BITS PILANI, DUBAI CAMPUS
Dubai International Academic City, Dubai
First Semester 2012-13
Comprehensive Examination(Closed Book)

No. of Questions:	17
No. of Pages	: 3

Course Number & Title: CS F214, Logic in Computer Science **Weightage: 40%**
Duration: 3 Hrs **Date: 06.01.2013** **Class: II year CS** **Marks: 40**

Answer All Questions Sequentially

1. Prove the validity of the following sequent using natural deduction in propositional logic. $\vdash ((p \to q) \to q) \to ((q \to p) \to p)$. 2.5 Marks

2. Draw the parse tree of the following propositional logic formula $\neg((\neg q \wedge (p \to r)) \wedge (r \to q))$ 1 Mark

3. Prove the validity of the following using natural deduction in propositional logic $(p \wedge q) \vee (p \wedge r) \dashv\vdash p \wedge (q \vee r)$ 2 Marks

4. Define a Horn formula. Find the CNF of $r \to (s \to (t \wedge s \to r))$ using the algorithm. 0.5 + 2 Marks

5. What is course of values induction? Prove using mathematical induction, for all n≥1,
$$1 + 4 + 7 + \cdots \ldots + (3n - 2) = \frac{n(3n-1)}{2}$$ 2 Marks

6. Prove the validity of the following argument using natural deduction in predicate logic.
John, a student in this class is 16 years old. Everyone who is 16 years old can get a driver's license. Therefore someone in this class can get a driver's license.
Assume the following predicates C(x) for "x is a student in this class", S(x) for "x is 16 years old" and D(x) for "x can get a driver's license". 2 Marks

7. Prove that the statements $\neg\exists x\, \forall y\, P(x,y)$ and $\forall x\, \exists y\, \neg P(x,y)$ have the same truth value. 2 Marks

8. Prove the validity of the following predicate logic sequent using natural deduction

$$\forall x\, (P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall x Q(x)$$ 2 Marks

9. Prove the validity of the following predicate logic sequent using natural deduction

$$\exists x \exists y \big(H(x,y) \vee H(y,x)\big), \neg\exists x H(x,x) \vdash \exists x \exists y \neg(x = y)$$ 4 Marks

10. A Kripke's structure $M = (S, \rightarrow, L)$ in CTL has $S = \{s_0, s_1, s_2, s_3\}$, $L(s_0) = \{p\}, L(s_1) = \{p, q\}, L(s_2) = \{q\}, L(s_3) = \{p\}$. The transition relations as $\{s_0 \rightarrow s_1, s_1 \rightarrow s_0, s_0 \rightarrow s_2, s_2 \rightarrow s_1, s_1 \rightarrow s_3, s_3 \rightarrow s_3\}$ .Draw the complete graph. For each of the following CTL formulas identify the states in which it holds in this model. 2 Marks

$AX\, p,\ EX\, p\,,\ A(p\ U\ q)$.

11. With respect to a LTL model M, and a path π in the model, define the temporal connectives U and R. Write the equivalence between them.

3 Marks

12. Given the following program

```
a = x;
y = 0;
while(a != 0) {
   y = y + 1;
   a = a – 1;
}
```

a) Show the variable bindings in each iteration with x = 6.
b) What is the loop invariant?
c) What does the program do? Give a specification for the program.
d) Give a proof for the loop invariant. 4 x 1 = 4 Marks

13. Prove the validity of the following partial correctness sequent
$\vdash_{par} (\!\top\!)\ y = x; y = x + x + y; (\!y = 3.x\!)$ 2 Marks

14. Prove the validity of the following modal logic sequent using natural deduction
$\vdash_K \Box(p \longrightarrow q) \wedge \Box(q \longrightarrow r) \longrightarrow \Box(p \longrightarrow r)$ 2.5 Marks

15. Find natural deduction rules for the following in modal logic KT45
$\Box \Diamond p \longleftrightarrow \Diamond p.$ 2 Marks

16. A Kripke's model in modal logic is defined with $W = \{u, v, w, t\}$. $L(u) = \{p\}$, $L(v) = \{r\}, L(w) = \{p\}, L(t) = \{p, q\}$. The accessibility relations are defined as $u \longrightarrow u$ $w \longrightarrow v, t$ $t \longrightarrow v, w$. With a neat graph draw the model and state whether each of the following modal logic formulas are valid/invalid in that model. 2.5 Marks
$M, u \vDash \Box p$, $M, t \vDash \neg\Box p$, $M, w \vDash \Diamond \Box \bot$ and $M, v \vDash \Box \bot$.

17. Write short notes on Prolog. 2 Mark.

*****************BEST OF LUCK ******************

BITS PILANI, DUBAI CAMPUS
Dubai International Academic City, Dubai
First Semester 2012-13
Test – 2(Open Book)

No. of Questions:	5
No. of Pages	: 2

Course Number & Title: CS F214, Logic in Computer Science **Weightage: 20%**
Duration: 50 minutes **Date: 09-12-2012** **Class: II year CS** **Marks: 20**
ONLY PRESCRIBED TEXT BOOK AND HANDWRITTEN CLASS NOTES ARE ALLOWED

Answer All Questions

1. Write the propositional sequent corresponding to the following predicate sequent. Also prove its validity.

   $\exists x \neg \emptyset \vdash \neg \forall x \, \emptyset.$ 4 M

2. Prove the validity of $\exists x(A \rightarrow B(x)) \vdash A \rightarrow \exists x \, B(x)$, where A has arity 0.

   3.5 M

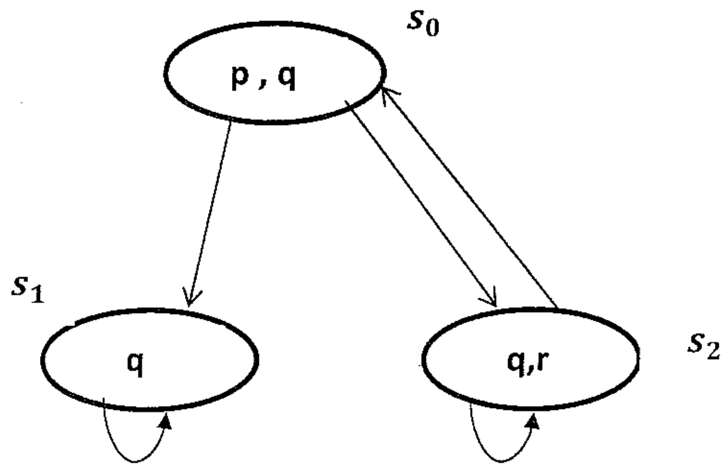
3. Assuming the words highlighted as atomic descriptions, express each of the following English sentences using an equivalent formula in CTL.
   a) Whenever a system gets into a state'**s**', it will sometimes be able to get out of it.
   b) The system may be in both **waiting** and **busy** state, but not both at the same time.
   c) If it ever **rains** it won't keep raining forever.
   d) An elevator should keep its doors **open**, until there is a **request** to use it.
   e) It should be able to get **lays-chips** and **fivestar** from the vending machine, but not both at the same time. 5 x 0.5 = 2.5 M

4. Does the following LTL equivalence hold? If so, prove it.

   $F \emptyset \equiv \top \, U \, \emptyset$ 4 M

5. For a model shown in the figure below, mention whether each of the following relations hold? In each case show the path/paths where the relation holds/does not hold. 6 x 1 = 6 M
   a) $M, s_0 \vDash AGq$

b) $M, s_0 \models AG(q \wedge p)$
c) $M, s_2 \models EG(q \wedge r)$
d) $M, s_0 \models AXr$
e) $M, s_0 \models E(p \, U \, q)$
f) $M, s_0 \models AF \, p$



********************** BEST OF LUCK************************

BITS PILANI, DUBAI CAMPUS
Dubai International Academic City, Dubai
First Semester 2012-13
Test – 1(Closed Book)

No. of Questions:	7
No. of Pages	: 1

Course Number & Title: CS F214, Logic in Computer Science — **Weightage: 25%**
Duration: 50 minutes — **Date: 15-10-2012** — **Class: II year CS** — **Marks: 25**

Answer All Questions

1. Consider the following argument:-
   If you use Linux and Mozilla as a browser, you avoid problems. In contrast, if you use internet explorer, you will have problems. Now you use Mozilla, but also Internet explorer sometimes. Therefore, you don't use Linux.

   a) What is the propositional logic sequent corresponding to the argument?
   b) Is the argument valid? 2 + 3 = 5 M

2. Prove the validity of the following sequents 3 + 4 + 2 = 9 M
   a) $\neg p \vee \neg q \vdash \neg(p \wedge q)$
   b) $(p \rightarrow q) \vee (q \rightarrow r)$ using $q \vee \neg q$ as LEM
   c) $p \wedge (q \vee r) \vdash (p \wedge q) \vee (p \wedge r)$

3. Prove that "It is raining" is a logical consequence of "If it is hot, then it will rain. It is hot today" both syntactically and semantically. 2 M

4. Define tautology. Is $(p \vee q) \rightarrow (p \wedge q)$ a tautology? 2 M

5. State the soundness and completeness theorem of propositional logic. 2 M

6. Draw the parse tree of the wff given : 2 M
   $\neg (s \rightarrow (\neg (p \rightarrow (q \vee \neg s))))$

7. Prove by mathematical induction, for any natural number n,
   $$2 + 4 + 6 + 8 + \ldots\ldots + 2n = n(n+1)$$
   3 M

*****************BEST OF LUCK ******************

BITS PILANI, DUBAI CAMPUS
Dubai International Academic City, Dubai
First Semester 2012-13
Quiz – 2(Closed Book)

No. of Questions: 5
No. of Pages : 3

R

Course Number & Title: CS F214, Logic in Computer Science **Weightage: 7%**
Duration: 20 minutes **Date: 21-11-2012** **Class: II year CS** **Marks: 7**

Name **ID No:**

Answer All Questions

1. Let $\emptyset$ be the formula $\forall x\ \forall y\ \exists z\ (R(x,y) \rightarrow R(y,z))$ where R is a binary predicate.
2 x 0.5 = 1Mark

a) For a model *M*, Let $A^M \stackrel{\text{def}}{=} \{a,b,c,d\}$, and $R^M \stackrel{\text{def}}{=} \{\ (b,c),(b,b),(b,a)\}$. Does $M \vDash \emptyset$ hold? Justify your answer in one line.
Ans:

b) For a model *M*, Let $A^M \stackrel{\text{def}}{=} \{a,b,c\}$, what should be R^M if $M \vDash \emptyset$ hold?
Ans:

2. Let S(x) be the predicate "x is a student", B(x) the predicate "x is a book", and H(x,y) the predicate "x has y". The universe of discourse is the set of all objects. Use quantifiers to express each of the following statements. 5 x 0.5 = 2.5 Marks

a) Every student has a book
Ans:

b) Some student does not have any book

Ans:

c) Some student has all the books

Ans:

d) Not every student has a book

Ans:

e) There is a book which every student has.

Ans:

3. Draw the parse tree corresponding to the predicate logic formula. Identify free and bound variables.

$\forall x\, ((\exists x\, (P(x) \rightarrow Q(x))) \wedge S(x, y))$ 1.5 Marks

Ans:

4. Let m be a constant, f a function with arity one, S and B be binary predicates. Identify the following strings as valid or invalid formulas in predicate logic. If invalid, justify in one line. 6 x 0.25 = 1.5 Marks

a) $S(m,x)$ **Ans:**

b) $B(m,f(m))$ **Ans:**

c) $f(m)$ **Ans:**

d) $B(B(m,x),y)$ **Ans:**

e) $(B(x,y) \rightarrow (\exists z\, S(z,y)))$ **Ans:**

f) $(B(x) \rightarrow B(B(x)))$ **Ans:**

5. Which natural deduction rules are used to prove the validity of the sequent $\forall \boldsymbol{x}\, \emptyset \vdash \exists \boldsymbol{x}\, \emptyset$ 0.5 Mark

Ans:

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First Semester 2012-13
Quiz – 1(Closed Book)

No. of Questions: 7
No. of Pages : 3

B

Course Number & Title: CS F214, Logic in Computer Science **Weightage: 8%**
Duration: 20 minutes **Date: 31-10-2012** **Class: II year CS** **Marks: 8**

Name **ID No:**

Answer All Questions

1. State the Lemma that is used to define the validity of a disjunction of literals
   Ans: 1 M

2. Construct a CNF for ϕ based on the following truth table: 1 M

r	s	q	ϕ
T	T	T	**F**
T	T	F	**T**
T	F	T	**F**
F	T	T	**F**
T	F	F	**T**
F	T	F	**F**
F	F	T	**F**
F	F	F	**T**

Ans:

3. Using quantifiers complete the following : 0.5 x 2 = 1 M
   a) $\neg \exists x\, P(x) \equiv$ ____________
   b) $\neg \forall x\, P(x) \equiv$ ____________

4. A formula **A** is satisfiable iff ¬**A** is ________________. 0.5 M

5. Suppose variables x and y denote people, L(x, y) is a binary predicate for x likes y. Translate the following English sentences to predicate logic

   a) There is somebody whom everybody likes. 0.5 x 4 = 2 M
      Ans:

   b) There is somebody who Ann doesn't like.
      Ans:

   c) There is somebody who no one likes.
      Ans:

   d) Everybody likes somebody.
      Ans:

6. If the predicate P(x, y) represent x < y. For any value of x and y, what is the truth value of the following. Reason in one sentence.
   a) $\forall x\, \exists y\, P(x, y)$ 0. 5 x 2 = 1 M

Ans:

b) $\exists y\, \forall x\, P(x, y)$
Ans:

7. State whether the following are true or false, where A and B are propositional logic formulae 0.5 x 3 = 1.5 M

a) $\boldsymbol{A} \rightarrow \bot \equiv \neg \boldsymbol{A}$
Ans:

b) $(\boldsymbol{A} \wedge \neg \boldsymbol{B}) \rightarrow (\boldsymbol{B} \vee \neg \boldsymbol{A}) \equiv \neg \boldsymbol{A} \vee \boldsymbol{B}$
Ans:

c) $\boldsymbol{A} \wedge \boldsymbol{B} \equiv \neg \boldsymbol{A} \vee \neg \boldsymbol{B}$
Ans:

*****************BEST OF LUCK ******************