM: 16 Marks DURATION: 20 Minutes Date: 10.10.2012

NAME: ______________________	ID: ______________

Answer all the questions:

1. Solve the differential equation by converting it in to an exact differential $\frac{x\,dx}{(x^2+y^2)^{\frac{1}{2}}} + \frac{y\,dy}{(x^2+y^2)^{\frac{1}{2}}} = 0$ (3M)

Find an Integrating factor which converts the following non-exact differential equation into an exact equation
$(xy - 1)dx + (x^2 - xy)dy = 0$ (3M)

3. Reduce the given Bernoulli's equation into a linear differential equation by using suitable substitution (do not solve the equation) $x\frac{dy}{dx} + y = x^4y^3$ (3M)

4. Find the particular solution of $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ with $y_1 = e^{-2x}$, $y_2 = e^{-3x}$ and with the initial conditions $\mathbf{y(0) = 1, y'(0) = 1}$ and check whether they are linearly independent. (3M)

5. Find general solution of the differential equation $x^2\frac{d^2y}{dx^2} - x(x+2)\frac{dy}{dx} + (x+2)y = 0$, if one solution is $y_1 = x$ (4M)